

Lecture
Music Processing

Tempo and Beat Tracking

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Introduction

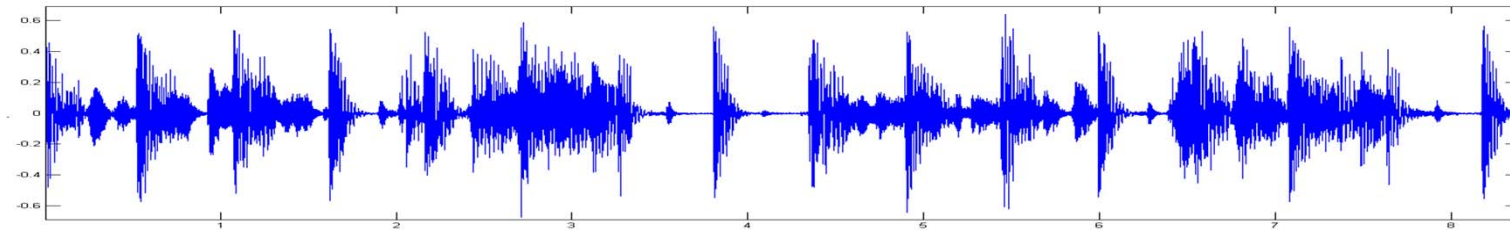
Basic beat tracking task:

Given an audio recording of a piece of music,
determine the periodic sequence of beat positions.

“Tapping the foot when listening to music”

Introduction

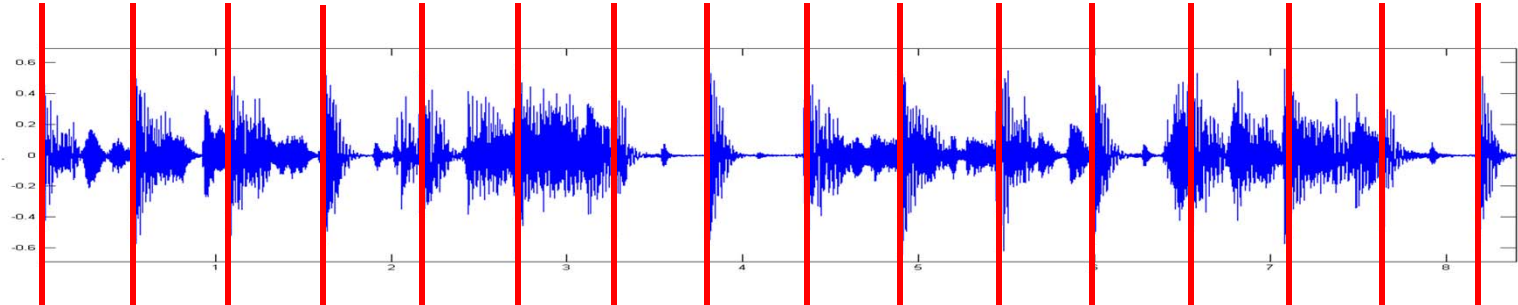
Example: Queen – Another One Bites The Dust



Time (seconds)

Introduction

Example: Queen – Another One Bites The Dust



Time (seconds)



Introduction

Example: Happy Birthday to you

Pulse level: **Measure**

The image shows two staves of musical notation for the song 'Happy Birthday to you'. The first staff is in 3/4 time and contains the lyrics: 'Hap - py Birth - day to you, Hap - py Birth - day to you, Hap - py'. The second staff continues with the lyrics: 'Birth - day dear _____, Hap - py Birth - day to you!'. Four red arrows point downwards to the first note of each of the four measures in the first staff, indicating the pulse level at the measure level.

Introduction

Example: Happy Birthday to you

Pulse level: **Tactus (beat)**

The image shows a musical score for the song "Happy Birthday to you" in 3/4 time. The first staff is marked with red arrows pointing to the downbeats of each measure, indicating the pulse level (Tactus). The lyrics are: "Hap - py Birth - day to you, Hap - py Birth - day to you, Hap - py Birth - day dear _____, Hap - py Birth - day to you!".

Hap - py Birth - day to you, Hap - py Birth - day to you, Hap - py
Birth - day dear _____, Hap - py Birth - day to you!

Introduction

Example: Chopin – Mazurka Op. 68-3

Pulse level: Quarter note


Tempo: ???



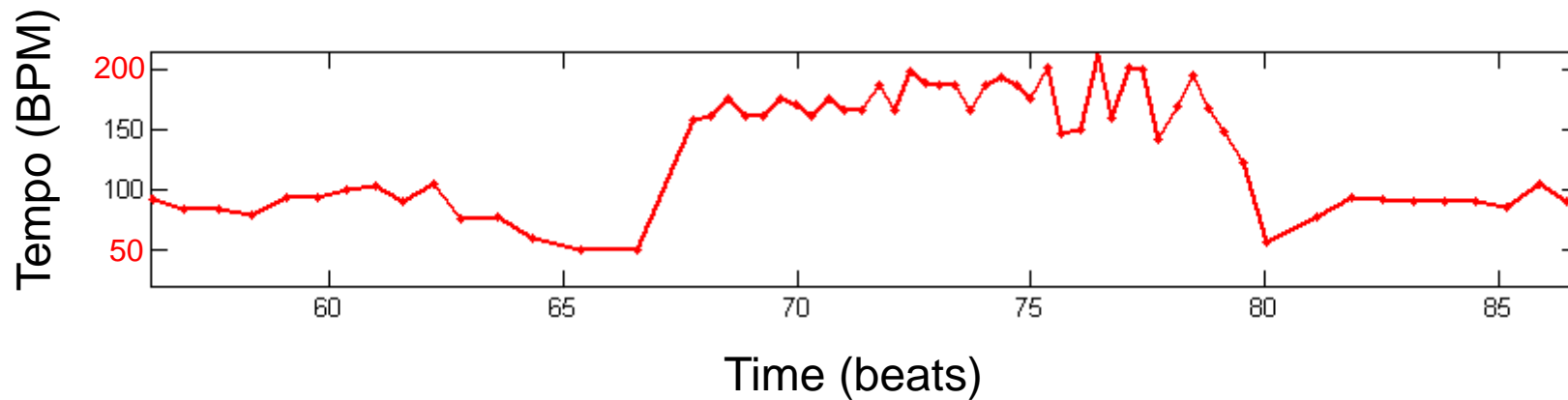
Introduction

Example: Chopin – Mazurka Op. 68-3

Pulse level: Quarter note

Tempo: **50-200 BPM** 

Tempo curve



Introduction

Example: Borodin – String Quartet No. 2

Pulse level: Quarter note

Tempo: 120-140 BPM (roughly)

Beat tracker without any prior knowledge



Beat tracker with prior knowledge on
rough tempo range



Introduction

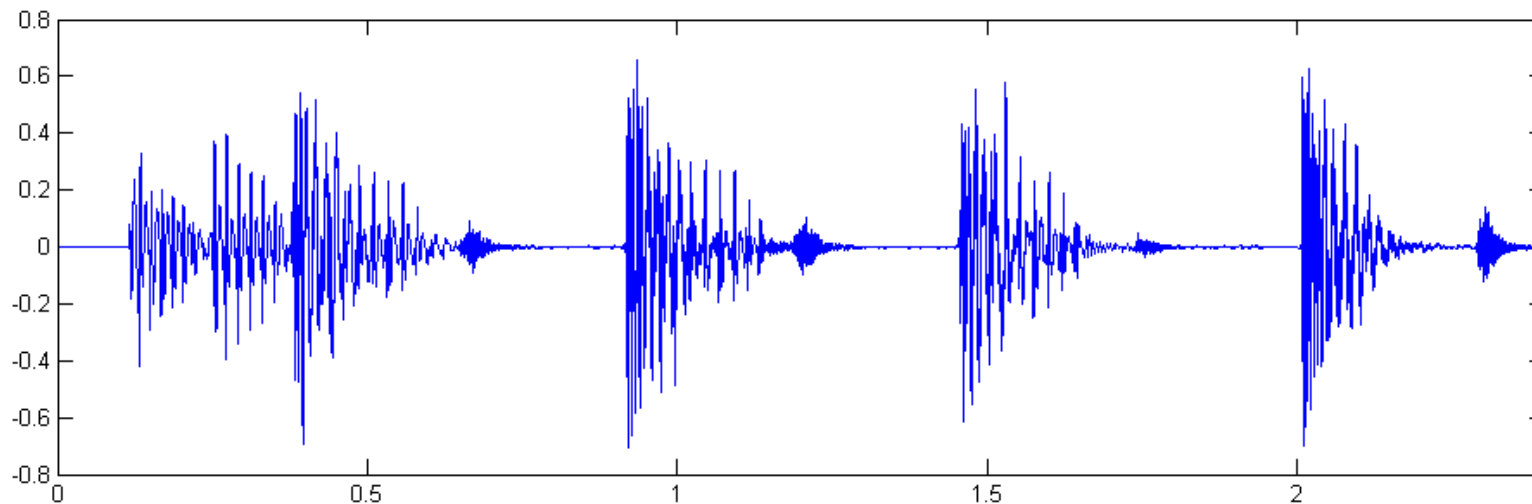
Challenges in beat tracking

- Pulse level often unclear
- Local/sudden tempo changes (e.g. rubato)
- Vague information
(e.g., soft onsets, extracted onsets corrupt)
- Sparse information
(often only note onsets are used)

Introduction

Tasks

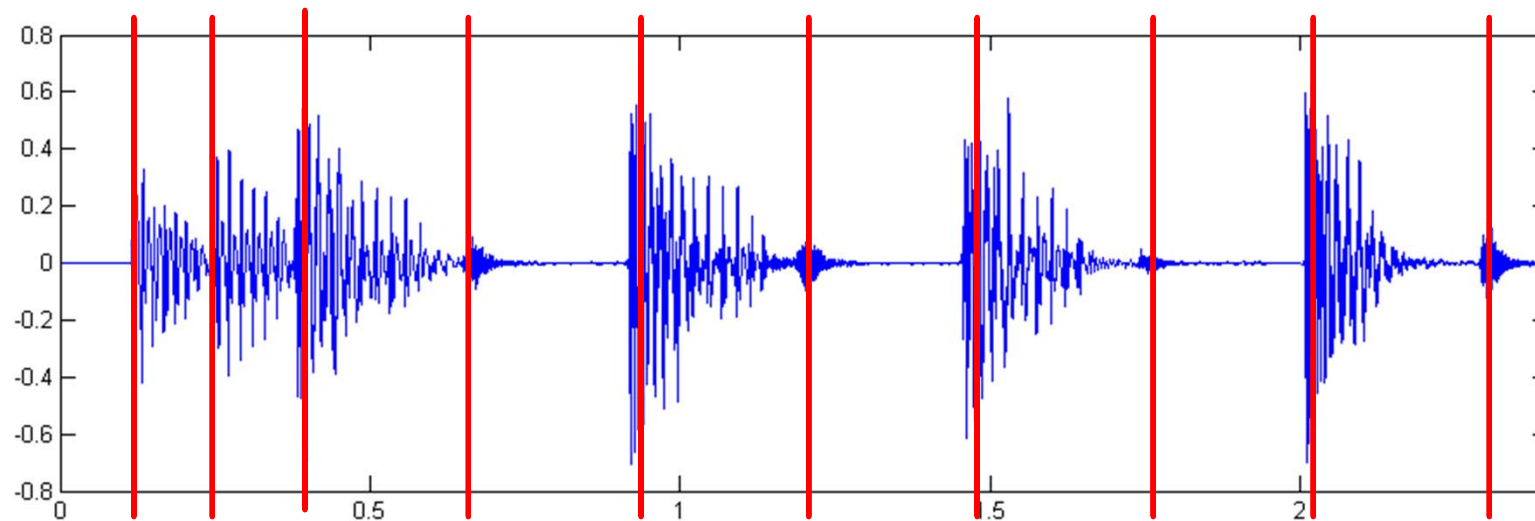
- Onset detection
- Beat tracking
- Tempo estimation



