

Lecture  
Music Processing

## Tempo and Beat Tracking

**Meinard Müller**  
International Audio Laboratories Erlangen  
meinard.mueller@audiolabs-erlangen.de

## Introduction

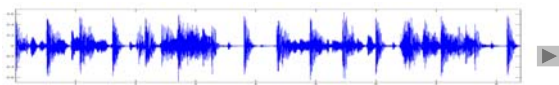
### Basic beat tracking task:

Given an audio recording of a piece of music,  
determine the periodic sequence of beat positions.

“Tapping the foot when listening to music”

## Introduction

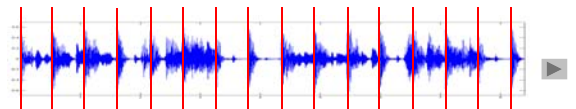
Example: Queen – Another One Bites The Dust



Time (seconds)

## Introduction

Example: Queen – Another One Bites The Dust



Time (seconds)



## Introduction

Example: Happy Birthday to you

Pulse level: **Measure**

Musical score for 'Happy Birthday to you' in 3/4 time. The top staff has lyrics: 'Hap - py Birth - day to you, Hap - py Birth - day to you, Hap - py'. The bottom staff has lyrics: 'Birth - day dear \_\_\_\_\_, Hap - py Birth - day to you!'. Red arrows point to the beginning of each measure in the top staff.

## Introduction

Example: Happy Birthday to you

Pulse level: **Tactus (beat)**

The same musical score as in the previous slide, but with red arrows pointing to every individual beat in the top staff.

## Introduction

Example: Happy Birthday to you

Pulse level: **Tatum (temporal atom)**



The image shows a musical score for 'Happy Birthday to you' in 3/4 time. Red arrows point to every note head in both staves, representing the 'tatum' or 'temporal atom' pulse level. The lyrics are: 'Hap - py Birth - day to you, Hap - py Birth - day to you, Hap - py Birth - day dear \_\_\_\_\_, Hap - py Birth - day to you!'.

## Introduction

Example: Chopin – Mazurka Op. 68-3

Pulse level: Quarter note

Tempo: ??? ▶

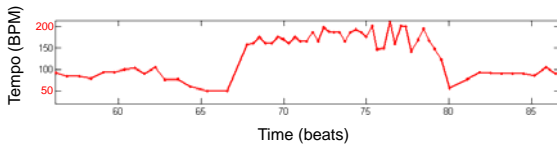
## Introduction

Example: Chopin – Mazurka Op. 68-3

Pulse level: Quarter note

Tempo: **50-200 BPM** ▶

Tempo curve



## Introduction

Example: Borodin – String Quartet No. 2

Pulse level: Quarter note

Tempo: 120-140 BPM (roughly)

Beat tracker without any prior knowledge ▶

Beat tracker with prior knowledge on rough tempo range ▶

## Introduction

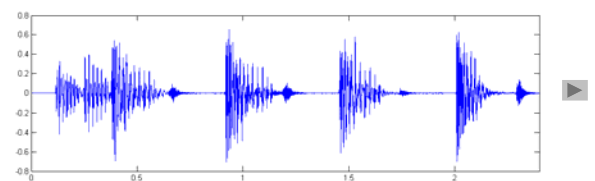
### Challenges in beat tracking

- Pulse level often unclear
- Local/sudden tempo changes (e.g. rubato)
- Vague information (e.g., soft onsets, extracted onsets corrupt)
- Sparse information (often only note onsets are used)

## Introduction

Tasks

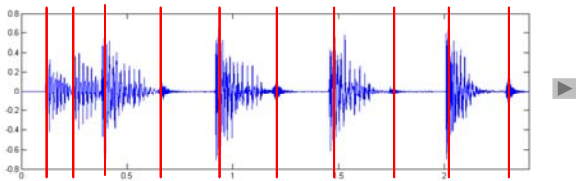
- Onset detection
- Beat tracking
- Tempo estimation



## Introduction

### Tasks

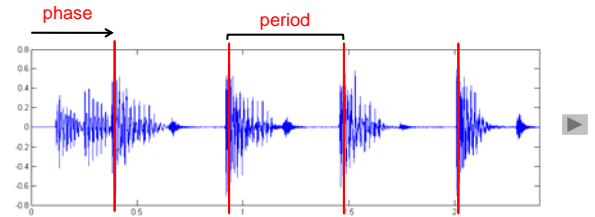
- Onset detection
- Beat tracking
- Tempo estimation



## Introduction

### Tasks

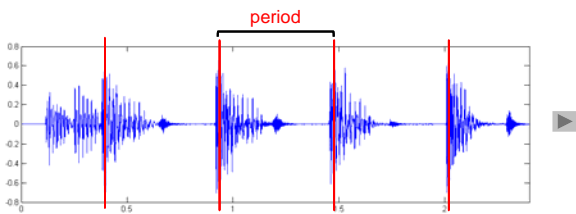
- Onset detection
- Beat tracking
- Tempo estimation



## Introduction

### Tasks

- Onset detection
  - Beat tracking
  - Tempo estimation
- Tempo := 60 / period  
Beats per minute (BPM)

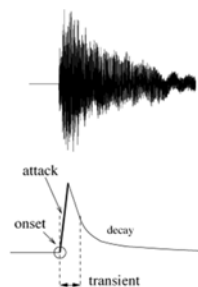


## Onset Detection

- Finding start times of perceptually relevant acoustic events in music signal
- Onset is the time position where a note is played
- Onset typically goes along with a change of the signal's properties:
  - energy or loudness
  - pitch or harmony
  - timbre

## Onset Detection

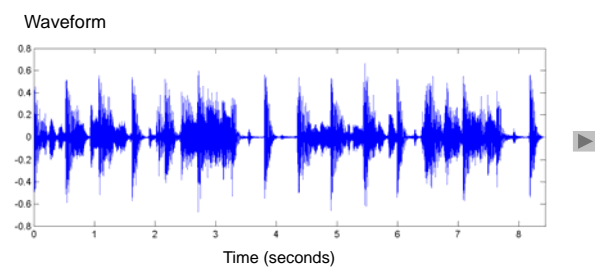
- Finding start times of perceptually relevant acoustic events in music signal
- Onset is the time position where a note is played
- Onset typically goes along with a change of the signal's properties:
  - energy or loudness
  - pitch or harmony
  - timbre



