Lecture

Music Processing

Music Synchronization

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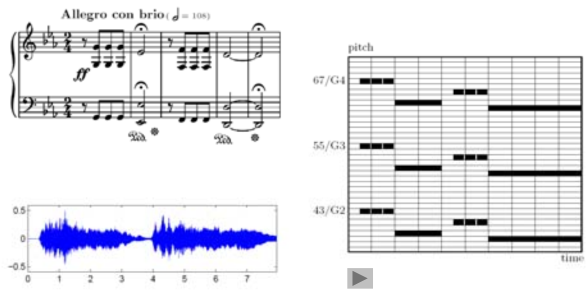
Music Data

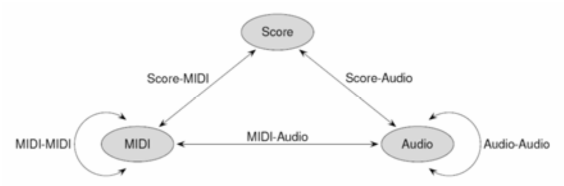


Music Data

Various interpretations – Beethoven's Fifth

Bernstein
Karajan
Scherbakov (piano)
MIDI (piano)

Music Synchronization



Schematic view of various synchronization tasks

Music Synchronization: Audio-Audio

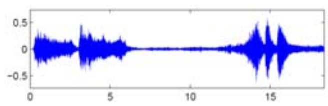
Given: Two different audio recordings of the same underlying piece of music.

Goal: Find for each position in one audio recording the musically corresponding position in the other audio recording.

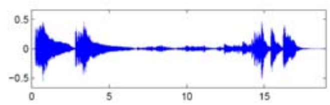
Music Synchronization: Audio-Audio

Beethoven's Fifth

Karajan



Scherbakov

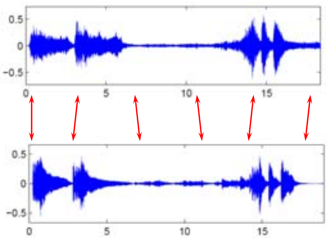


Music Synchronization: Audio-Audio

Beethoven's Fifth

Karajan

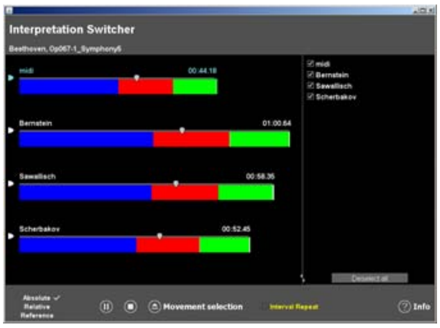
Scherbakov



Synchronization: Karajan → Scherbakov

Music Synchronization: Audio-Audio

Application: Interpretation Switcher



Music Synchronization: Audio-Audio

Two main steps:

1.) Audio features
- Robust but discriminative
- Chroma features
- Robust to variations in instrumentation, timbre, dynamics
- Correlate to harmonic progression

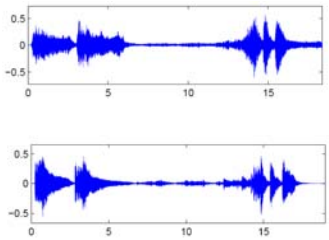
2.) Alignment procedure
- Deals with local and global tempo variations
- Needs to be efficient

Music Synchronization: Audio-Audio

Beethoven's Fifth

Karajan

Scherbakov



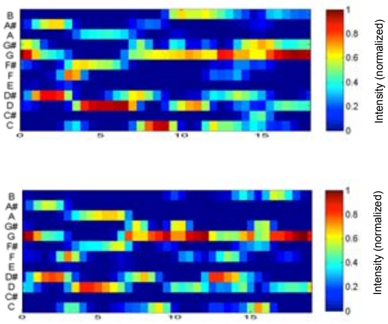
Time (seconds)

Music Synchronization: Audio-Audio

Beethoven's Fifth

Karajan

Scherbakov



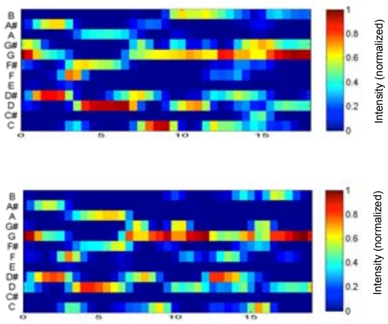
Time (seconds)