Introduction

Tasks

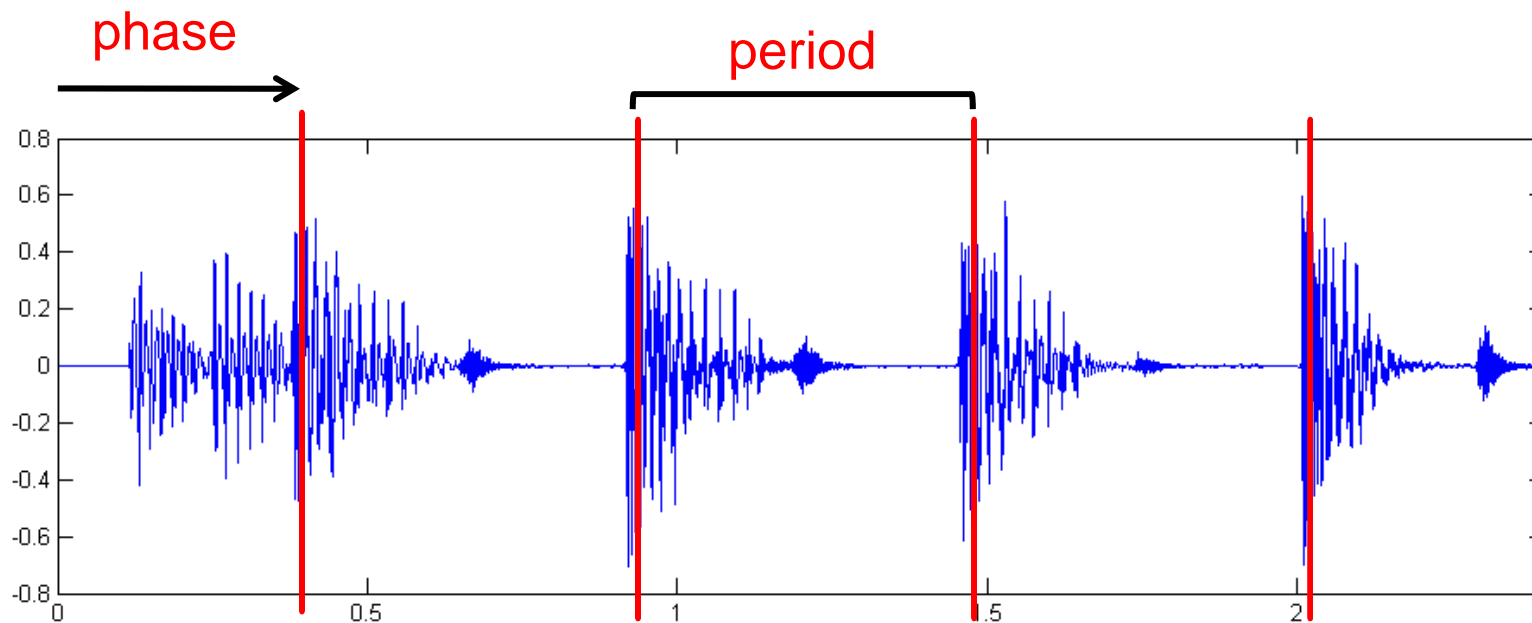
- Onset detection
- Beat tracking
- Tempo estimation



Introduction

Tasks

- Onset detection
- **Beat tracking**
- Tempo estimation



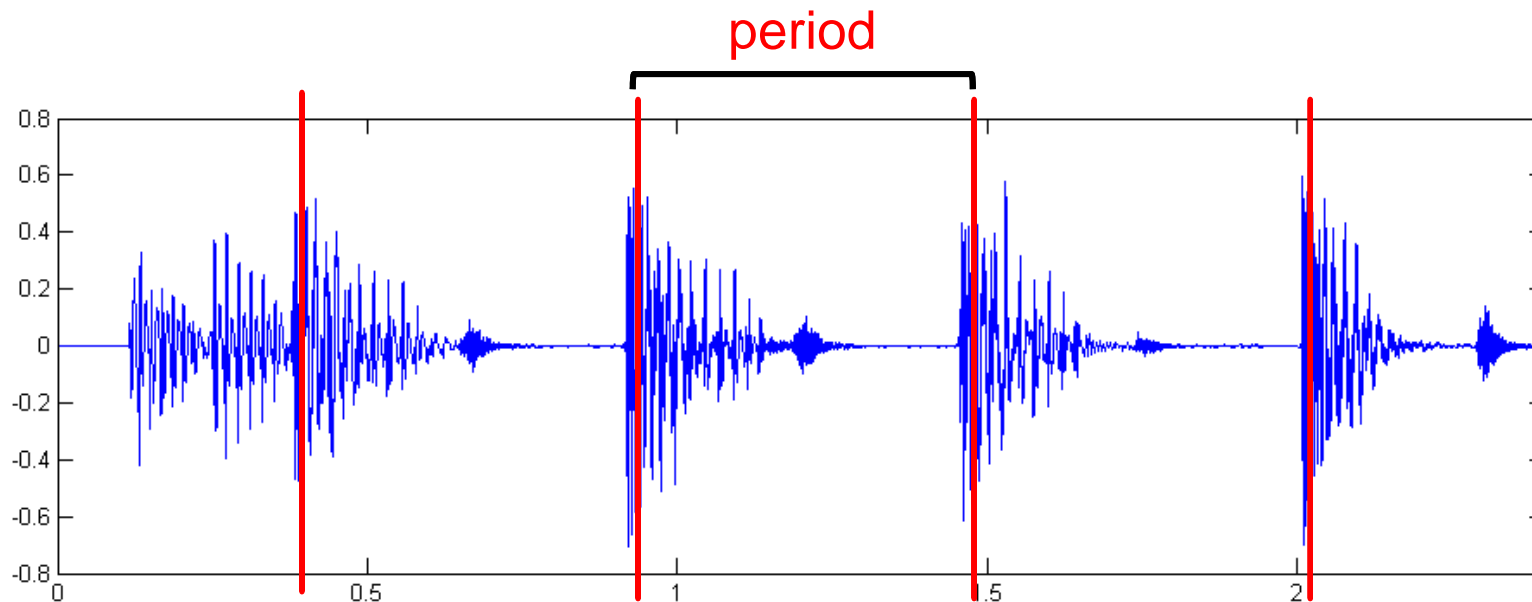
Introduction

Tasks

- Onset detection
- Beat tracking
- Tempo estimation

Tempo := $60 / \text{period}$

Beats per minute (BPM)

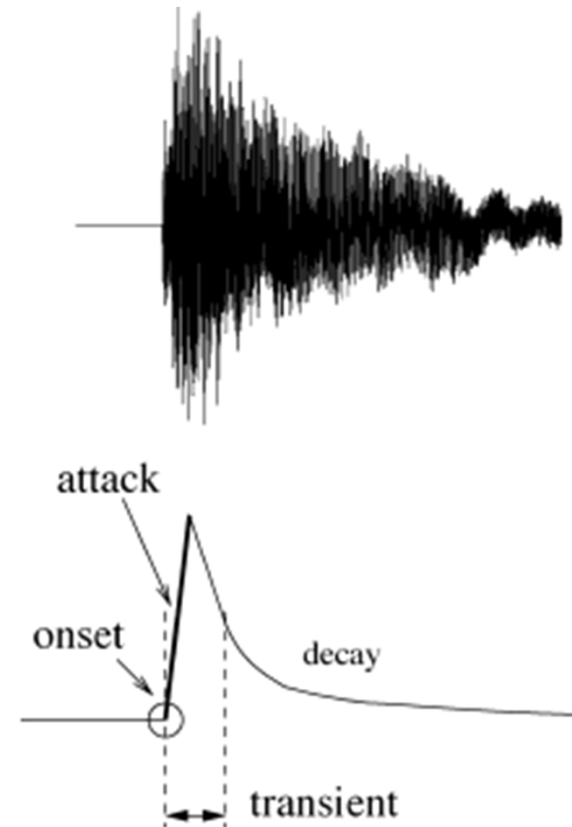


Onset Detection

- Finding start times of perceptually relevant acoustic events in music signal
- Onset is the time position where a note is played
- Onset typically goes along with a change of the signal's properties:
 - energy or loudness
 - pitch or harmony
 - timbre

Onset Detection

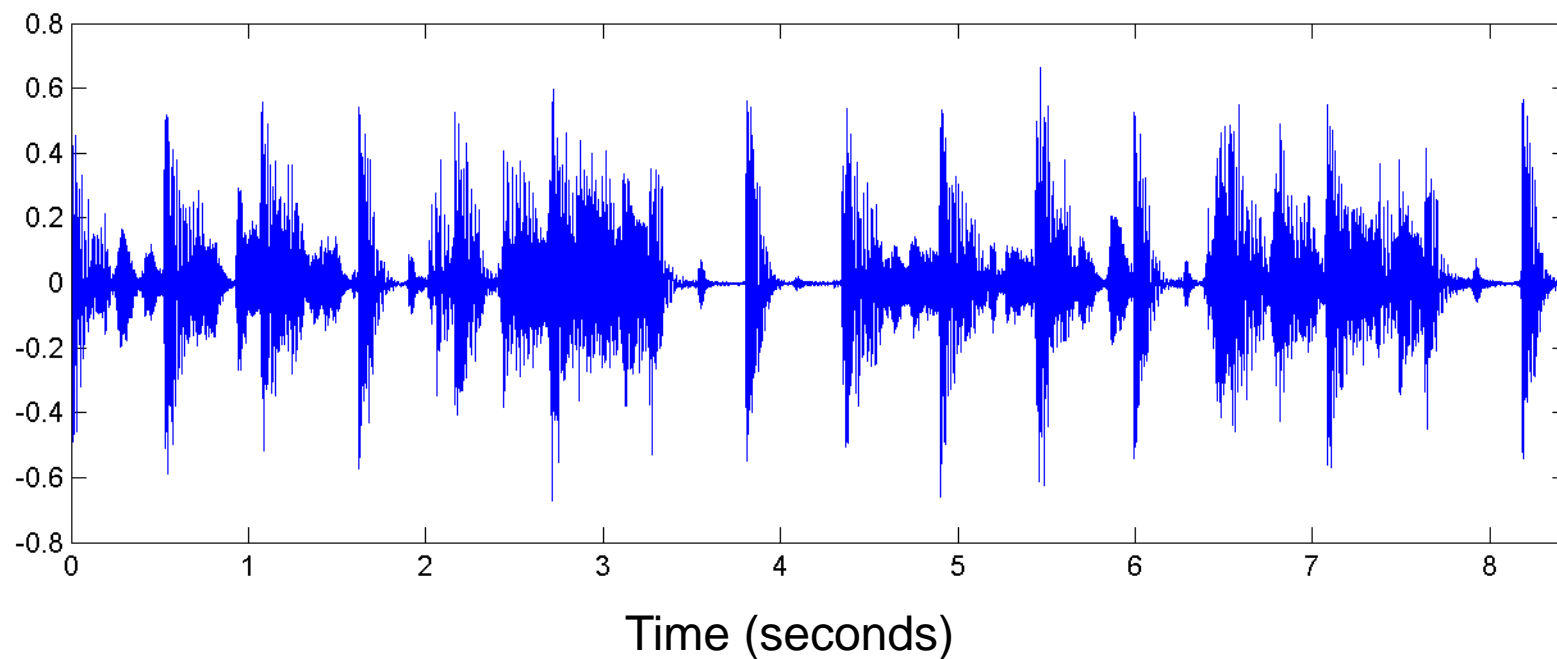
- Finding start times of perceptually relevant acoustic events in music signal
- Onset is the time position where a note is played
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 - timbre