[Bello et al., IEEE-TASLP 2005]

## Onset Detection (Energy-Based)

### Steps

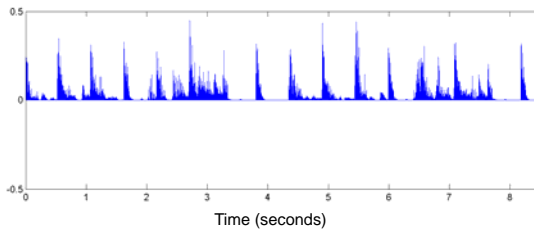


## Onset Detection (Energy-Based)

### Steps

1. Amplitude squaring

Squared waveform

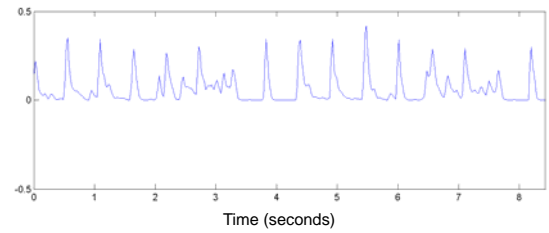


## Onset Detection (Energy-Based)

### Steps

1. Amplitude squaring
2. Windowing

Energy envelope

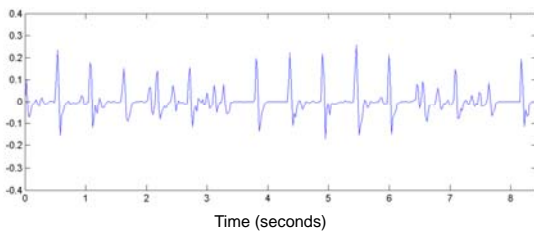


## Onset Detection (Energy-Based)

### Steps

1. Amplitude squaring
2. Windowing
3. Differentiation Capturing energy changes

Differentiated energy envelope

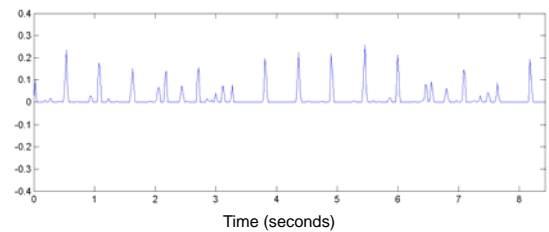


## Onset Detection (Energy-Based)

### Steps

1. Amplitude squaring
2. Windowing
3. Differentiation Only energy increases are relevant for note onsets
4. Half wave rectification Only energy increases are relevant for note onsets

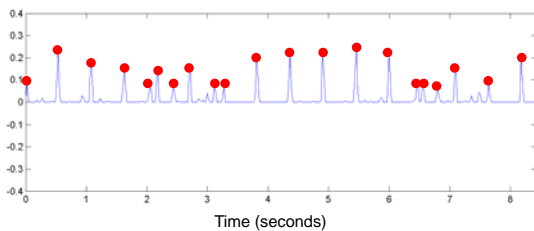
Novelty curve



## Onset Detection (Energy-Based)

### Steps

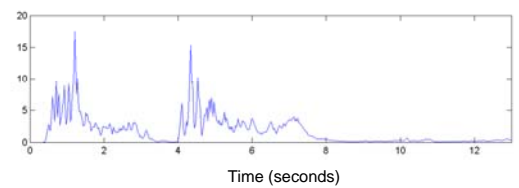
1. Amplitude squaring
2. Windowing
3. Differentiation Peak positions indicate note onset candidates
4. Half wave rectification
5. Peak picking



## Onset Detection (Energy-Based)



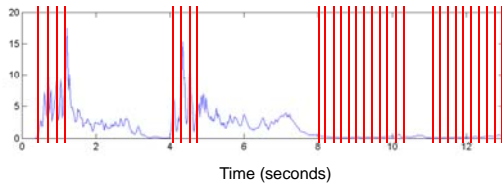
Energy envelope



## Onset Detection (Energy-Based)



Energy envelope / note onsets positions

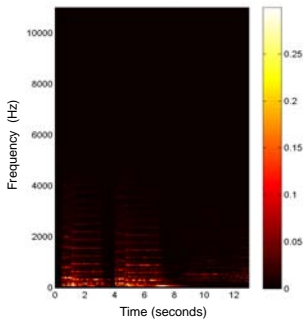


## Onset Detection

- Energy curves often only work for percussive music
- Many instruments such as strings have weak note onsets
- No energy increase may be observable in complex sound mixtures
- More refined methods needed that capture
  - changes of spectral content
  - changes of pitch
  - changes of harmony

## Onset Detection (Spectral-Based)

Magnitude spectrogram  $|X|$



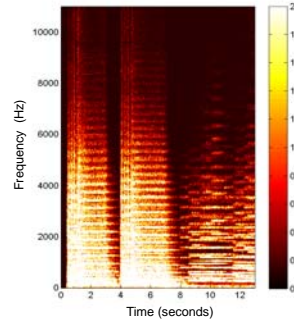
### Steps:

1. Spectrogram

- Aspects concerning pitch, harmony, or timbre are captured by spectrogram
- Allows for detecting local energy changes in certain frequency ranges

## Onset Detection (Spectral-Based)

Compressed spectrogram  $Y$



### Steps:

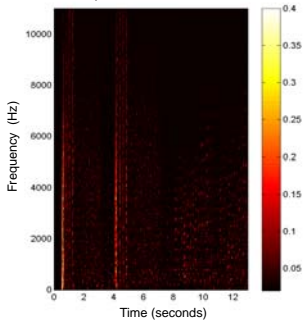
1. Spectrogram
2. Logarithmic compression

$$Y = \log(1 + C \cdot |X|)$$

- Accounts for the logarithmic sensation of sound intensity
- Dynamic range compression
- Enhancement of low-intensity values
- Often leading to enhancement of high-frequency spectrum