Music Synchronization: Audio-Audio

Beethoven's Fifth

Karajan

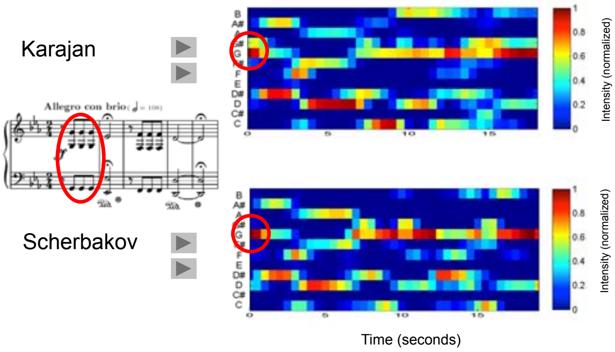
Scherbakov



Time (seconds)

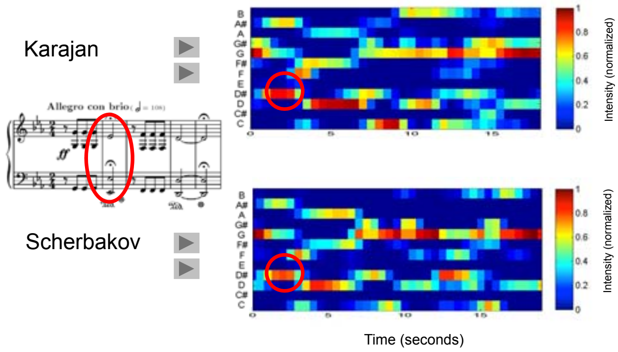
Music Synchronization: Audio-Audio

Beethoven's Fifth

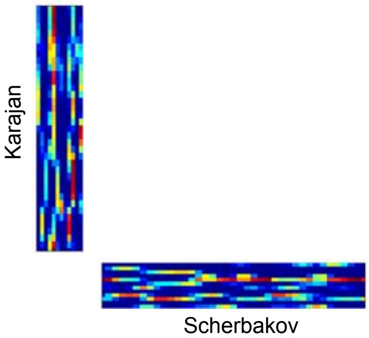


Music Synchronization: Audio-Audio

Beethoven's Fifth

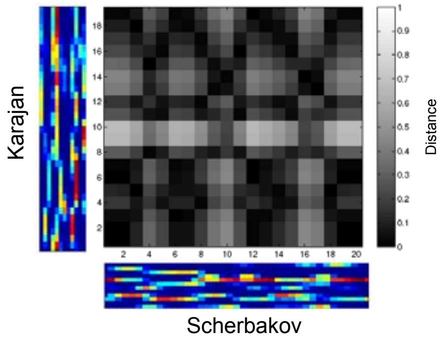


Music Synchronization: Audio-Audio



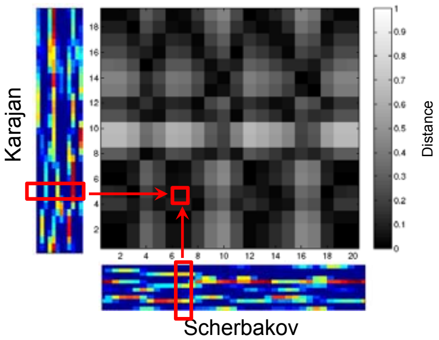
Music Synchronization: Audio-Audio

Cost matrix



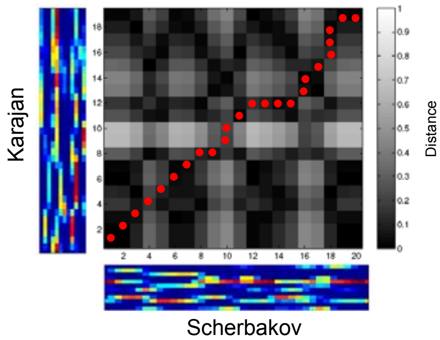
Music Synchronization: Audio-Audio

Cost matrix



Music Synchronization: Audio-Audio

Cost-minimizing alignment path



Music Synchronization: Audio-Audio

Beethoven's Fifth

Karajan

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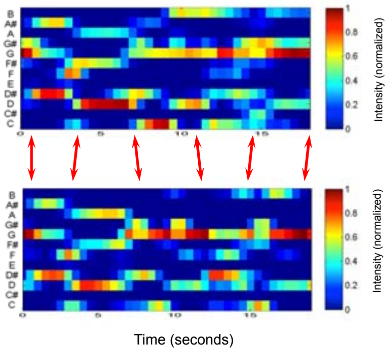


Music Synchronization: Audio-Audio

How to compute the alignment?