Onset Detection (Energy-Based)

Steps

Waveform

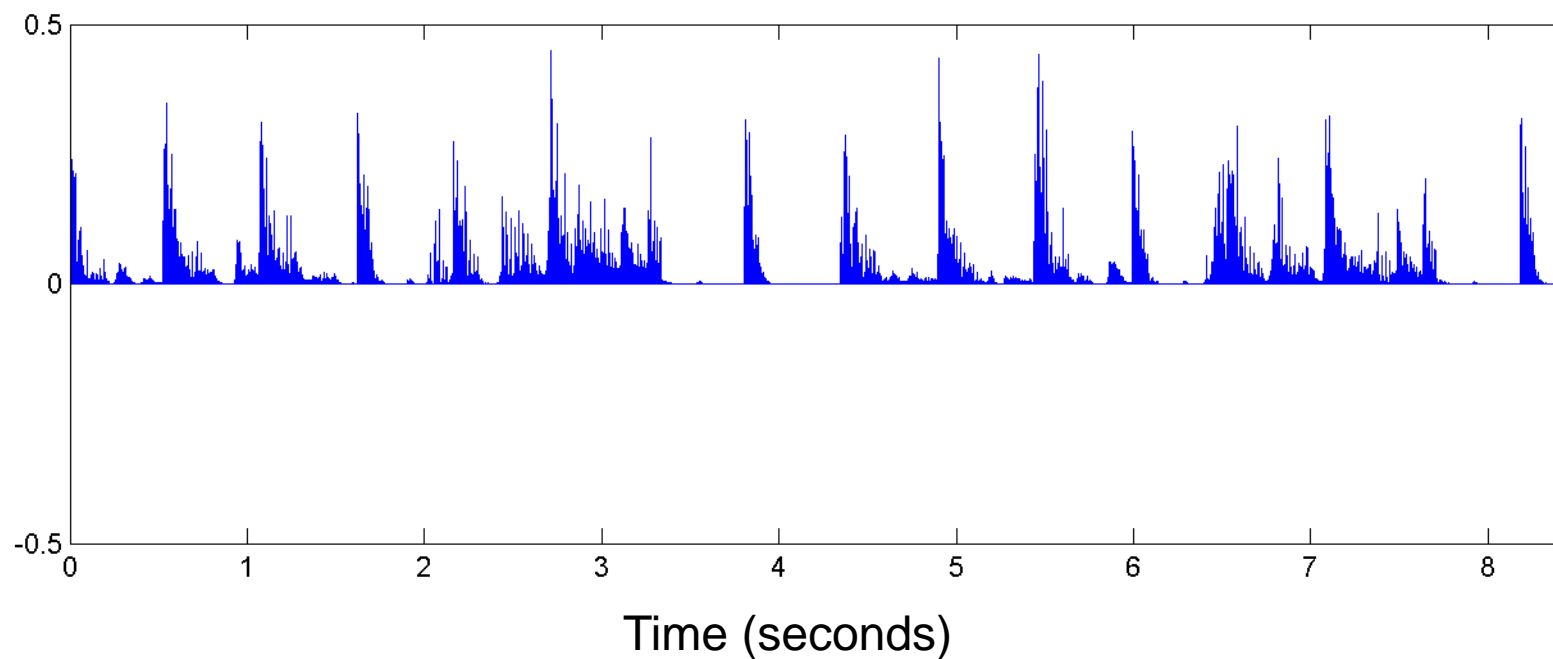


Onset Detection (Energy-Based)

Steps

1. Amplitude squaring

Squared waveform

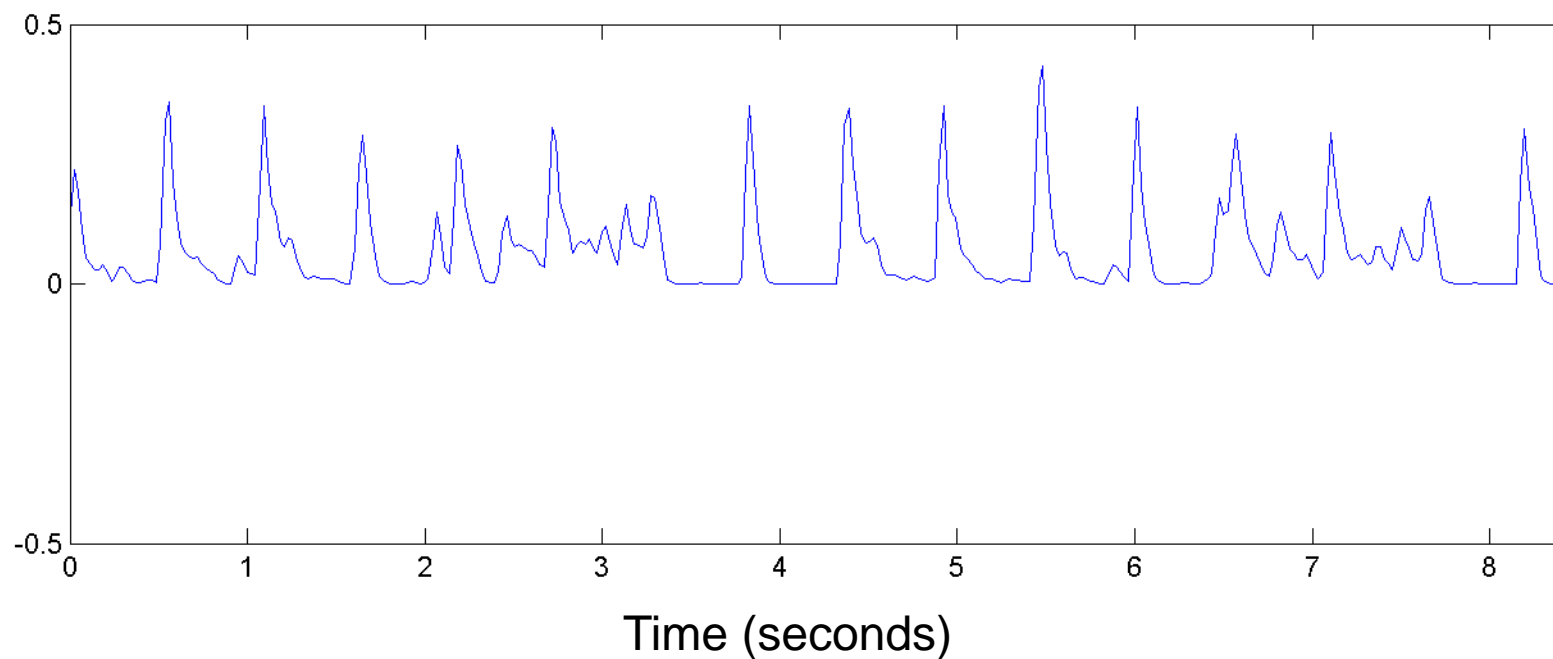


Onset Detection (Energy-Based)

Steps

1. Amplitude squaring
2. Windowing

Energy envelope



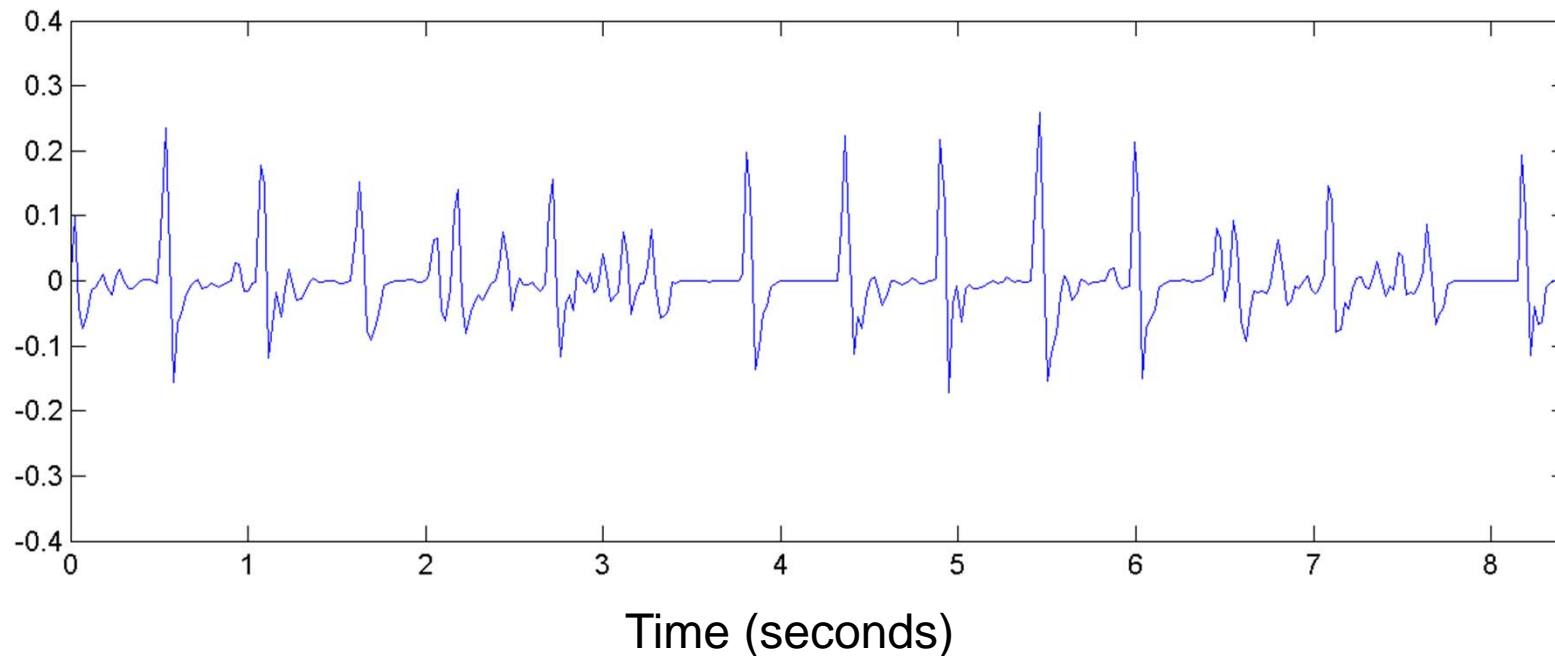
Onset Detection (Energy-Based)