## Onset Detection (Spectral-Based)

Spectral difference



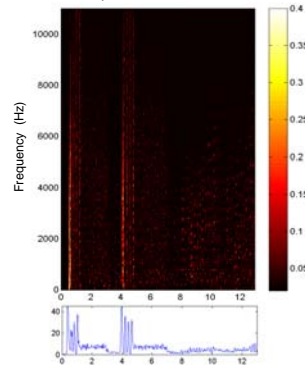
### Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation

- First-order temporal difference
- Captures changes of the spectral content
- Only positive intensity changes considered

## Onset Detection (Spectral-Based)

Spectral difference



### Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation

- Frame-wise accumulation of all positive intensity changes
- Encodes changes of the spectral content

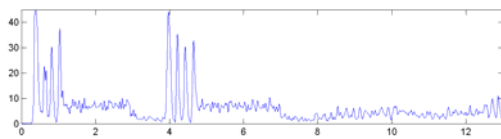
Novelty curve

## Onset Detection (Spectral-Based)

### Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation

Novelty curve



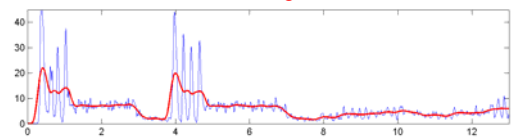
## Onset Detection (Spectral-Based)

### Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation
5. Normalization

Novelty curve

Substraction of local average

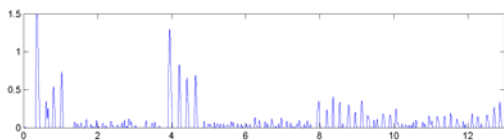


## Onset Detection (Spectral-Based)

### Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation
5. Normalization

Normalized novelty curve

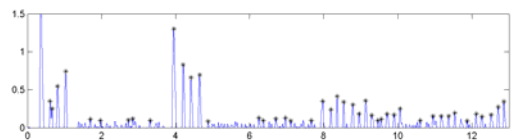


## Onset Detection (Spectral-Based)

### Steps:

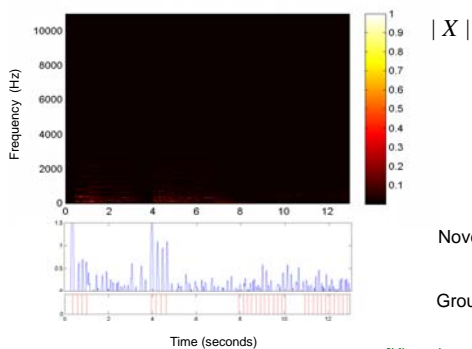
1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation
5. Normalization
6. Peak picking

Normalized novelty curve



## Onset Detection (Spectral-Based)

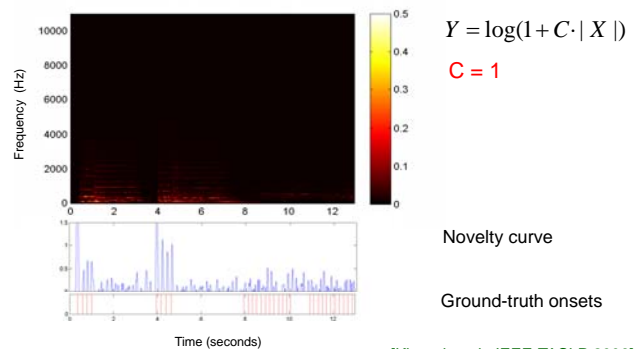
Logarithmic compression is essential



[Klapuri et al., IEEE-TASLP 2006]

## Onset Detection (Spectral-Based)

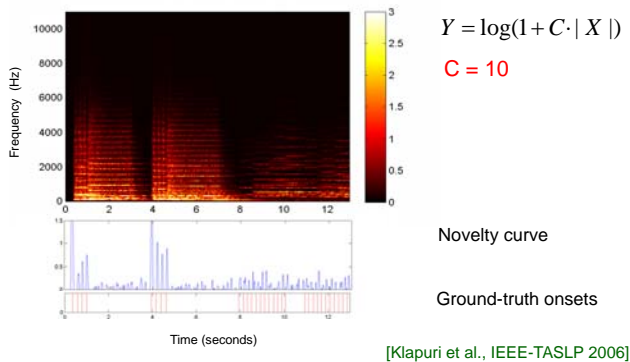
Logarithmic compression is essential



[Klapuri et al., IEEE-TASLP 2006]

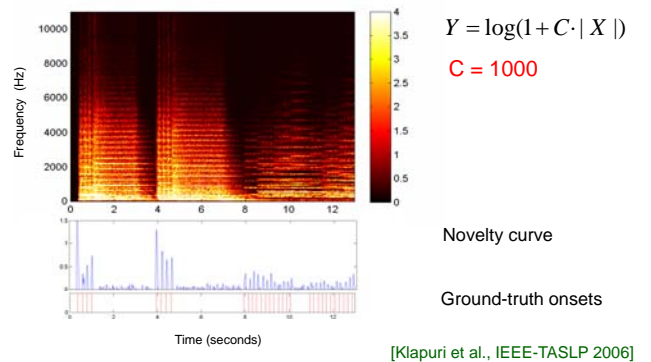
## Onset Detection (Spectral-Based)

Logarithmic compression is essential



## Onset Detection (Spectral-Based)

Logarithmic compression is essential



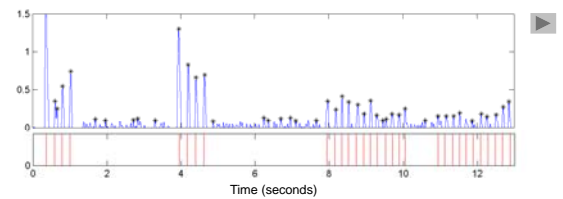
## Onset Detection (Spectral-Based)

