Comments on "Generating sensor signals in isotropic noise fields"

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Abstract

In this report provides comments on Algorithm 1 as described in E. A. P. Habets and S. Gannot, Generating sensor signals in isotropic noise fields, J. Acoust. Soc. Am., vol. 122, no. 6, pp. 3464–3470, Dec. 2007. Specifically, it has recently come to our attention that the source positions generated and used by Algorithm 1 are not uniformly distributed on a unit sphere. Nevertheless, it has been shown in the original manuscript that for the tested array configuration and sufficient number of sources the desired spatial coherence can be obtained. In order to provide a more accurate and possibly more efficient solution, two alternative methods to compute the position of the sources are investigated in this report.

1 Uniformly distributed points over $\mathcal{S}^2$

As mentioned in [1] it is paramount that the positions of the mutually independent noise sources are uniformly distributed over $\mathcal{S}^2 = \{\mathbf{p} \in \mathbb{R}^3 : \|\mathbf{p}\|_2 = 1\}$. Algorithm 1 in [1] uses $N = N_\phi \, N_\theta$ points where the values for θ are equally spaced between 0 and $(N_\theta - 1)/N_\phi$ and the values for ϕ are equally spaced over $\sin(\phi)$ with values between 0 and π. The resulting points (ϕ, θ) are not uniformly distributed on a unit sphere and hence might not always yield the desired spatial coherence corresponding to an isotropically spherical noise field. Nevertheless, it has been shown in [1] that accurate results can be achieved when $N_\phi \geq 96$ and $N_\theta \geq 32$.

The distribution of points on a unit sphere is a subject area which has many applications and which gives rise to a number of problems, many of which are unsolved or are hard in the sense of computational complexity. Various stochastic methods have been developed to generate random points on the sphere that are uniformly distributed. When such a stochastic method is used a sufficiently large number of points is required to achieve accurate and consistent results. Alternatively, a geometric method can be used to compute the location of N sources using a specific algorithm.

To uniformly distribute N points on a unit sphere is a challenging problem by itself. While uniformly distribution a number of points such as, 4, 6, 8, 12 and 20 over $\mathcal{S}^2$ is rather straightforward, the distribution of an arbitrary number is a challenging problem. In the following two subsection we subsequently describe a stochastic method and a geometric method to uniformly distribute N points on a unit sphere.

Algorithm 1: Creating sensor signals for an arbitrary 3D array that result from a spherical noise field. The source positions are computed using a stochastic method.

```
Data: $\mathbf{P}$, $M$, $L$, $N$, $f_s$
Result: $\mathbf{z}$
$L' = 2^{\lceil \log_2(L) \rceil}$;
$\omega = \pi f_s\,[0 : 1/L' : 1]$;
$u = -1 + 2\,\mathrm{rand}(N, 1)$;
$\theta = 2\pi\,\mathrm{rand}(N, 1)$;
for $k = 1 : N$ do
    $\mathbf{X}' = \mathrm{randn}(1, L'/2 + 1) + i\,\mathrm{randn}(1, L'/2 + 1)$;
    $\mathbf{X}(1, :) = \mathbf{X}(1, :) + \mathbf{X}'$;
    $\mathbf{v} = \begin{bmatrix} \sqrt{1 - u^2(k)}\,\cos(\theta(k)) \\ \sqrt{1 - u^2(k)}\,\sin(\theta(k)) \\ u(k) \end{bmatrix}$;
    for $m = 2 : M$ do
        $\Delta = \mathbf{v}^T \mathbf{P}(:, m)$;
        $\mathbf{X}(m, :) = \mathbf{X}(m, :) + \mathbf{X}' \exp(-j\Delta\omega/c)$;
    end
end
$\mathbf{X} = \Big[\sqrt{L'}\,\mathrm{Re}\{\mathbf{X}(:, 1)\}, \sqrt{L'/2}\,\mathbf{X}(:, 2 : L'/2),$
    $\sqrt{L'}\,\mathrm{Re}\{\mathbf{X}(:, L'/2 + 1), \sqrt{L'/2}\,\mathrm{conj}(\mathbf{X}(:, L'/2 : -1 : 2))\Big]$;
$\mathbf{z} = \mathrm{ifft}(\mathbf{X}/\sqrt{N}, L', 2)$;
$\mathbf{z} = \mathbf{z}(:, 1 : L)$;
```