Steps

1. Amplitude squaring
2. Windowing
3. Differentiation

Capturing energy changes

Differentiated energy envelope



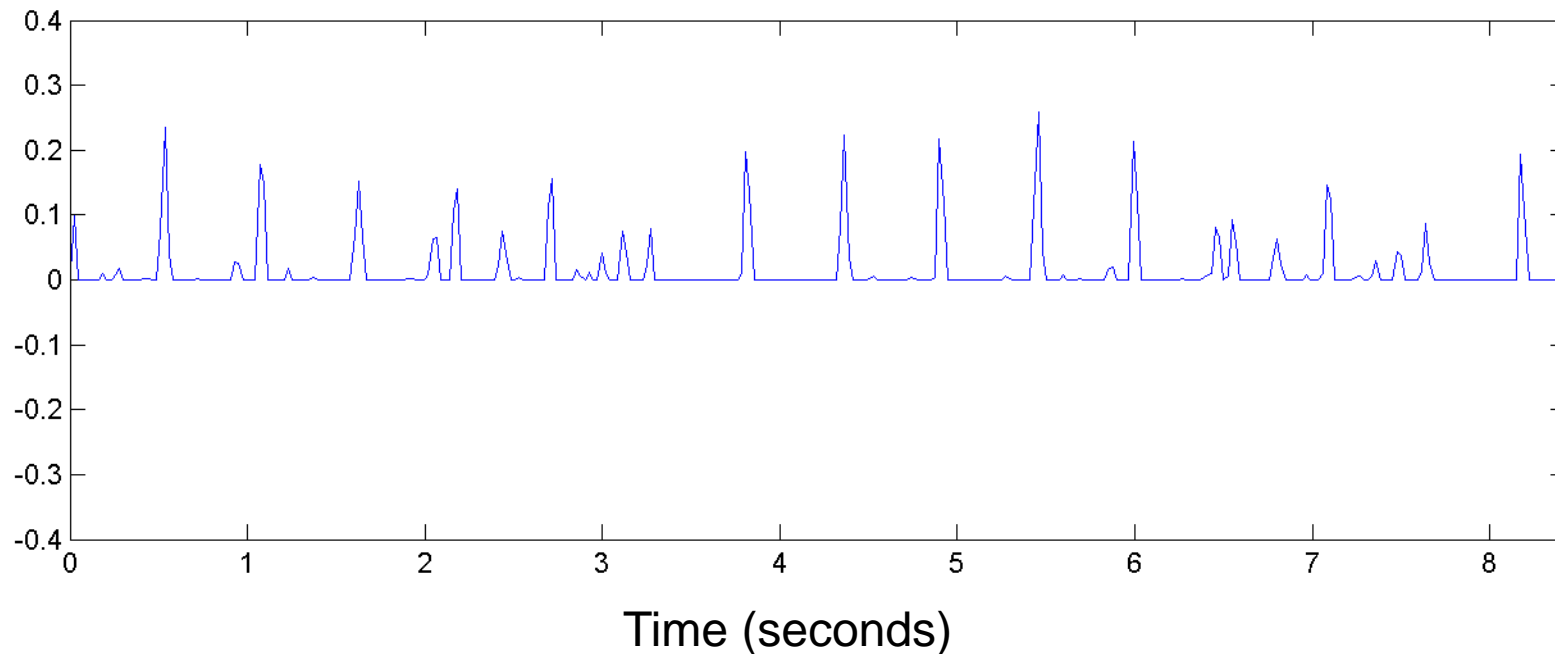
Onset Detection (Energy-Based)

Steps

1. Amplitude squaring
2. Windowing
3. Differentiation
4. Half wave rectification

Only energy increases are relevant for note onsets

Novelty curve

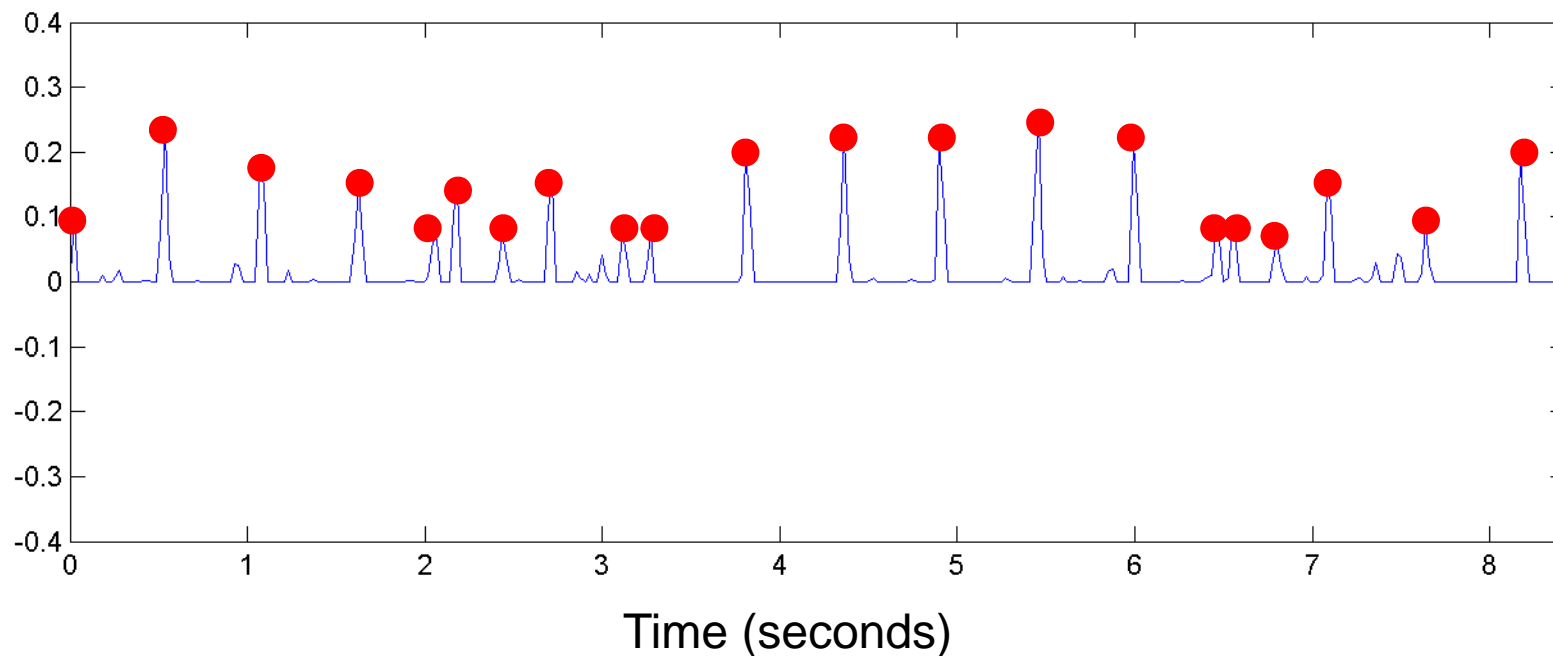


Onset Detection (Energy-Based)

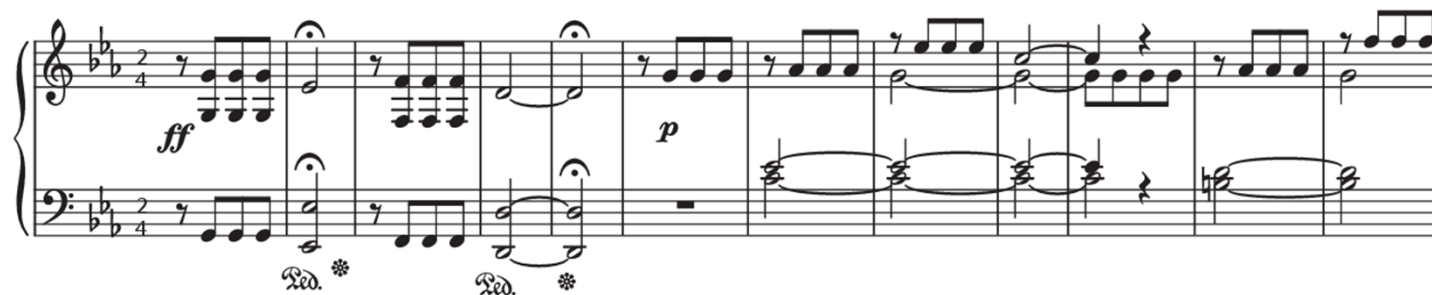
Steps

1. Amplitude squaring
2. Windowing
3. Differentiation
4. Half wave rectification
5. Peak picking

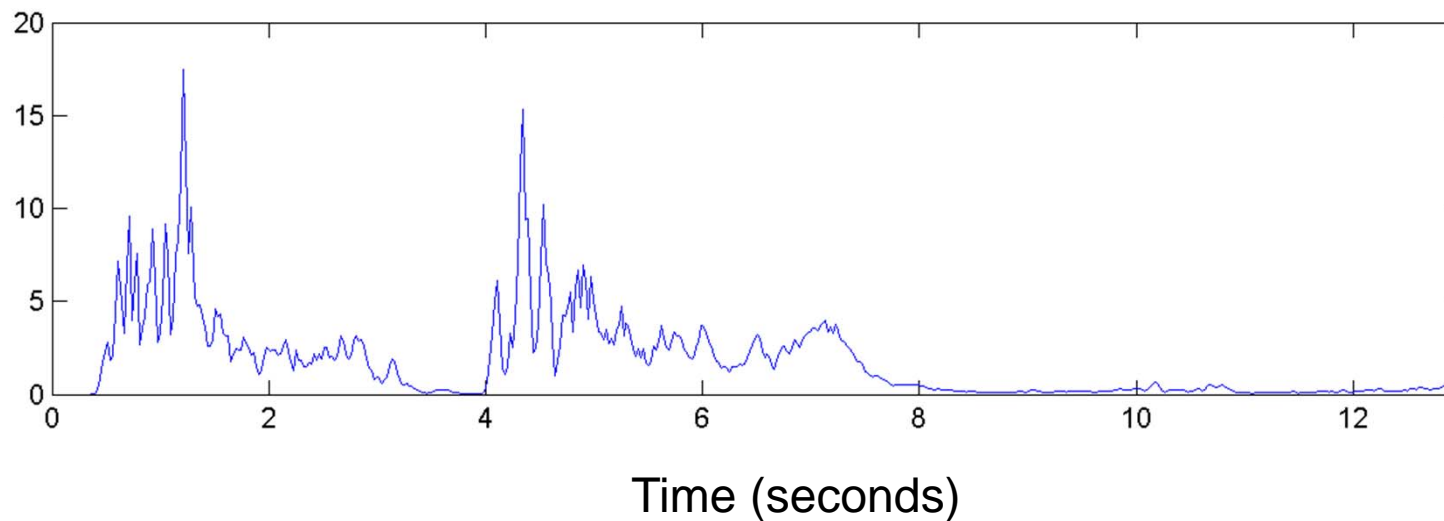
Peak positions indicate
note onset candidates



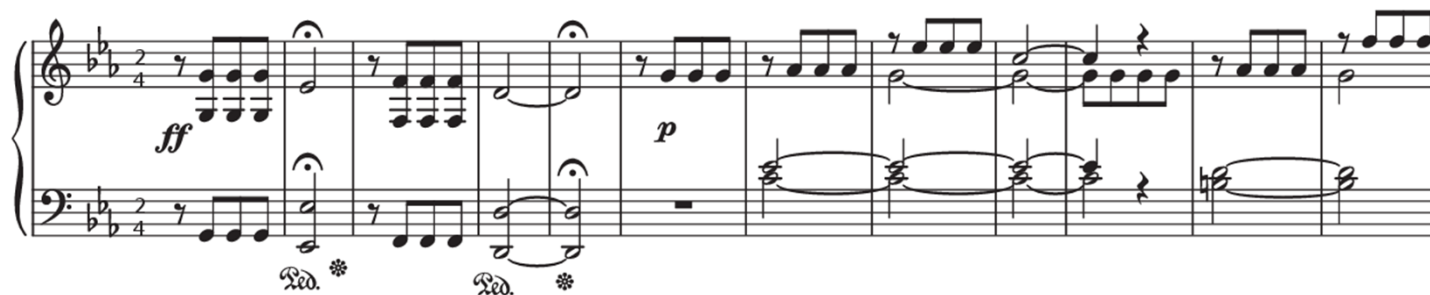
Onset Detection (Energy-Based)