$\Rightarrow$ Cost matrices

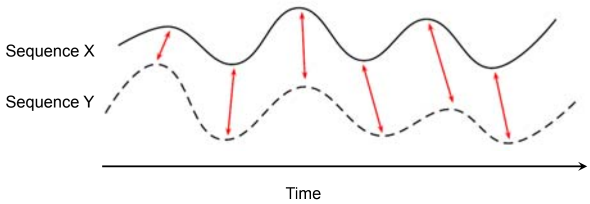
$\Rightarrow$ Dynamic programming

$\Rightarrow$ Dynamic Time Warping (DTW)

Dynamic Time Warping

- Well-known technique to find an optimal alignment between two given (time-dependent) sequences under certain restrictions.
- Intuitively, sequences are warped in a non-linear fashion to match each other.
- Originally used to compare different speech patterns in automatic speech recognition

Dynamic Time Warping



Time alignment of two time-dependent sequences, where the aligned points are indicated by the arrows.

Dynamic Time Warping

The objective of DTW is to compare two (time-dependent) sequences

$$X := (x_1, x_2, \ldots, x_N)$$

of length $N \in \mathbb{N}$ and

$$Y := (y_1, y_2, \ldots, y_M)$$

of length $M \in \mathbb{N}$. Here,

$$x_n, y_m \in \mathcal{F},\ n \in [1:N],\ m \in [1:M],$$

are suitable features that are elements from a given feature space denoted by $\mathcal{F}$.

Dynamic Time Warping

To compare two different features $x, y \in \mathcal{F}$ one needs a local cost measure which is defined to be a function

$$c : \mathcal{F} \times \mathcal{F} \to \mathbb{R}_{\geq 0}$$

Typically, $c(x, y)$ is small (low cost) if x and y are similar to each other, and otherwise $c(x, y)$ is large (high cost).

Dynamic Time Warping

Evaluating the local cost measure for each pair of elements of the sequences X and Y one obtains the cost matrix

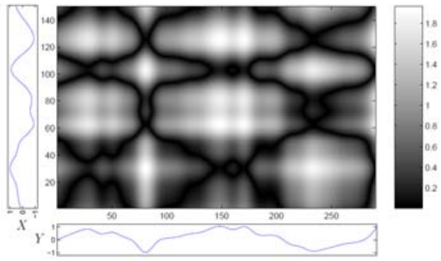
$$C \in \mathbb{R}^{N \times M}$$

denfined by

$$C(n, m) := c(x_n, y_m).$$

Then the goal is to find an alignment between X and Y having minimal overall cost. Intuitively, such an optimal alignment runs along a "valley" of low cost within the cost matrix C.

Dynamic Time Warping



Cost matrix of the two real-valued sequences X and Y using the Manhattan distance (absolute value of the difference) as local cost measure c.

Dynamic Time Warping

The next definition formalizes the notion of an alignment.

A **warping path** is a sequence $p = (p_1, \ldots, p_L)$ with

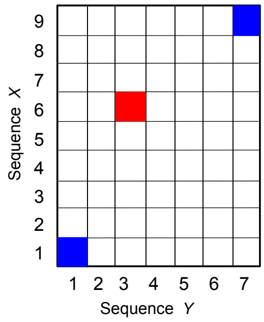
$$p_\ell = (n_\ell, m_\ell) \in [1:N] \times [1:M]$$

for $\ell \in [1:L]$ satisfying the following three conditions:

- Boundary condition: $p_1 = (1,1)$ and $p_L = (N, M)$
- Monotonicity condition: $n_1 \leq n_2 \leq \ldots \leq n_L$ and $m_1 \leq m_2 \leq \ldots \leq m_L$
- Step size condition: $p_{\ell+1} - p_\ell \in \{(1,0), (0,1), (1,1)\}$ for $\ell \in [1:L-1]$

Dynamic Time Warping

Warping path



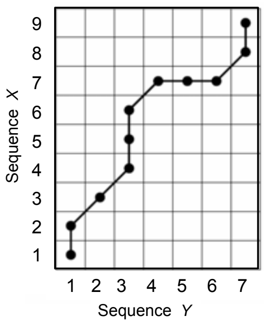
Each matrix entry (cell) corresponds to a pair of indices.