- Spectrogram  $X = (X(t, k))_{t, k}$   $t \in [1 : T]$   
 $k \in [1 : K]$
- Compressed Spectrogram  $Y := \log(1 + C \cdot |X|)$   $C > 1$ .
- Novelty curve  $\Delta : [1 : T - 1] \rightarrow \mathbb{R}$

$$\Delta(t) := \sum_{k=1}^K |Y(t+1, k) - Y(t, k)|_{\geq 0}$$

## Onset Detection

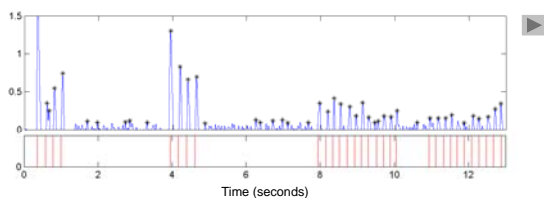
Peak picking



- Peaks of the novelty curve indicate note onset candidates

## Onset Detection

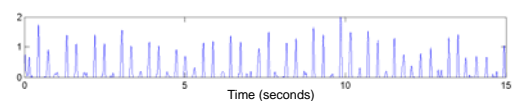
Peak picking



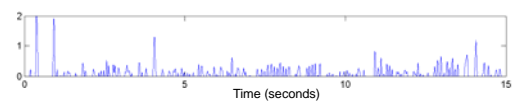
- Peaks of the novelty curve indicate note onset candidates
- In general many spurious peaks
- Usage of local thresholding techniques
- Peak-picking very fragile step in particular for soft onsets

## Onset Detection

Shostakovich – 2<sup>nd</sup> Waltz



Borodin – String Quartet No. 2



## Onset Detection

- Drumbeat ▶
- Going Home ▶
- Lyphard melodie ▶
- Por una cabeza ▶
- Donau ▶

## Beat and Tempo

### What is a beat?

- Steady pulse that drives music forward and provides the temporal framework of a piece of music [Parncutt 1994]  
[Sethares 2007]  
[Large/Palmer 2002]
- Sequence of perceived pulses that are equally spaced in time [Lerdah/ Jackendoff 1983]
- The pulse a human taps along when listening to the music [Fitch/ Rosenfeld 2007]

The term **tempo** then refers to the speed of the pulse.

## Beat and Tempo

### Strategy

- Analyze the novelty curve with respect to reoccurring or quasi-periodic patterns
- Avoid the explicit determination of note onsets (no peak picking)

## Beat and Tempo

### Strategy

- Analyze the novelty curve with respect to reoccurring or quasi-periodic patterns
- Avoid the explicit determination of note onsets (no peak picking)

[Scheirer, JASA 1998]

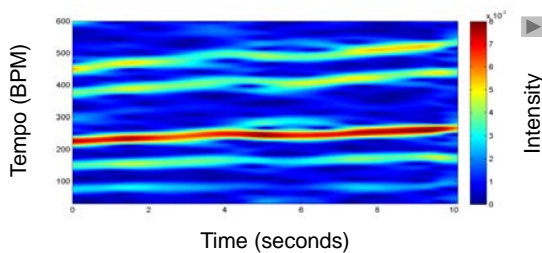
[Ellis, JNMR 2007]

### Methods

- Comb-filter methods [Davies/Plumbley, IEEE-TASLP 2007]
- Autocorrelation [Peeters, JASP 2007]
- Fourier transform [Grosche/Müller, ISMIR 2009]  
[Grosche/Müller, IEEE-TASLP 2011]

## Tempogram

Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.



## Tempogram (Fourier)

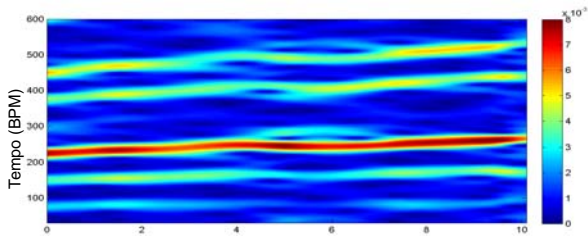
Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.

### Fourier-based method

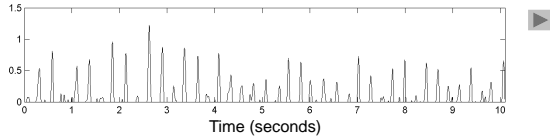
- Compute a spectrogram (STFT) of the novelty curve
- Convert frequency axis (given in Hertz) into tempo axis (given in BPM)
- Magnitude spectrogram indicates local tempo



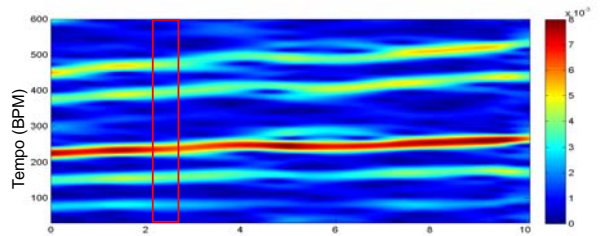
### Tempogram (Fourier)



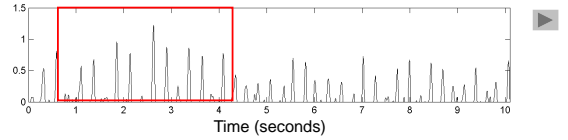
#### Novelty curve



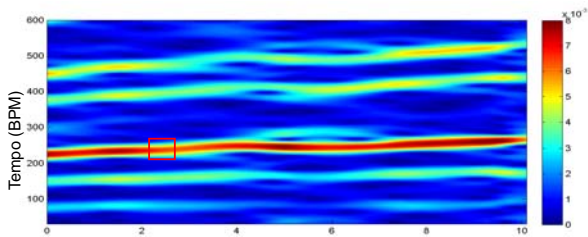
### Tempogram (Fourier)



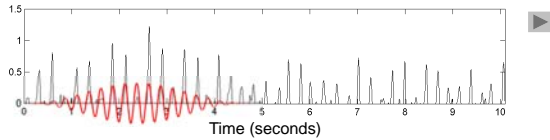
#### Novelty curve (local section)



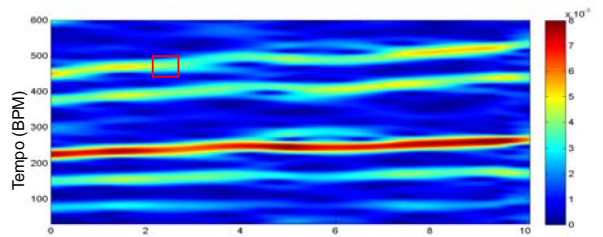
### Tempogram (Fourier)