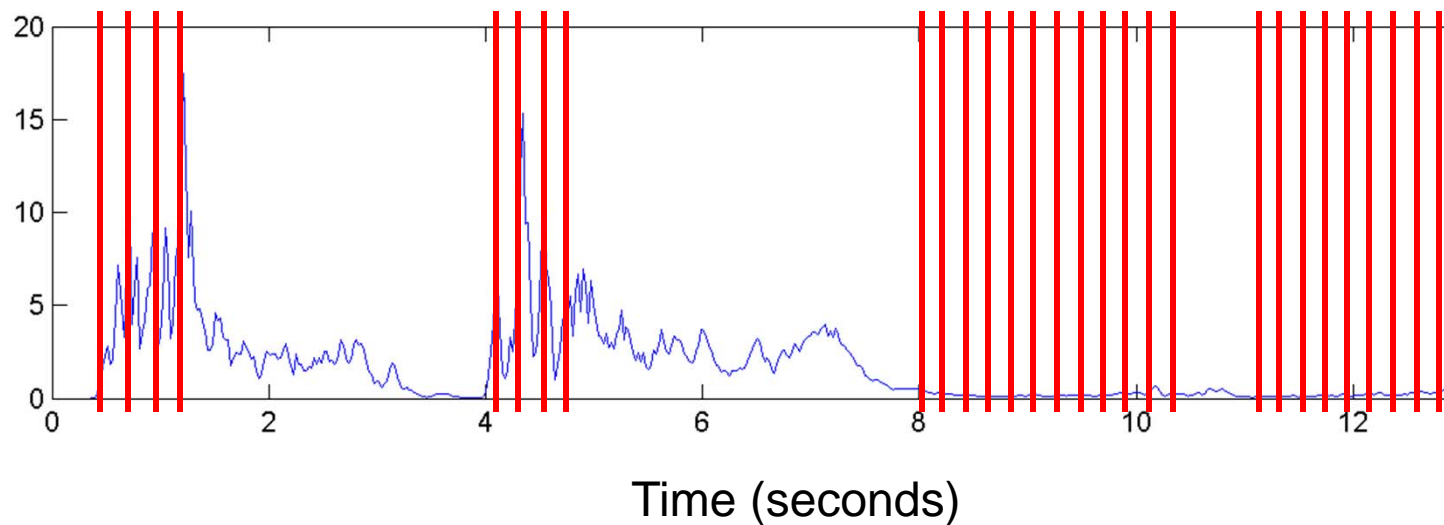
Energy envelope



Onset Detection (Energy-Based)



Energy envelope / note onsets positions

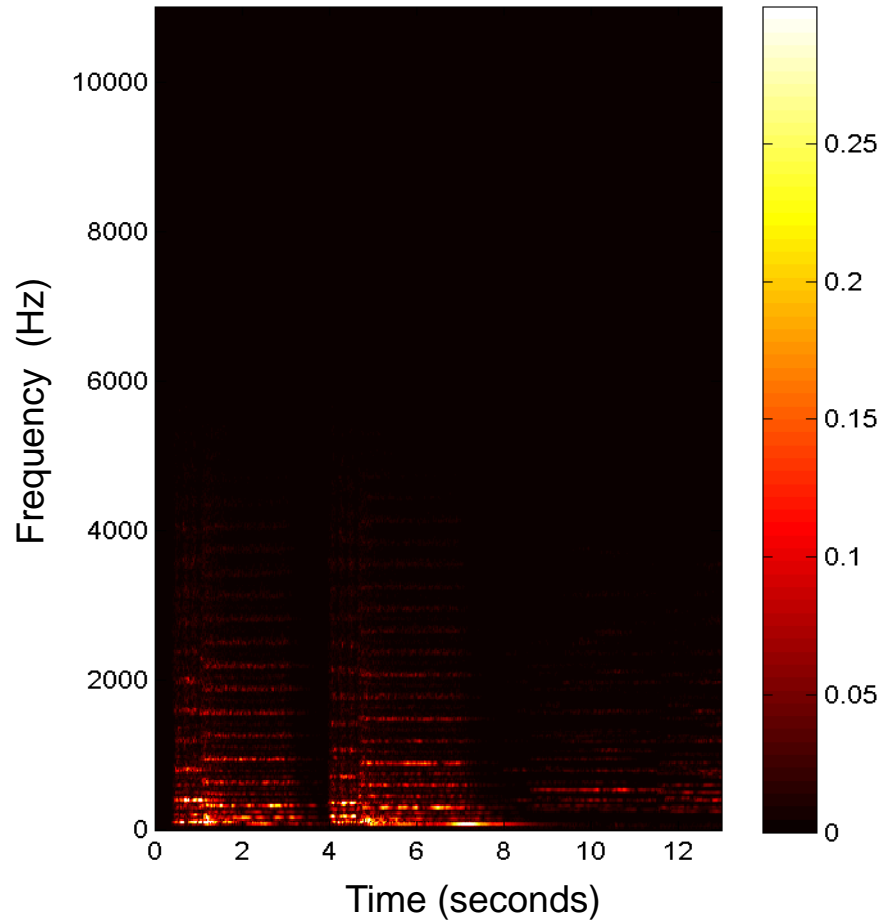


Onset Detection

- Energy curves often only work for percussive music
- Many instruments such as strings have weak note onsets
- No energy increase may be observable in complex sound mixtures
- More refined methods needed that capture
 - changes of spectral content
 - changes of pitch
 - changes of harmony

Onset Detection (Spectral-Based)

Magnitude spectrogram $|X|$



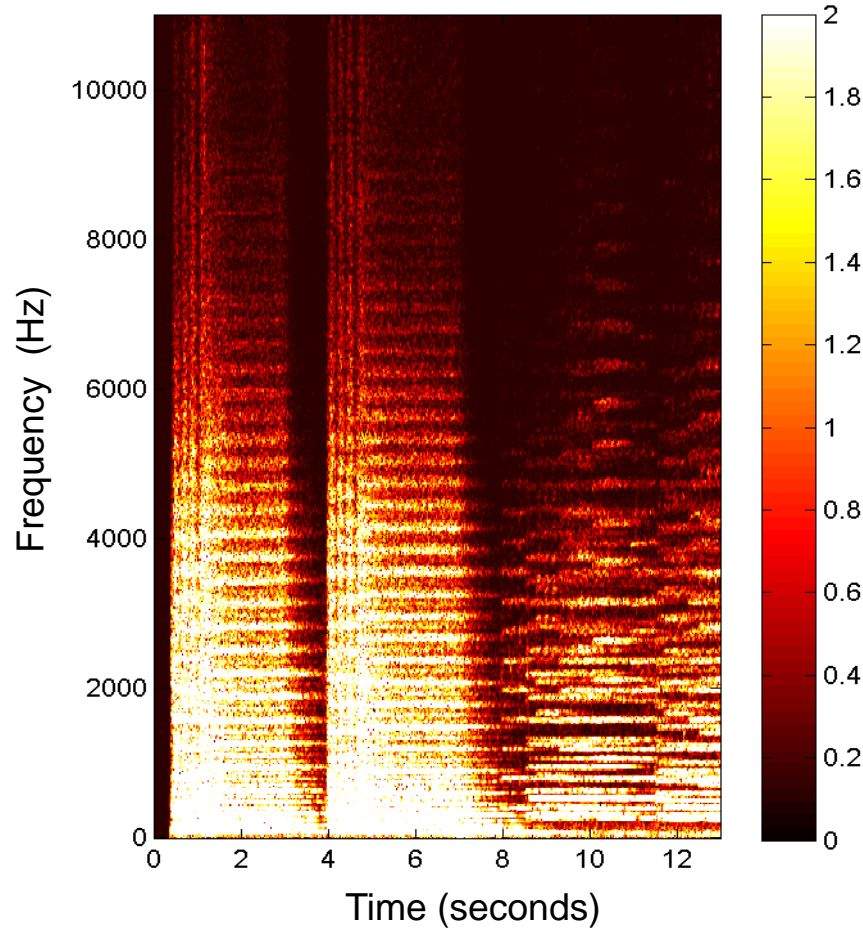
Steps:

1. Spectrogram

- *Aspects concerning pitch, harmony, or timbre are captured by spectrogram*
- *Allows for detecting local energy changes in certain frequency ranges*

Onset Detection (Spectral-Based)

Compressed spectrogram Y



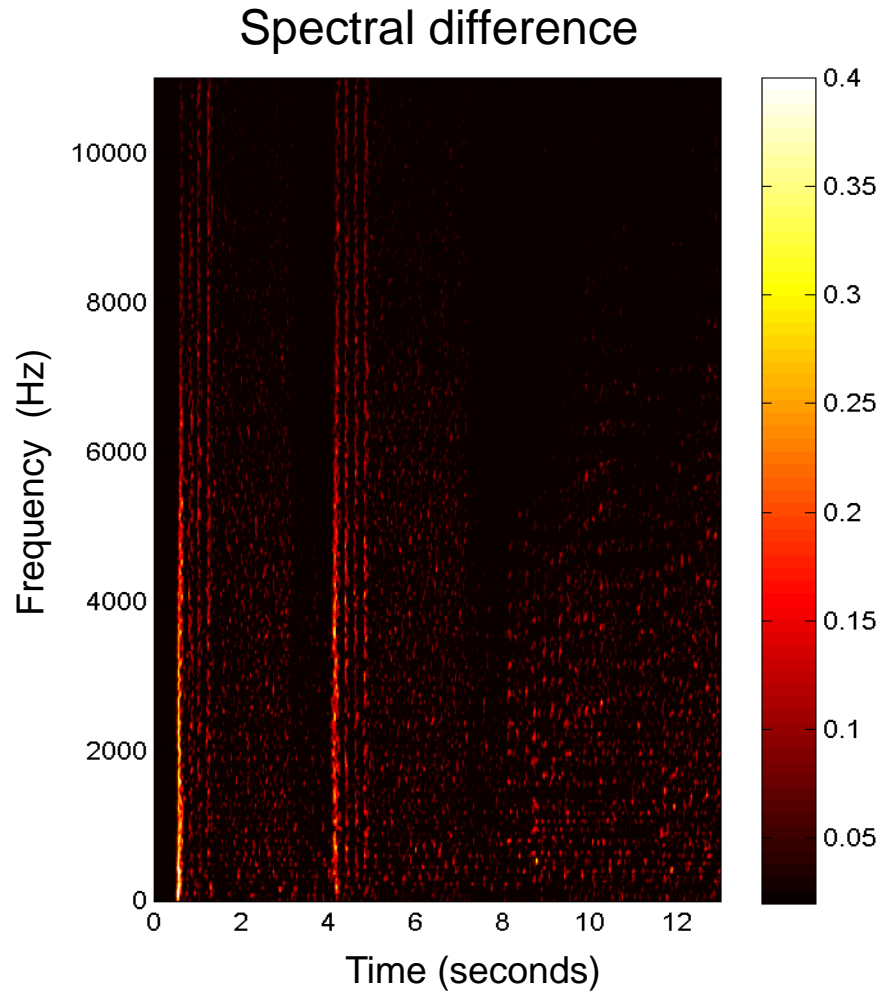
Steps:

1. Spectrogram
2. Logarithmic compression

$$Y = \log(1 + C \cdot |X|)$$

- *Accounts for the logarithmic sensation of sound intensity*
- *Dynamic range compression*
- *Enhancement of low-intensity values*
- *Often leading to enhancement of high-frequency spectrum*

Onset Detection (Spectral-Based)