Cell = (6,3)

Boundary cells:
p_1 = (1,1)
p_L = (N,M) = (9,7)

Dynamic Time Warping

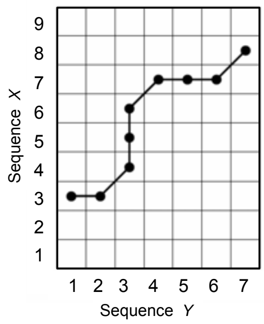
Warping path



Correct warping path

Dynamic Time Warping

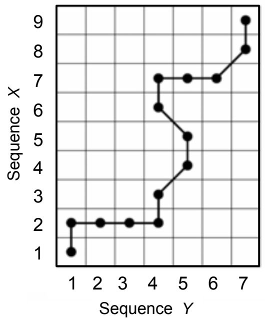
Warping path



Violation of boundary condition

Dynamic Time Warping

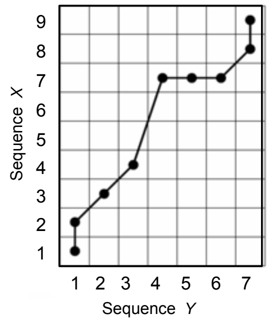
Warping path



Violation of monotonicity condition

Dynamic Time Warping

Warping path



Violation of step size condition

Dynamic Time Warping

The total cost $c_p(X,Y)$ of a warping path p between X and Y with respect to the local cost measure c is defined as

$$c_p(X,Y) := \sum_{\ell=1}^{L} c(x_{n_\ell}, y_{m_\ell})$$

Furthermore, an optimal warping path between X and Y is a warping path p^* having minimal total cost among all possible warping paths. The DTW distance $\mathrm{DTW}(X,Y)$ between X and Y is then defined as the total cost o p^*

$$\begin{aligned} \mathrm{DTW}(X,Y) &:= c_{p^*}(X,Y) \\ &= \min\{c_p(X,Y) \mid p \text{ is a warping path}\} \end{aligned}$$

Dynamic Time Warping

- The warping path p^* is not unique (in general).
- DTW does (in general) not definne a metric since it may not satisfy the triangle inequality.
- There exist exponentially many warping paths.
- How can p^* be computed efficiently?

Dynamic Time Warping

Notation:
$$\begin{aligned} X(1:n) &:= (x_1,\ldots,x_n), \quad 1 \le n \le N \\ Y(1:m) &:= (y_1,\ldots,y_m), \quad 1 \le m \le M \\ D(n,m) &:= \mathrm{DTW}(X(1:n), Y(1:m)) \end{aligned}$$

The matrix D is called the accumulated cost matrix.

The entry $D(n,m)$ specifies the cost of an optimal warping path that aligns $X(1:n)$ with $Y(1:m)$.

Dynamic Time Warping

Lemma:

$$\begin{array}{lll} (i) & D(N,M) & = \mathrm{DTW}(X,Y) \\ (ii) & D(1,1) & = C(1,1) \\ (iii) & D(n,1) & = \sum_{k=1}^{n} C(k,1) \\ & D(1,m) & = \sum_{k=1}^{m} C(1,k) \\ (iv) & D(n,m) & = \min \begin{pmatrix} D(n-1,m-1) \\ D(n-1,m) \\ D(n,m-1) \end{pmatrix} + C(n,m) \end{array}$$

for $n > 1,\ m > 1$

Proof: *(i)* – *(iii)* are clear by definition

Dynamic Time Warping

Proof of *(iv)*: Induction via n, m :

Let $n > 1,\ m > 1$ and $q = (q_1, \ldots, p_{L-1}, p_L)$ be an optimal warping path for $X(1:n))$ and $Y(1:m))$.
Then $q_L = (n, m)$ (boundary condition).

Let $q_{L-1} = (a, b)$. The step size condition implies

$$(a, b) \in \{(n-1, m-1), (n-1, m), (n, m-1)\}$$

The warping path $(q_1, \ldots, q_{L-1})$ must be optimal for $X(1:a),\ Y(1:b)$. Thus,

$$D(n, m) = c_{(q_1,\ldots,q_{L-1})}(X(1:a), Y(1:b)) + C(n, m)$$

■

Dynamic Time Warping

Accumulated cost matrix

Given the two feature sequences X and Y, the matrix D is computed recursively.

- Initialize D using *(ii)* and *(iii) of the lemma.*
- Compute $D(n, m)$ for $n > 1,\ m > 1$ using *(iv)*.
- $\mathrm{DTW}(X, Y) = D(N, M)$ *using (i).*

Note:

- Complexity *O(NM)*.
- Dynamic programming: “overlapping-subproblem property”