1.1 Stochastic method

One way to generate random points that are uniformly distributed over $\mathcal{S}^2$ is as follows. Let $u = \cos(\phi)$ denote an independent and identically distributed (i.i.d.) sample taken from a uniform distribution $u \in [-1, 1]$ and θ an i.i.d. sample taken from the uniform distribution $\theta \in [0, 2\pi)$. Now the x, y and z coordinate of the point is given by

$$\begin{aligned} x &= \sqrt{1 - u^2}\,\cos(\theta), \\ y &= \sqrt{1 - u^2}\,\sin(\theta), \\ z &= u. \end{aligned}$$

When sufficient points are generated they will be uniformly distributed over $\mathcal{S}^2$. Employing the above method results in Algorithm 1.

1.2 Geometric method

Alternatively, we can generate N uniformly distributed points using a geometric method. A thorough but understandable survey appears in [2]. Here we employ the spiral point method which has been proposed by Rakhmanov et al. [3] and is motivated by hexagonal tilings and numerical experimentation. Using spherical coordinates (ϕ, θ), $0 \leq \phi \leq \pi$, $0 \leq \theta \leq 2\pi$, we set

$$\phi(k) = \arccos\left(h(k)\right), \quad \text{for} \quad 1 \leq k \leq N, \tag{1}$$

with $h(k) = -1 + 2(k - 1)/(N - 1)$. We then define $\theta(k) = 0$ for $k \in \{1, N\}$ and compute the remaining values $\theta(k)$ using

$$\theta(k) = \quad \mathrm{mod}\left(\theta(k-1) + \frac{3.6}{\sqrt{N(1 - h(k)^2)}}, 2\pi\right), \quad \text{for} \quad 1 < k < N. \tag{2}$$

Algorithm 2: Creating sensor signals for an arbitrary 3D array that result from a spherical noise field. The source positions are computed using the spiral method [3].

```
Data: $\mathbf{P}$, $M$, $L$, $N$, $f_s$
Result: $\mathbf{z}$
$L' = 2^{\lceil \log_2(L) \rceil}$;
$\omega = \pi f_s \, [0 : 1/L' : 1]$;
for $k = 1 : N$ do
    $h = -1 + 2(k-1)/(N-1)$;
    $\phi(k) = \arccos(h)$;
    if $k = 1$ or $k = N$ then
        $\theta(k) = 0$;
    else
        $\theta(k) = \mod\left(\theta(k-1) + \frac{3.6}{\sqrt{N(1-h^2)}}, 2\pi\right)$;
    end
end
for $k = 1 : N$ do
    $\mathbf{X}' = \mathrm{randn}(1, L'/2+1) + i\,\mathrm{randn}(1, L'/2+1)$;
    $\mathbf{X}(1,:) = \mathbf{X}(1,:) + \mathbf{X}'$;
    $\mathbf{v} = \begin{bmatrix} \cos(\theta(k))\sin(\phi(k)) \\ \sin(\theta(k))\sin(\phi(k)) \\ \cos(\phi(k)) \end{bmatrix}$;
    for $m = 2 : M$ do
        $\Delta = \mathbf{v}^T \mathbf{P}(:,m)$;
        $\mathbf{X}(m,:) = \mathbf{X}(m,:) + \mathbf{X}' \exp(-j\Delta\omega/c)$;
    end
end
$\mathbf{X} = \Big[\sqrt{L'}\,\mathrm{Re}\{\mathbf{X}(:,1)\}, \sqrt{L'/2}\,\mathbf{X}(:,2:L'/2),$
    $\sqrt{L'}\,\mathrm{Re}\{\mathbf{X}(:,L'/2+1), \sqrt{L'/2}\,\mathrm{conj}(\mathbf{X}(:,L'/2:-1:2))\Big]$;
$\mathbf{z} = \mathrm{ifft}(\mathbf{X}/\sqrt{N}, L', 2)$;
$\mathbf{z} = \mathbf{z}(:,1:L)$;
```

Employing this method results in Algorithm 2. Compared to Algorithm 1 in [1] only the values of θ have been changed.