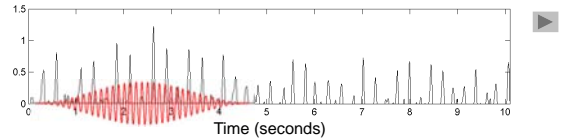
#### Windowed sinusoidal



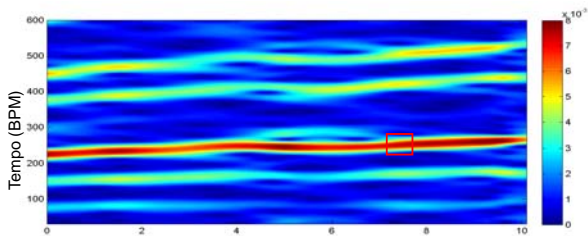
### Tempogram (Fourier)



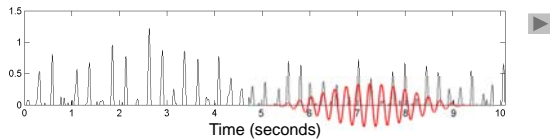
#### Windowed sinusoidal



### Tempogram (Fourier)



#### Windowed sinusoidal



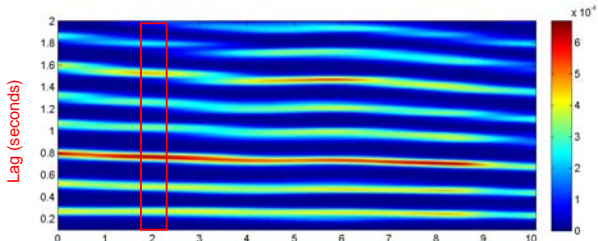
### Tempogram (Autocorrelation)

Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.

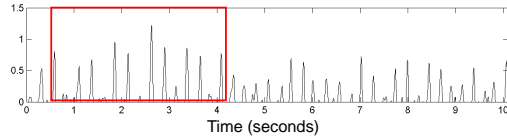
#### Autocorrelation-based method

- Compare novelty curve with time-lagged local sections of itself
- Convert lag-axis (given in seconds) into tempo axis (given in BPM)
- Autocorrelogram indicates local tempo

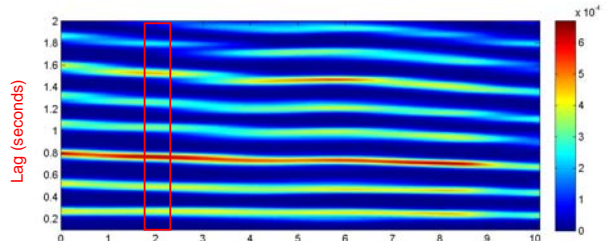
Tempogram (Autocorrelation)



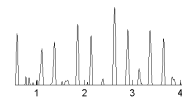
Novelty curve (local section)



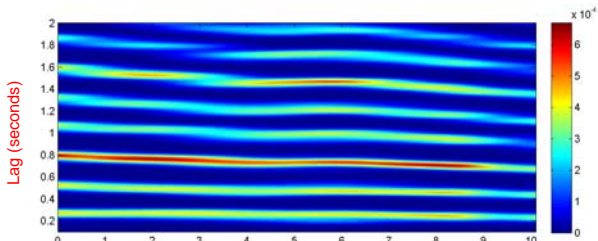
Tempogram (Autocorrelation)



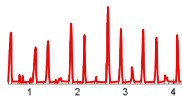
Windowed autocorrelation



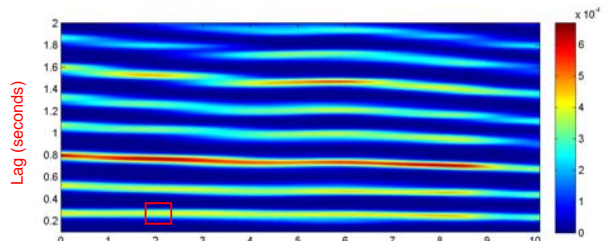
Tempogram (Autocorrelation)



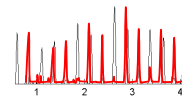
Lag = 0 (seconds)



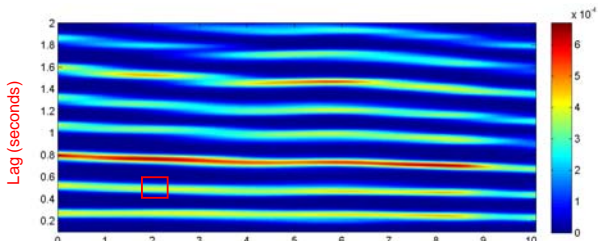
Tempogram (Autocorrelation)



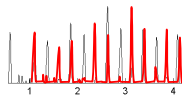
Lag = 0.26 (seconds)



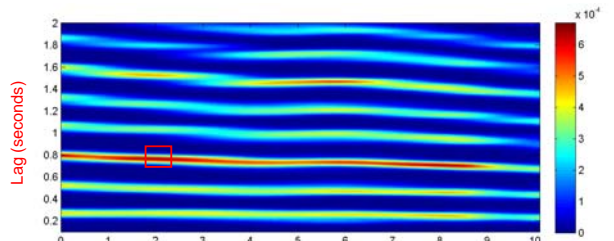
Tempogram (Autocorrelation)



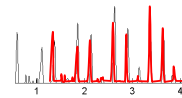
Lag = 0.52 (seconds)



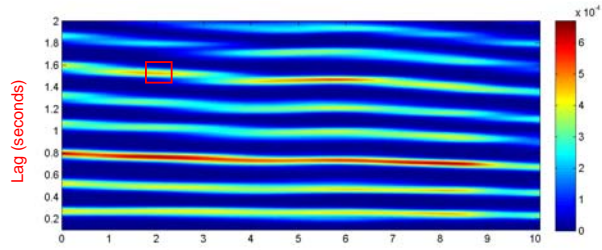
Tempogram (Autocorrelation)



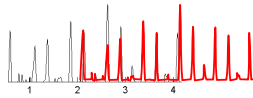
Lag = 0.78 (seconds)