Dynamic Time Warping

Optimal warping path

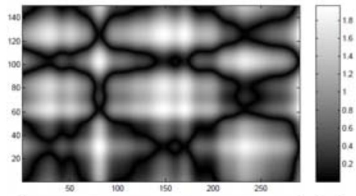
Given to the algorithm is the accumulated cost matrix D.
The optimal path $p^* = (p_1, \ldots, p_L)$ is computed in reverse order of the indices starting with $p_L = (N, M)$.
Suppose $p_\ell = (n, m)$ has been computed. In case $(n, m) = (1, 1)$, one must have $\ell = 1$ and we are done.
Otherwise,

$$p_{\ell-1} := \begin{cases} (1, m-1), & \text{if } n = 1 \\ (n-1, 1), & \text{if } m = 1 \\ \operatorname{argmin}\{D(n-1, m-1), & \\ \qquad D(n-1, m), D(n, m-1)\}, & \text{otherwise,} \end{cases}$$

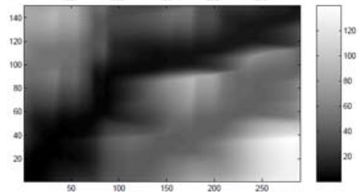
where we take the lexicographically smallest pair in case “argmin” is not unique.

Dynamic Time Warping

Cost matrix C



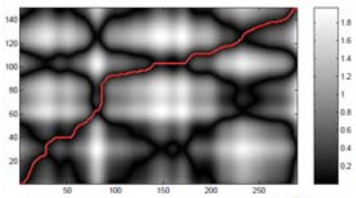
Accumulated cost martrix D



Dynamic Time Warping

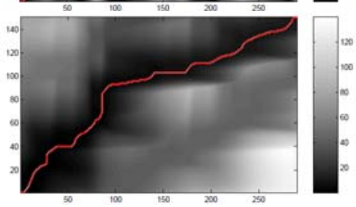
Cost matrix C

Optimal warping path



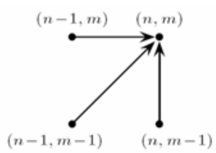
Accumulated cost martrix D

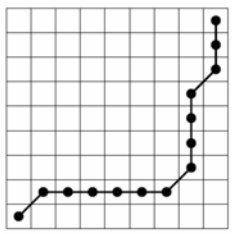
Optimal warping path



Dynamic Time Warping

Variation of step size condition





Dynamic Time Warping

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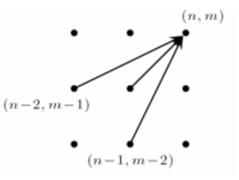

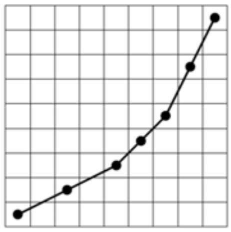

Dynamic Time Warping

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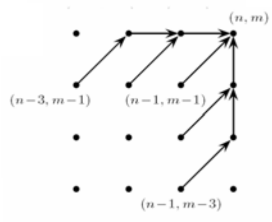
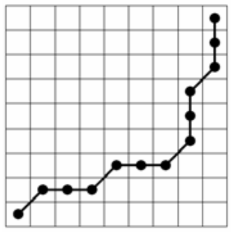




Dynamic Time Warping

- Computation via dynamic programming
- Memory requirements and running time: $O(NM)$
- Problem: Infeasible for large N and M
- Example: Feature resolution 10 Hz, pieces 15 min

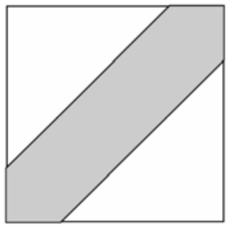
  $\Rightarrow$ $N, M \sim 10{,}000$

  $\Rightarrow$ $N \cdot M \sim 100{,}000{,}000$

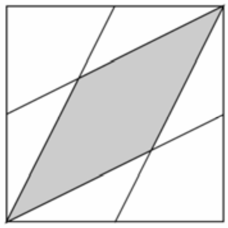
Dynamic Time Warping

Strategy: Global constraints

Sakoe-Chiba band



Itakura parallelogram



Dynamic Time Warping

Strategy: Global constraints

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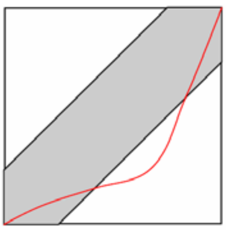

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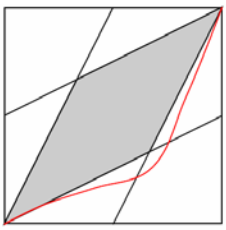
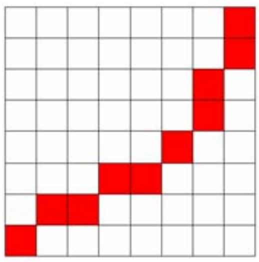


