Lecture

Music Processing

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Dynamic Time Warping (DTW)

Problem 1

Let $\mathcal{F} = \{\alpha, \beta, \gamma\}$ be a feature space. Furthermore, let $c : \mathcal{F} \times \mathcal{F} \to \mathbb{R}_{\geq 0}$ be local cost measure defined by

$$c(x, y) := 1 - \delta_{xy} = \begin{cases} 0 & \text{falls } x = y, \\ 1 & \text{falls } x \neq y, \end{cases}$$

for $x, y \in \mathcal{F}$. Specify the DTW distances $\mathrm{DTW}(X, Y)$, $\mathrm{DTW}(X, Z)$ und $\mathrm{DTW}(Y, Z)$ for the following sequences:

$$X = \alpha\beta\gamma\alpha\beta\gamma, \quad Y = \gamma\alpha\beta, \quad Z = \alpha\alpha\gamma\alpha.$$

Problem 2

Let $F = \mathbb{R}$ be a feature space and $c : \mathcal{F} \times \mathcal{F} \to \mathbb{R}_{\geq 0}$ the local cost measure defined by $c(x, y) = |x - y|$, $x, y \in \mathbb{R}$. Furthermore, let $X = (1, 7, 4, 4, 6)$ and $Y = (1, 2, 2, 7)$. Execute the algorithm by hand as described in the lecture for computing $\mathrm{DTW}(X, Y)$. In particular, specify the cost matrix C and the accumulated cost matrix D. Furthermore, specify an optimal warping path. Is there another optimal warping path?

Problem 3

Let $\mathcal{F}$ be a feature space and $c : \mathcal{F} \times \mathcal{F} \to \mathbb{R}_{\geq 0}$ be a symmetric local cost measure, i. e., $c(x, y) = c(y, x)$ for all $x, y \in \mathcal{F}$. Show that in this case the DTW distance is symmetric as well. Furthermore, show that the DTW distance generally does not satisfy the triangle inequality.

Problem 4

In this problem, the multiscale approach for DTW (MsDTW) is to be analyzed in more detail. Let $X = (x_1, x_2, \ldots, x_N)$ and $Y = (y_1, y_2, \ldots, y_M)$ be sequences of length N and M. For simplicity, we assume that $N = M = 2^n$ for a natural number $n \in \mathbb{N}$. Let $A^{\mathrm{DTW}}(N) = N^2$ denote the number of evaluations of the local cost measure that are required in the classical DTW algorithm.

In the following, we assume that the MsDTW-approach is performed recursively with $f_1 = f_2 = \ldots = f_n = 2$. Let $A^{\mathrm{MsDTW}}(N)$ denote the number of evaluations of the local cost measure that are required in the MsDTW algorithm. Specify a possibly small upper bound for $A^{\mathrm{MsDTW}}(N)$. What can be said about the memory requirements?

(Note: In the analysis, the operations required for computing the coarsened sequences $X_1, X_2, \ldots, X_n$ und $Y_1, Y_2, \ldots, Y_n$ are left unconsidered.)