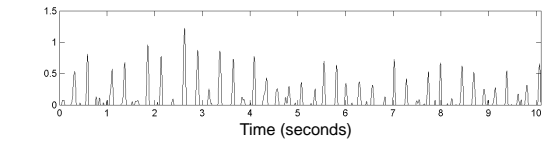
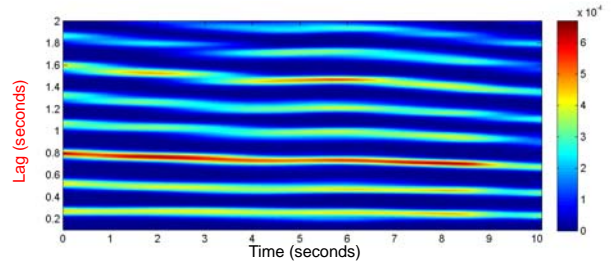
### Tempogram (Autocorrelation)



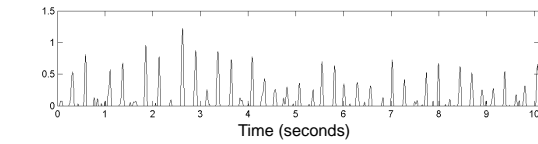
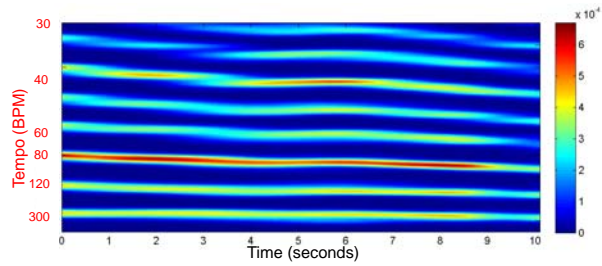
Lag = 1.56 (seconds)



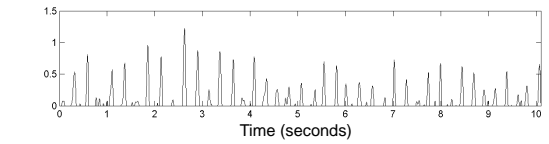
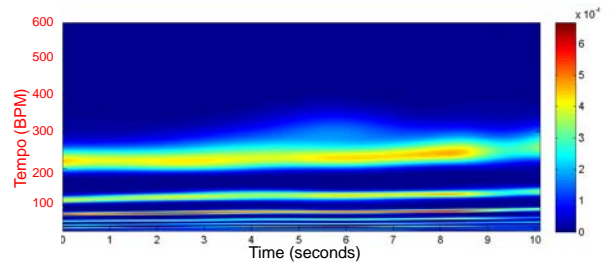
### Tempogram (Autocorrelation)



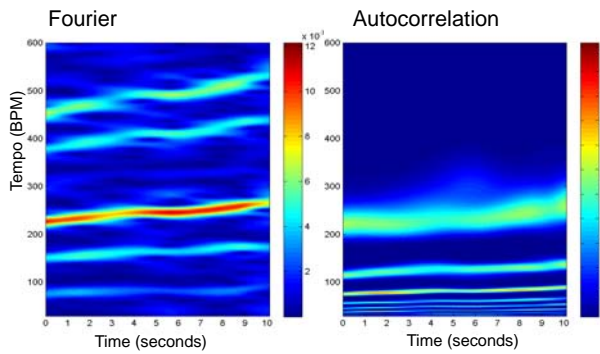
### Tempogram (Autocorrelation)



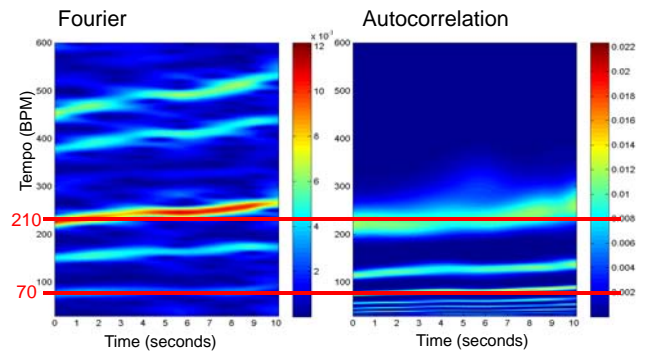
### Tempogram (Autocorrelation)



### Tempogram



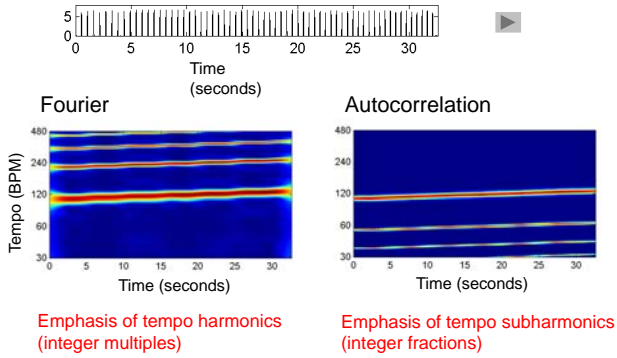
### Tempogram



Tempo@Tatum = 210 BPM

Tempo@Measure = 70 BPM

## Tempogram



[Peeters, JASP 2007][Grosche et al., ICASSP 2010]

## Tempogram (Summary)

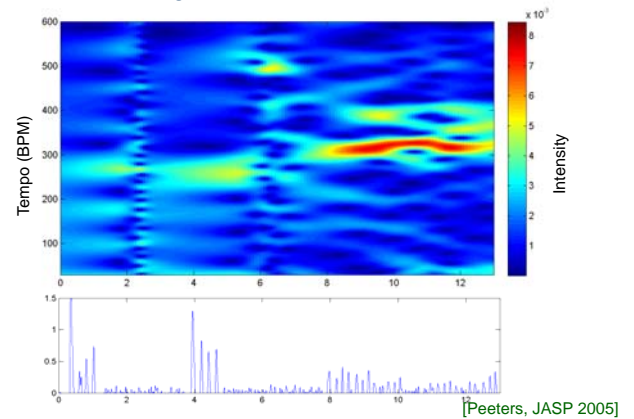
Fourier	Autocorrelation
Novelty curve is compared with sinusoidal kernels each representing a specific tempo	Novelty curve is compared with time-lagged local (windowed) sections of itself
Convert frequency (Hertz) into tempo (BPM)	Convert time-lag (seconds) into tempo (BPM)
Reveals novelty periodicities	Reveals novelty self-similarities
Emphasizes harmonics	Emphasizes subharmonics
Suitable to analyze tempo on tatum and tactus level	Suitable to analyze tempo on tactus and measure level