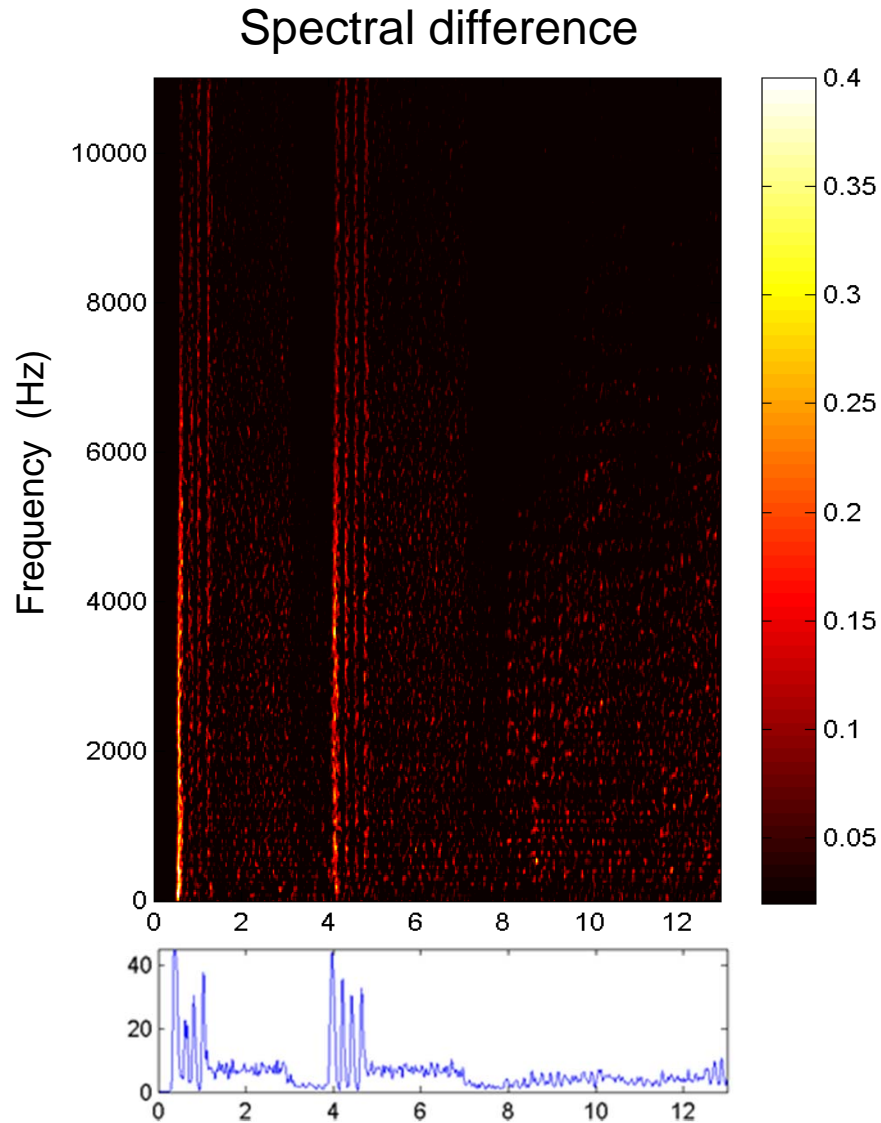


Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation

- *First-order temporal difference*
- *Captures changes of the spectral content*
- *Only positive intensity changes considered*

Onset Detection (Spectral-Based)



Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation

- *Frame-wise accumulation of all positive intensity changes*
- *Encodes changes of the spectral content*

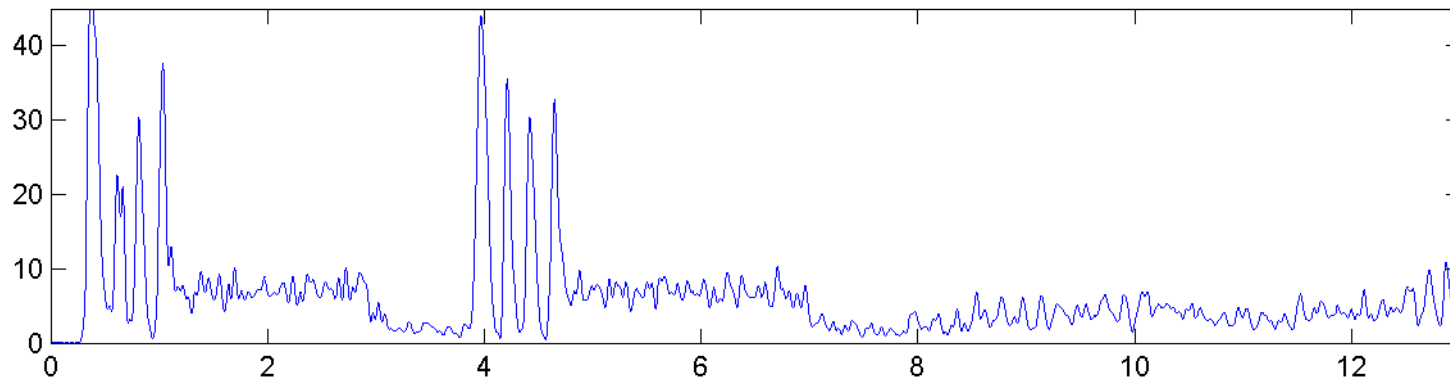
Novelty curve

Onset Detection (Spectral-Based)

Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation

Novelty curve



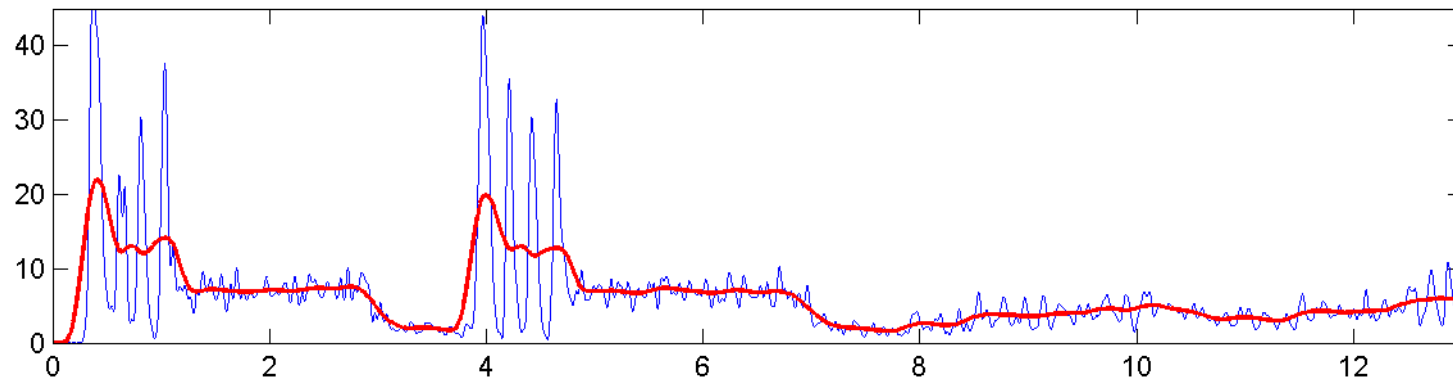
Onset Detection (Spectral-Based)

Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation
5. Normalization

Novelty curve

Substraction of local average

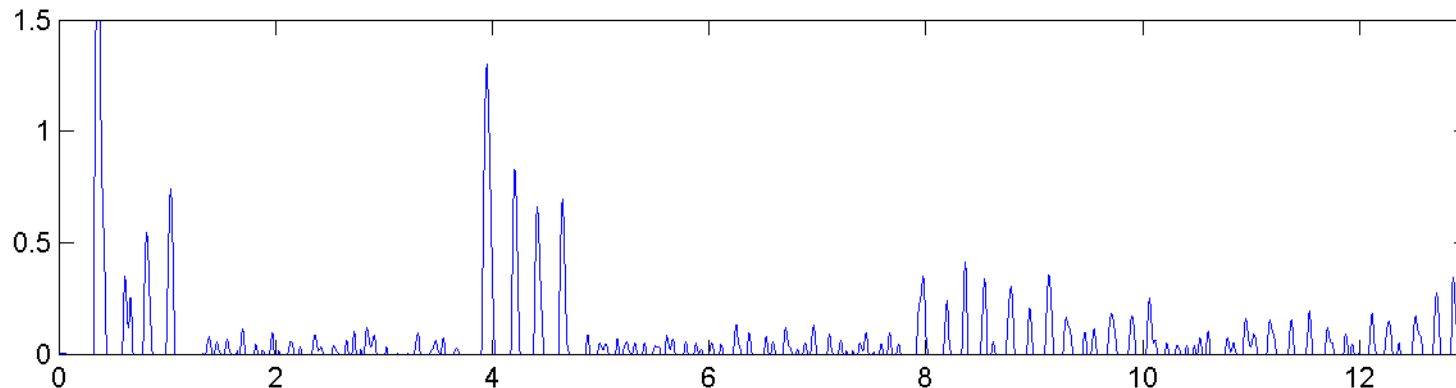


Onset Detection (Spectral-Based)

Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation
5. Normalization

Normalized novelty curve

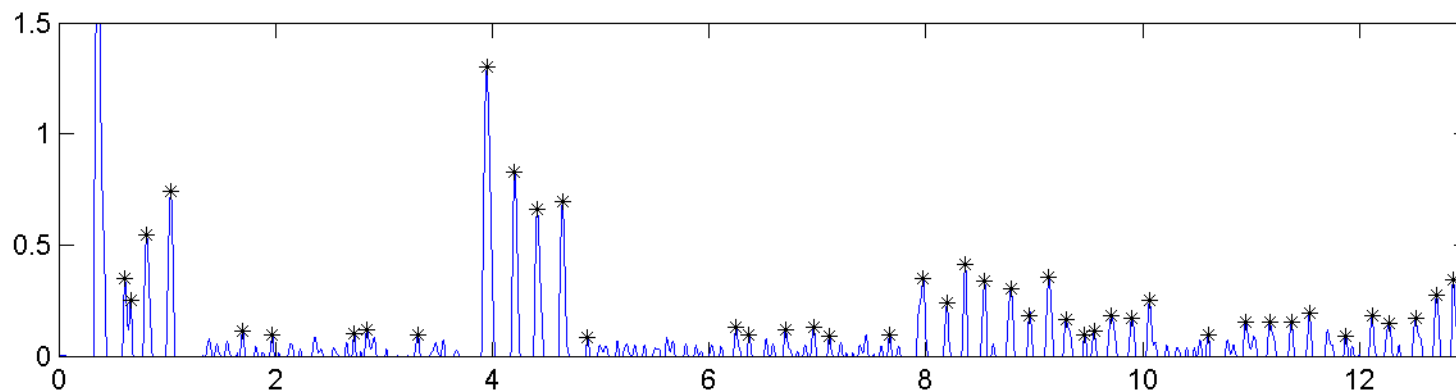


Onset Detection (Spectral-Based)

Steps:

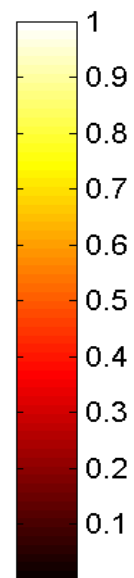
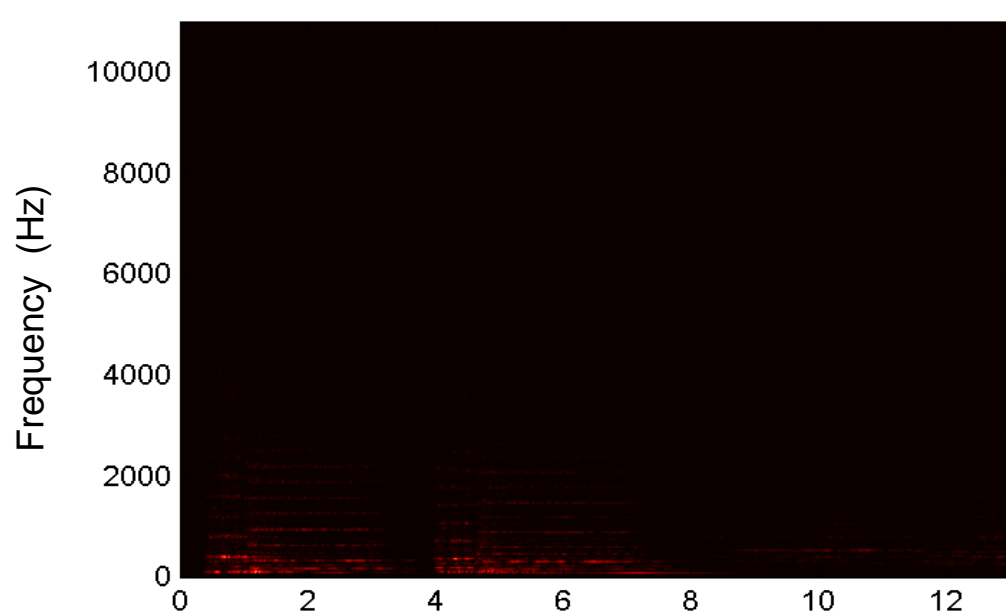
1. Spectrogram
2. Logarithmic compression
3. Differentiation
4. Accumulation
5. Normalization
6. Peak picking