Problem: Optimal warping path not in constraint region

Dynamic Time Warping

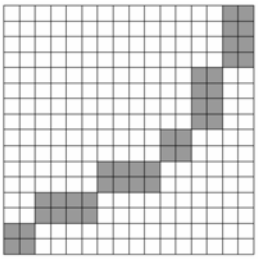
Strategy: Multiscale approach



Compute optimal warping path on coarse level

Dynamic Time Warping

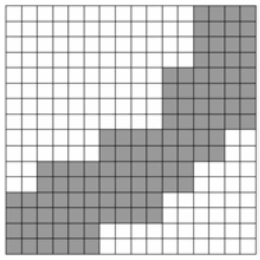
Strategy: Multiscale approach



Project on fine level

Dynamic Time Warping

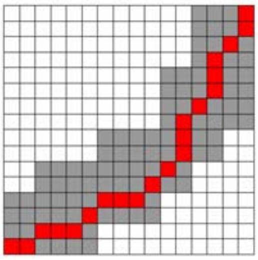
Strategy: Multiscale approach



Specify constraint region

Dynamic Time Warping

Strategy: Multiscale approach



Compute *constrained* optimal warping path

Dynamic Time Warping

Strategy: Multiscale approach

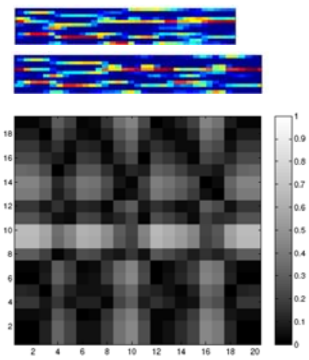
- Suitable features?
- Suitable resolution levels?
- Size of constraint regions?

Good trade-off between efficiency and robustness?

Suitable parameters depend very much on application!

Music Synchronization: Audio-Audio

- Transform audio recordings into chroma vector sequences
  $\rightsquigarrow X := (x_1, x_2, \ldots, x_N)$
  $\rightsquigarrow Y := (y_1, y_2, \ldots, y_M)$
- Compute cost matrix $C(n,m) := c(x_n, y_m)$ with respect to local cost measure c



Music Synchronization: Audio-Audio

- Transform audio recordings into chroma vector sequences
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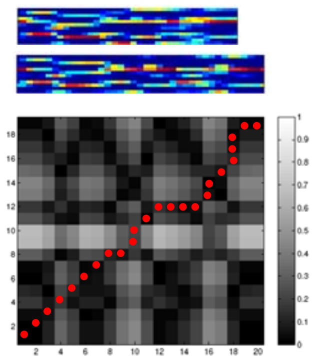
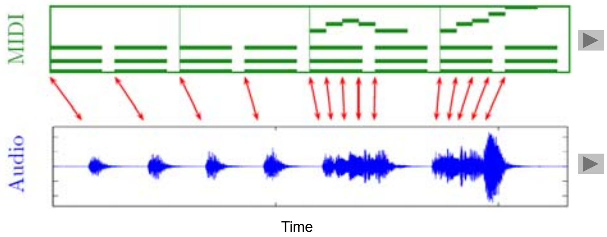
  $\rightsquigarrow Y := (y_1, y_2, \ldots, y_M)$
- Compute cost matrix $C(n,m) := c(x_n, y_m)$ with respect to local cost measure c
- Compute cost-minimizing warping path from C



Music Synchronization: MIDI-Audio



Music Synchronization: MIDI-Audio

MIDI = meta data

Automated annotation

Audio recording

Sonification of annotations

Music Synchronization: MIDI-Audio

MIDI = reference (score)

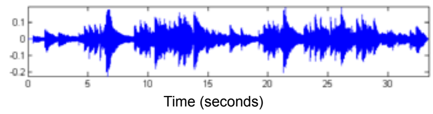
Tempo information

Audio recording

Performance Analysis: Tempo Curves

Schumann: Träumerei

Performance:

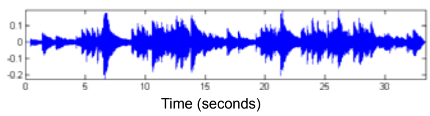


Performance Analysis: Tempo Curves

Schumann: Träumerei

Score (reference):

Performance:



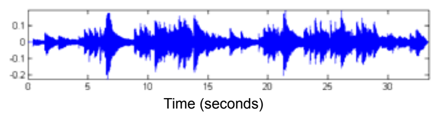
Performance Analysis: Tempo Curves

Schumann: Träumerei

Score (reference):

Strategy: Compute score-audio synchronization and derive tempo curve

Performance:

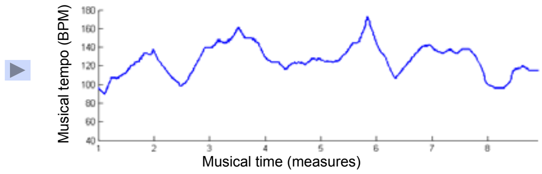


Performance Analysis: Tempo Curves

Schumann: Träumerei

Score (reference):

Tempo curve:

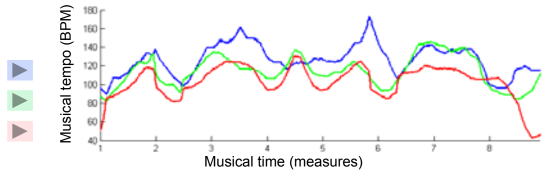


Performance Analysis: Tempo Curves

Schumann: Träumerei

Score (reference):

Tempo curves:

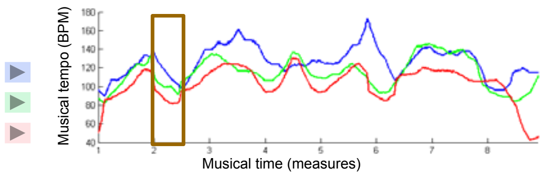


Performance Analysis: Tempo Curves

Schumann: Träumerei

Score (reference):

Tempo curves:

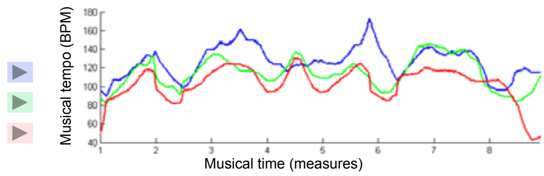


Performance Analysis: Tempo Curves

Schumann: Träumerei

What can be done if no reference is available?

Tempo curves:

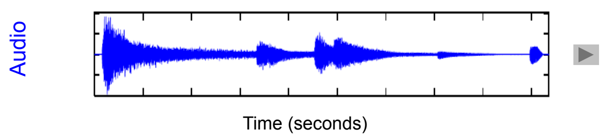


Music Synchronization: MIDI-Audio

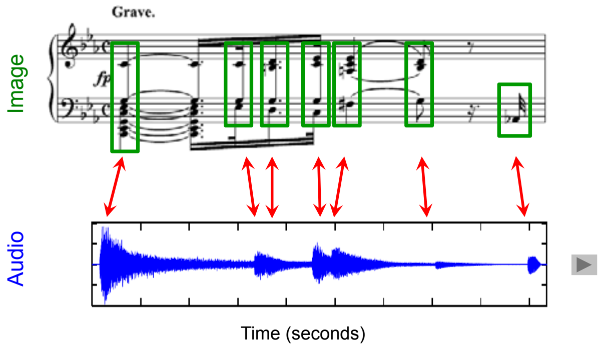
Applications

- Automated audio annotation
- Accurate audio access after MIDI-based retrieval
- Automated tracking of MIDI note parameters during audio playback
- Performance Analysis

Music Synchronization: Image-Audio

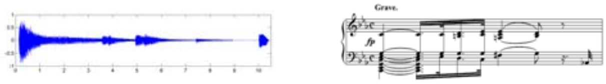


Music Synchronization: Image-Audio



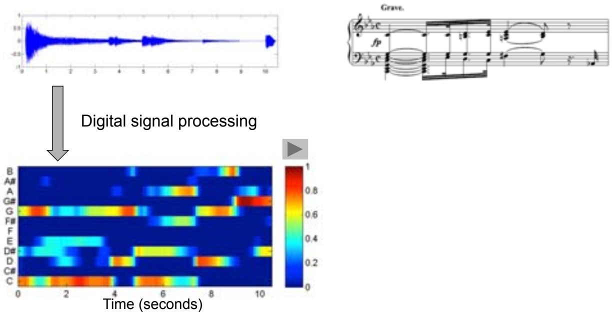
Music Synchronization: Image-Audio

Convert into common mid-level feature representation



Music Synchronization: Image-Audio

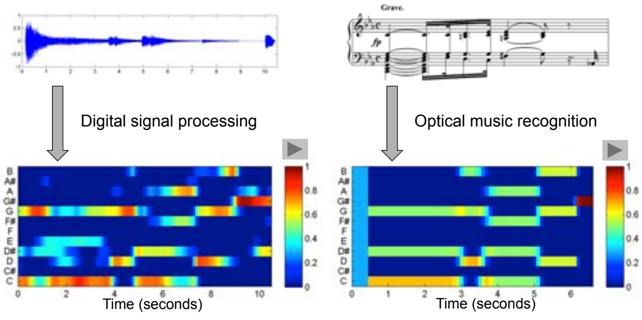
Convert into common mid-level feature representation