2 Evaluation

Similar to the experiment conducted in [1] we compute the mean square error (MSE) between the measured and theoretical spatial coherence function. For both methods we conduct a Monte Carlo simulation with 25 trials. In Fig. 1 (solid line) the mean and standard deviation of the MSE for different values of N are shown. As expected, both the mean and the standard deviation (denoted by the error bars) are decreasing monotonically when N increases. Similar to the findings in [1] we see that the MSE reaches an asymptotic value of approximately -30 dB. In Fig. 1 (dashed line) the results are shown for the geometric method. Both in term of the mean and standard deviation of the MSE the geometric method outperformances the stochastic method. While Algorithm 1 in [1] requires $N \geq 160 \cdot 32 = 5120$ points to achieve an average MSE of -30 dB, the spiral method achieves around the same MSE value for $N \geq 256$. Hence, by ensuring that the points are near-uniformly distributed we require less noise sources

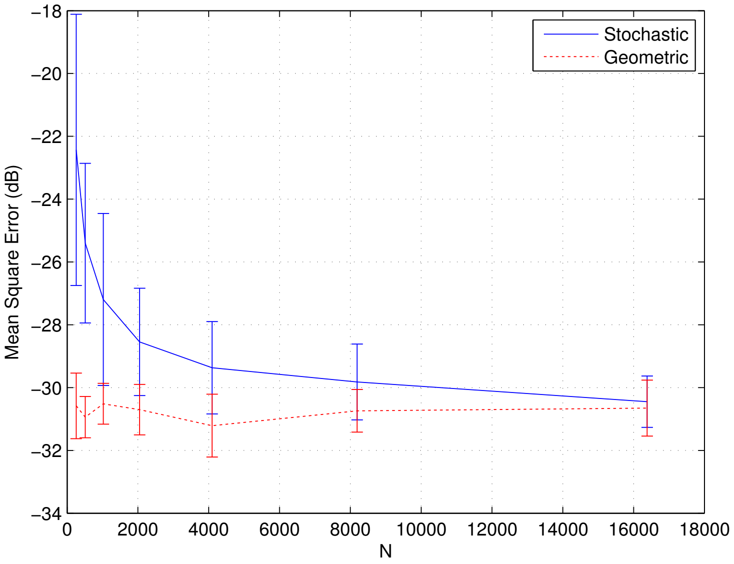

Figure 1: Mean square error in dB between the measured and theoretical spatial coherence for $N = 2^n$, with $n = 8, 9, \ldots, 14$.

and hence reduce the computational complexity of the generator.

3 Conclusion

In this report we employed two alternative methods to generate N uniformly distributed points on a unit sphere and studied the influence of these methods on the accuracy of our generator. For the considered array configuration (as described in [1]), Algorithm 2 achieves an average MSE smaller than −30 dB when using $N \geq 256$ and is therefore preferred over the method used in [1]. Finally, the MALAB function *sinf_3D.m* [4] is now based on Algorithm 2.

References

[1] E. A. P. Habets and S. Gannot, “Generating sensor signals in isotropic noise fields,” *J. Acoust. Soc. Am.*, vol. 122, no. 6, pp. 3464–3470, Dec. 2007.

[2] E. Saff and A. Kuijlaars, “Distributing many points on a sphere,” *Mathematical Intelligencer*, vol. 19.1, pp. 5–11, 1997.

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[4] E. A. P. Habets and S. Gannot. (2010, Sep.) MATLAB implementation for: Generating sensor signals in isotropic noise fields. [Online]. Available: http://home.tiscali.nl/ehabets/ginfs.html