Normalized novelty curve

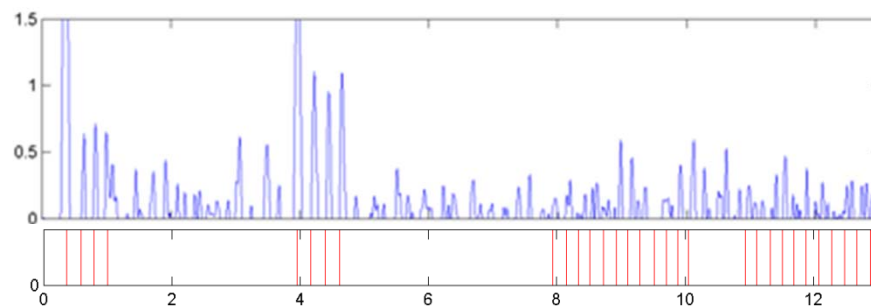


Onset Detection (Spectral-Based)

Logarithmic compression is essential



$$|X|$$



Novelty curve

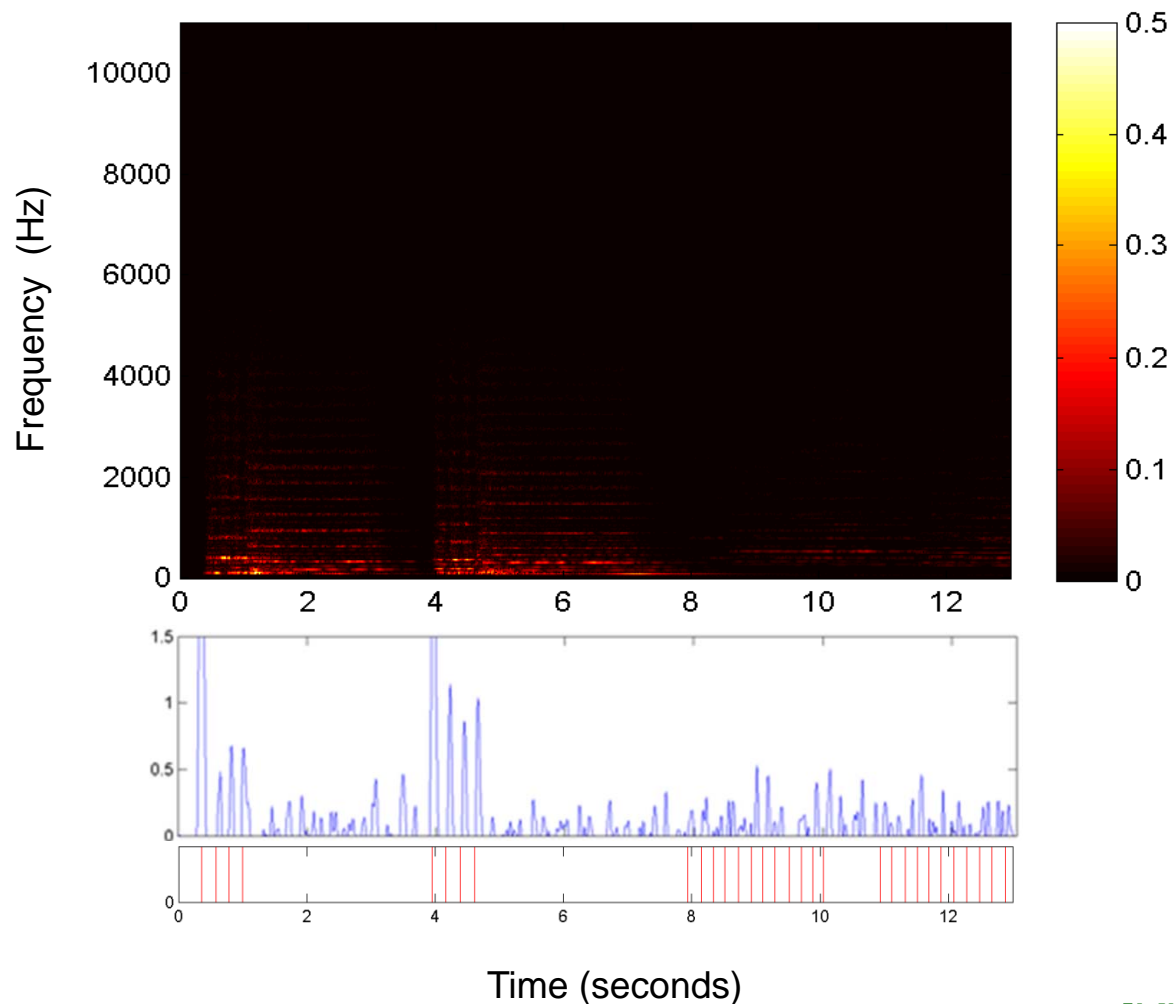
Ground-truth onsets

Time (seconds)

[Klapuri et al., IEEE-TASLP 2006]

Onset Detection (Spectral-Based)

Logarithmic compression is essential



$$Y = \log(1 + C \cdot |X|)$$

$$C = 1$$

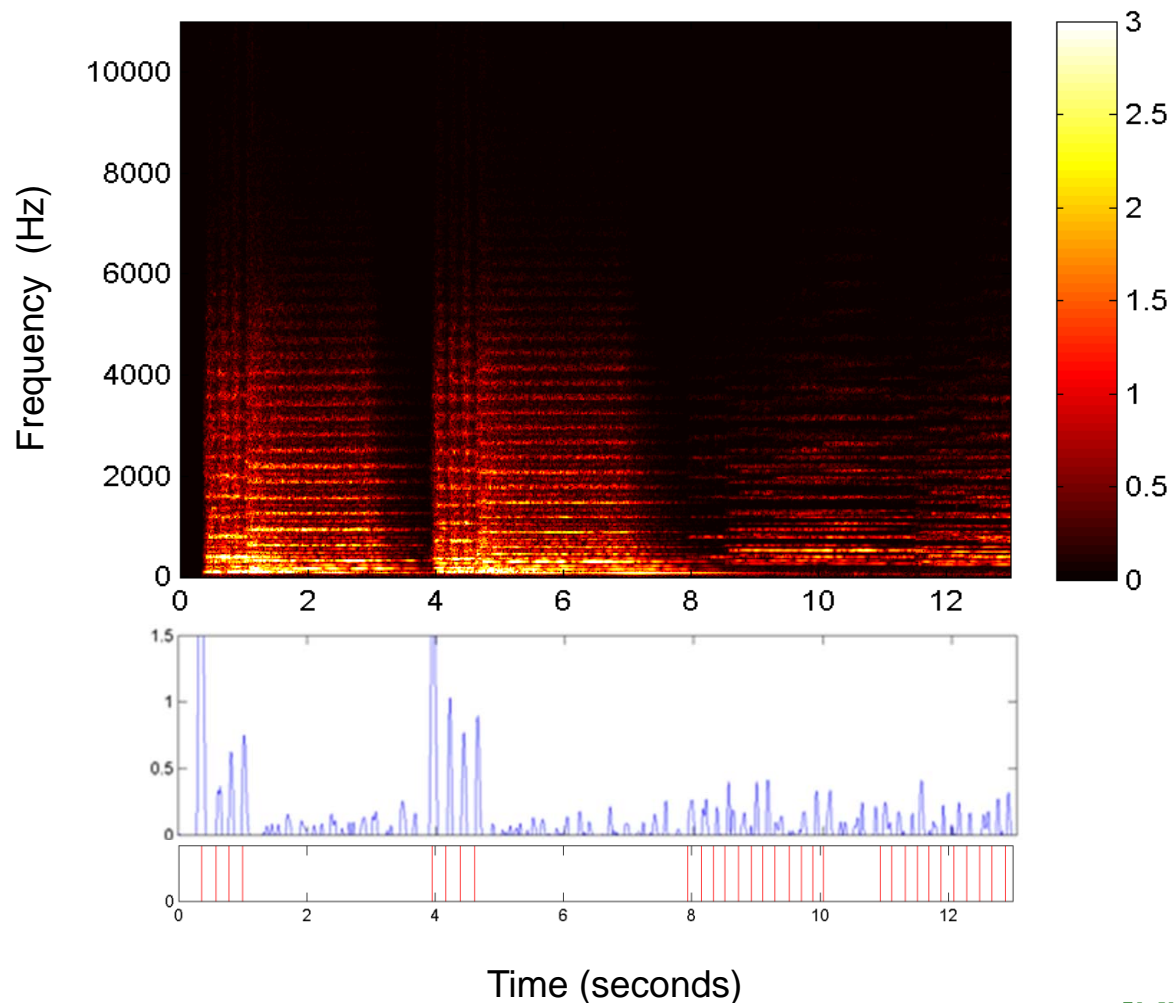
Novelty curve

Ground-truth onsets

[Klapuri et al., IEEE-TASLP 2006]

Onset Detection (Spectral-Based)

Logarithmic compression is essential



$$Y = \log(1 + C \cdot |X|)$$

$$C = 10$$

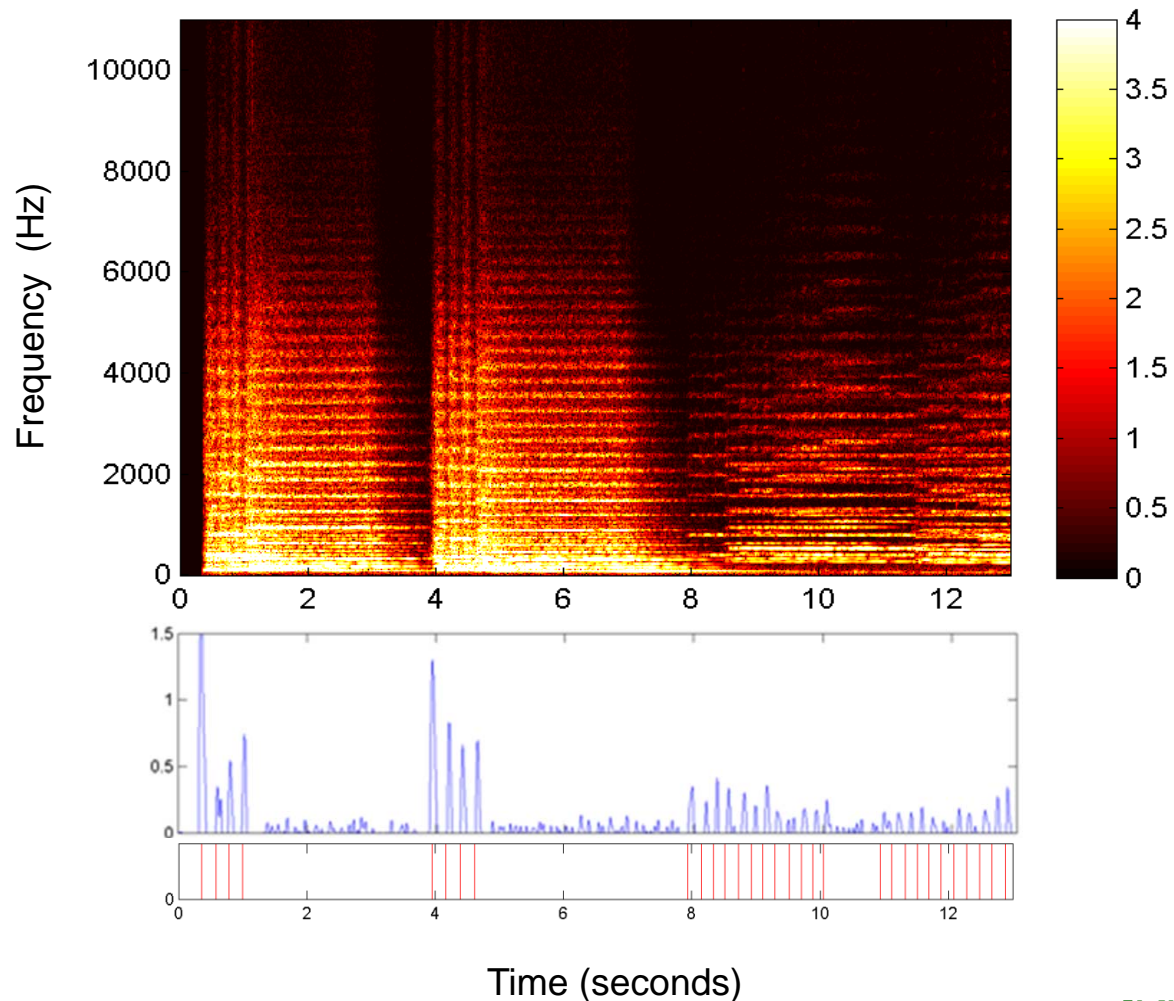
Novelty curve

Ground-truth onsets

[Klapuri et al., IEEE-TASLP 2006]