## Beat Tracking

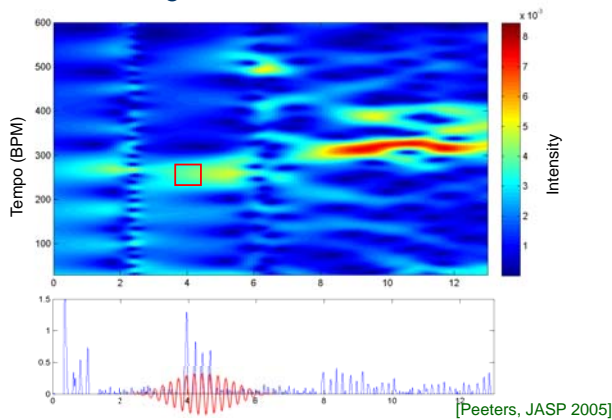
- Given the tempo, find the best sequence of beats
- Complex Fourier tempogram contains **magnitude** and **phase** information
- The **magnitude** encodes how well the novelty curve resonates with a sinusoidal kernel of a specific tempo
- The **phase** optimally aligns the sinusoidal kernel with the peaks of the novelty curve

[Peeters, JASP 2005]

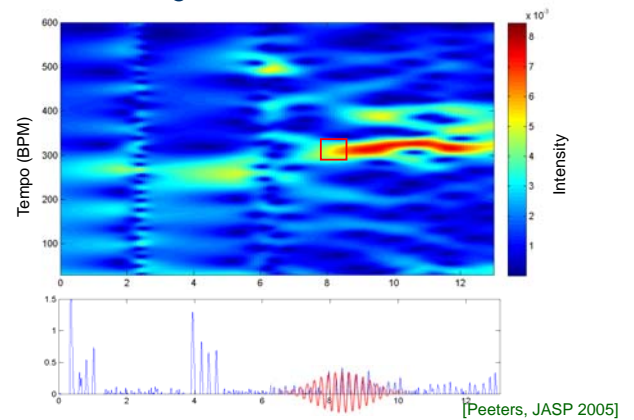
## Beat Tracking



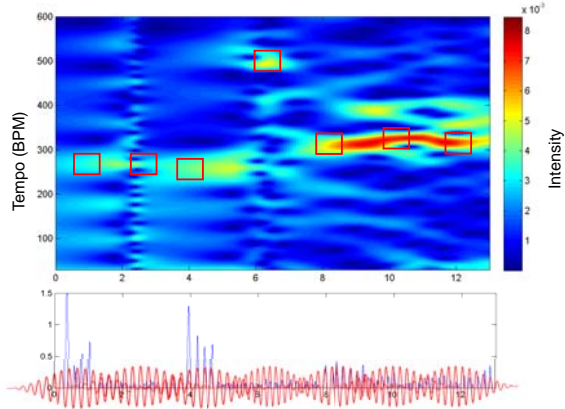
## Beat Tracking



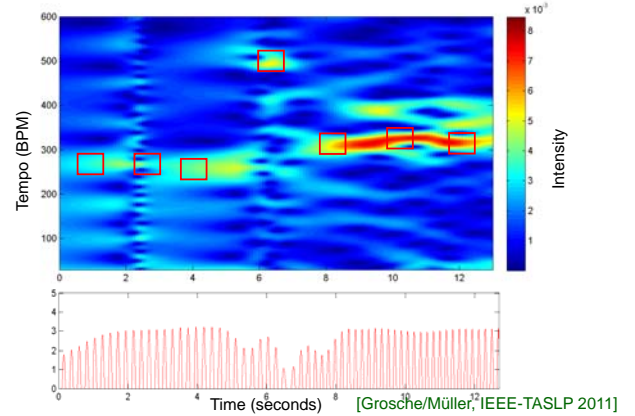
## Beat Tracking



## Beat Tracking



## Beat Tracking



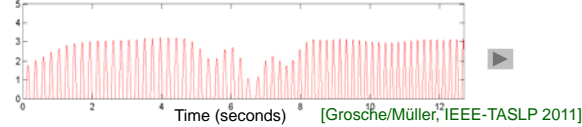
## Beat Tracking



### Novelty Curve



### Predominant Local Pulse (PLP)



## Beat Tracking

### Novelty Curve

- Indicates note onset candidates
- Extraction errors in particular for soft onsets
- Simple peak-picking problematic

### Predominant Local Pulse (PLP)

- Periodicity enhancement of novelty curve
- Accumulation introduces error robustness
- Locality of kernels handles tempo variations

## Beat Tracking

- Local tempo at time  $t$  :  $\tau_t \in \Theta$   $\Theta = [60:240]$  BPM

- Phase  $\varphi_t := \frac{1}{2\pi} \arccos\left(\frac{\text{Re}(\mathcal{T}(t, \tau_t))}{|\mathcal{T}(t, \tau_t)|}\right)$

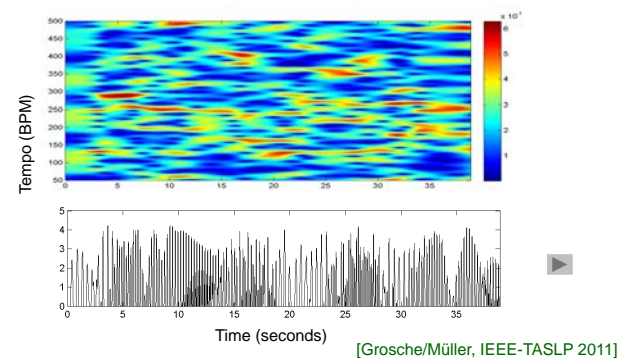
- Sinusoidal kernel  $\kappa_t : \mathbb{Z} \rightarrow \mathbb{R}$   
 $\kappa_t(n) := W(n-t) \cos(2\pi(\tau_t/60 \cdot n - \varphi_t))$   $n \in \mathbb{Z}$