Audio chroma representation

Music Synchronization: Image-Audio

Convert into common mid-level feature representation



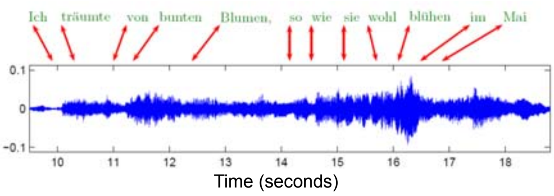
Audio chroma representation

Image chroma representation

Music Synchronization: Image-Audio

Application: Score Viewer

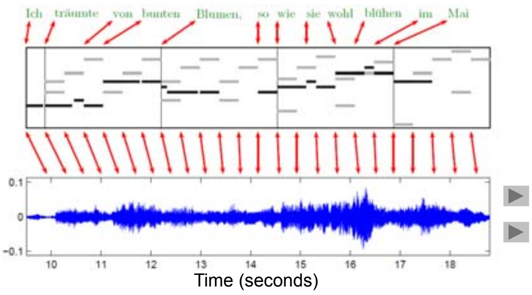
Music Synchronization: Lyrics-Audio



Difficult task!

Music Synchronization: Lyrics-Audio

Lyrics-Audio → Lyrics-MIDI + MIDI-Audio



Music Synchronization: Lyrics-Audio

Application: SyncPlayer/LyricsSeeker

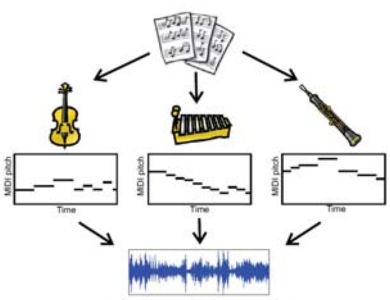
Source Separation

- Decomposition of audio stream into different sound sources
- Central task in digital signal processing
- “Cocktail party effect”
- Sources are often assumed to be statistically independent
- This is often not the case in music

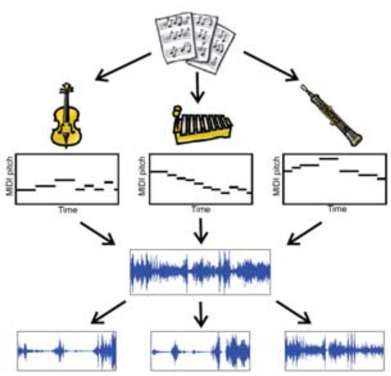
Strategy: Exploit additional information (e.g. musical score) to support the seperation process

Score-Informed Source Separation

Score-Informed Source Separation

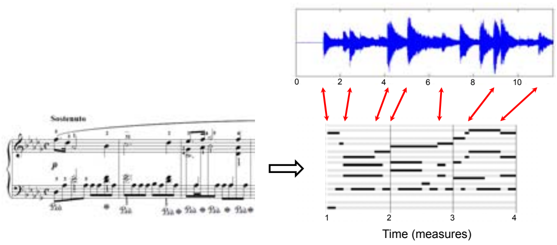


Score-Informed Source Separation



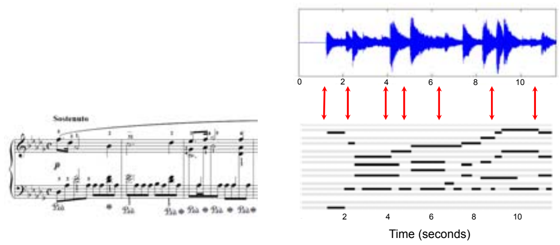
Score-Informed Source Separation

First step: Use music synchronization techniques to generate an audio-synchronous piano roll representation from the score.



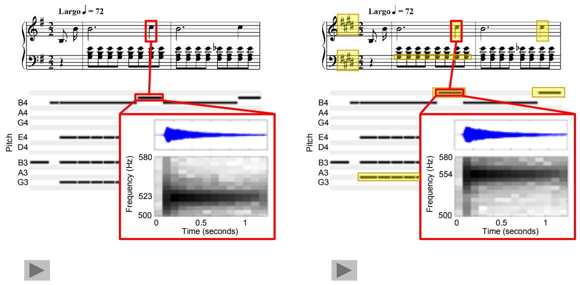
Score-Informed Source Separation

First step: Use music synchronization techniques to generate an audio-synchronous piano roll representation from the score.



Score-Informed Source Separation

Application: Audio editing



Score-Informed Source Separation

Application: Instrument equalization