- Periodicity curve  $\Gamma : [1:T] \rightarrow \mathbb{R}_{\geq 0}$   
 $\Gamma(n) = \left| \sum_{t \in [1:T]} \kappa_t(n) \right|_{\geq 0}$   $n \in [1:T]$

[Grosche/Müller, IEEE-TASLP 2011]

## Beat Tracking

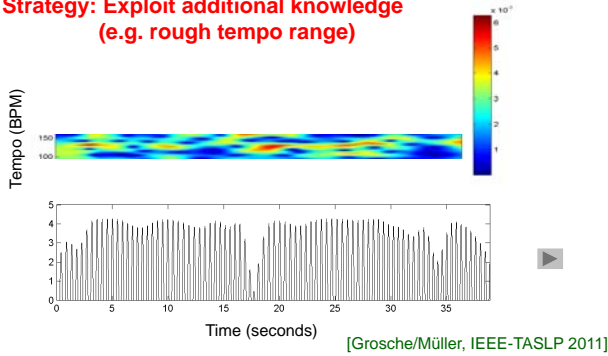
Borodin – String Quartet No. 2



## Beat Tracking

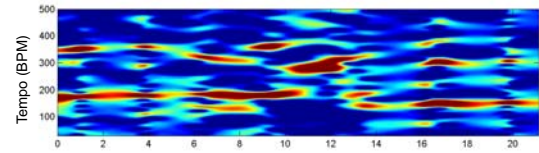
Borodin – String Quartet No. 2

**Strategy: Exploit additional knowledge  
(e.g. rough tempo range)**



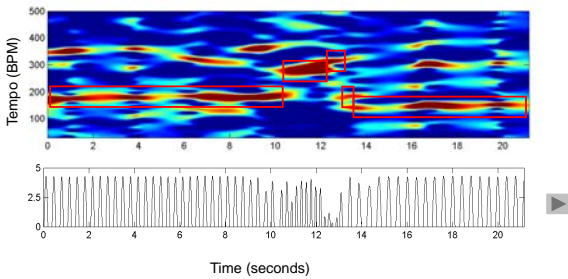
## Beat Tracking

Brahms Hungarian Dance No. 5



## Beat Tracking

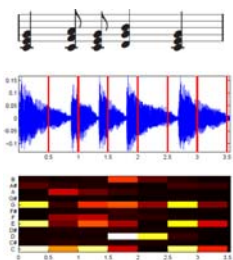
Brahms Hungarian Dance No. 5



## Applications

- Feature design  
(beat-synchronous features, adaptive windowing)
- Digital DJ / audio editing  
(mixing and blending of audio material)
- Music classification
- Music recommendation
- Performance analysis  
(extraction of tempo curves)

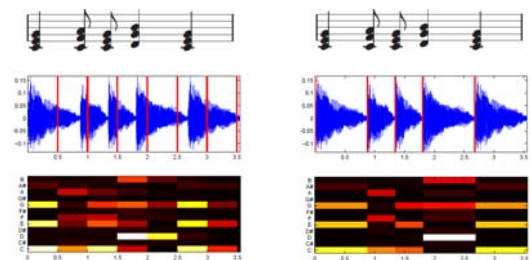
## Application: Feature Design



**Fixed window size**

[Ellis et al., ICASSP 2008] [Bello/Pickens, ISMIR 2005]

## Application: Feature Design

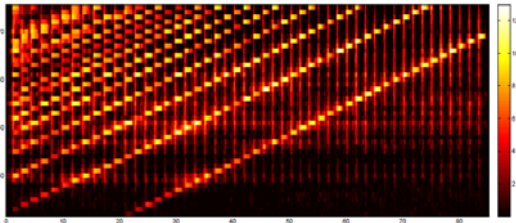


**Fixed window size**

**Adaptive window size**

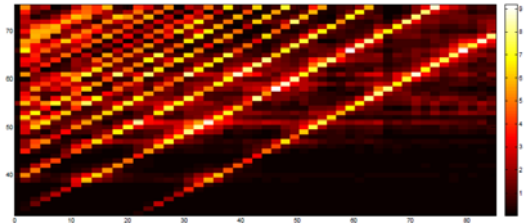
[Ellis et al., ICASSP 2008] [Bello/Pickens, ISMIR 2005]

## Application: Feature Design



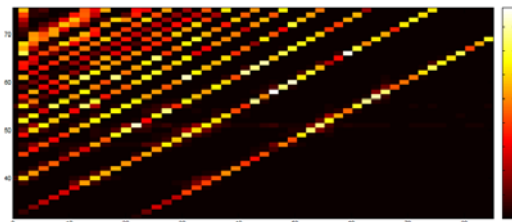
Fixed window size (100 ms)

## Application: Feature Design



Adaptive window size (roughly 1200 ms)  
Note onset positions define boundaries

## Application: Feature Design



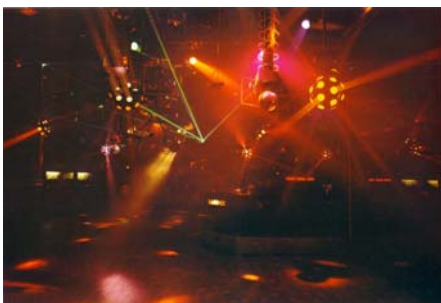
Adaptive window size (roughly 1200 ms)  
Note onset positions define boundaries  
Denoising by excluding boundary neighborhoods

## Application: Audio Editing (Digital DJ)



<http://www.mixxx.org/>

## Application: Beat-Synchronous Light Effects



## Summary

1. Onset Detection
  - Novelty curve (*something is changing*)
  - Indicates note onset candidates
  - Hard task for non-percussive instruments (strings)
2. Tempo Estimation
  - Fourier tempogram
  - Autocorrelation tempogram
  - Musical knowledge (tempo range, continuity)
3. Beat tracking
  - Find most likely beat positions
  - Exploiting phase information from Fourier tempogram