Onset Detection (Spectral-Based)

Logarithmic compression is essential



$$Y = \log(1 + C \cdot |X|)$$

$$C = 1000$$

Novelty curve

Ground-truth onsets

[Klapuri et al., IEEE-TASLP 2006]

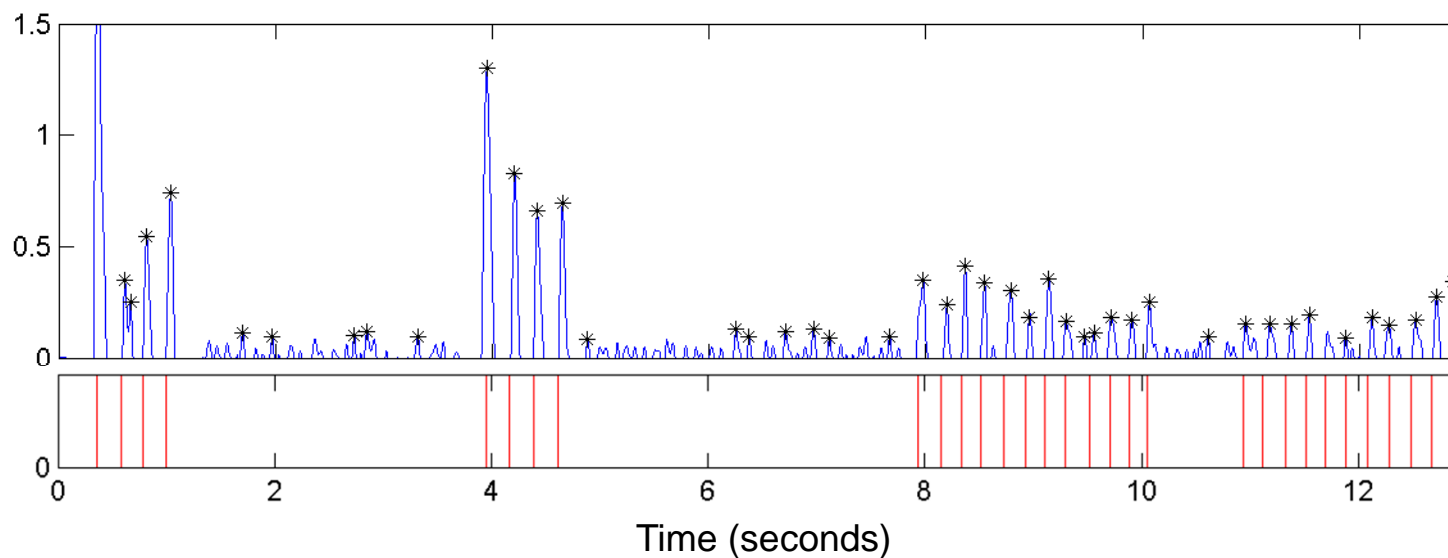
Onset Detection (Spectral-Based)

- Spectrogram $X = (X(t, k))_{t, k}$ $t \in [1 : T]$
 $k \in [1 : K]$
- Compressed Spectrogram $Y := \log(1 + C \cdot |X|)$ $C > 1$.
- Novelty curve $\Delta : [1 : T - 1] \rightarrow \mathbb{R}$:

$$\Delta(t) := \sum_{k=1}^K |Y(t+1, k) - Y(t, k)|_{\geq 0}$$

Onset Detection

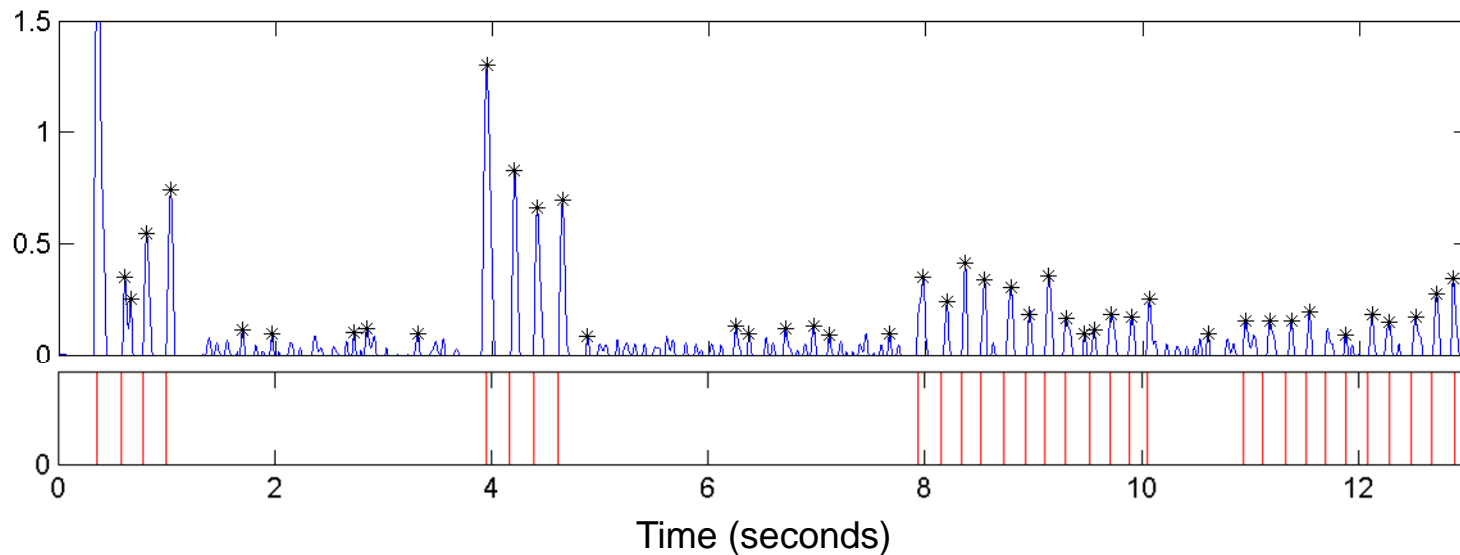
Peak picking



- Peaks of the novelty curve indicate note onset candidates

Onset Detection

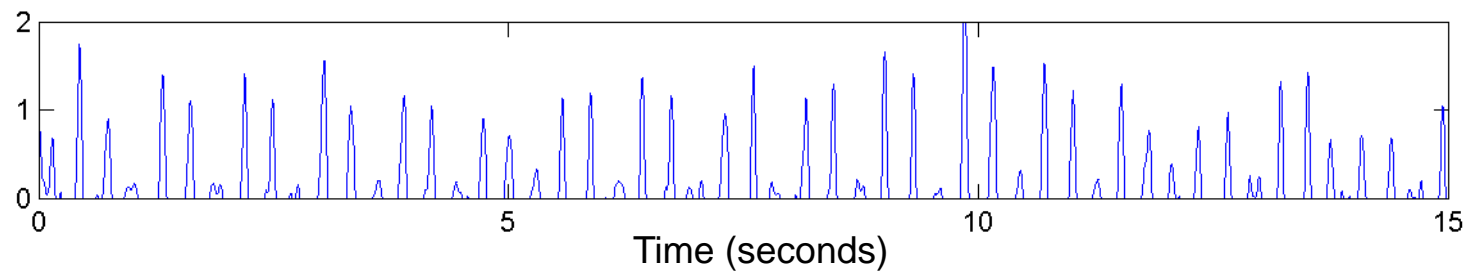
Peak picking



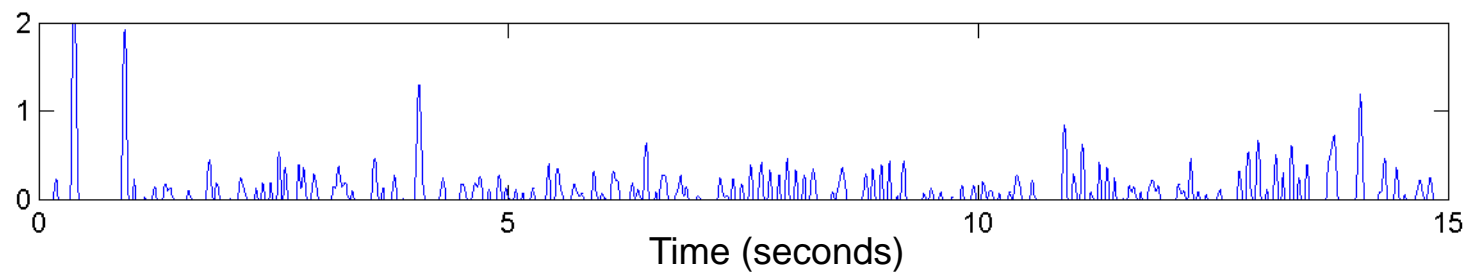
- Peaks of the novelty curve indicate note onset candidates
- In general many spurious peaks
- Usage of local thresholding techniques
- **Peak-picking very fragile step in particular for soft onsets**

Onset Detection

Shostakovich – 2nd Waltz



Borodin – String Quartet No. 2



Onset Detection

Drumbeat



Going Home



Lyphard melodie



Por una cabeza



Donau



Beat and Tempo

What is a beat?

- Steady pulse that drives music forward and provides the temporal framework of a piece of music
- Sequence of perceived pulses that are equally spaced in time
- The pulse a human taps along when listening to the music

[Parncutt 1994]

[Sethares 2007]

[Large/Palmer 2002]

[Lerdahl/ Jackendoff 1983]

[Fitch/ Rosenfeld 2007]

The term **tempo** then refers to the speed of the pulse.

Beat and Tempo

Strategy

- Analyze the novelty curve with respect to reoccurring or quasi-periodic patterns
- Avoid the explicit determination of note onsets (no peak picking)

Beat and Tempo

Strategy

- Analyze the novelty curve with respect to reoccurring or quasi-periodic patterns
- Avoid the explicit determination of note onsets (no peak picking)

[Scheirer, JASA 1998]

Methods

- Comb-filter methods
- Autocorrelation
- Fourier transform

[Ellis, JNMR 2007]

[Davies/Plumbley, IEEE-TASLP 2007]

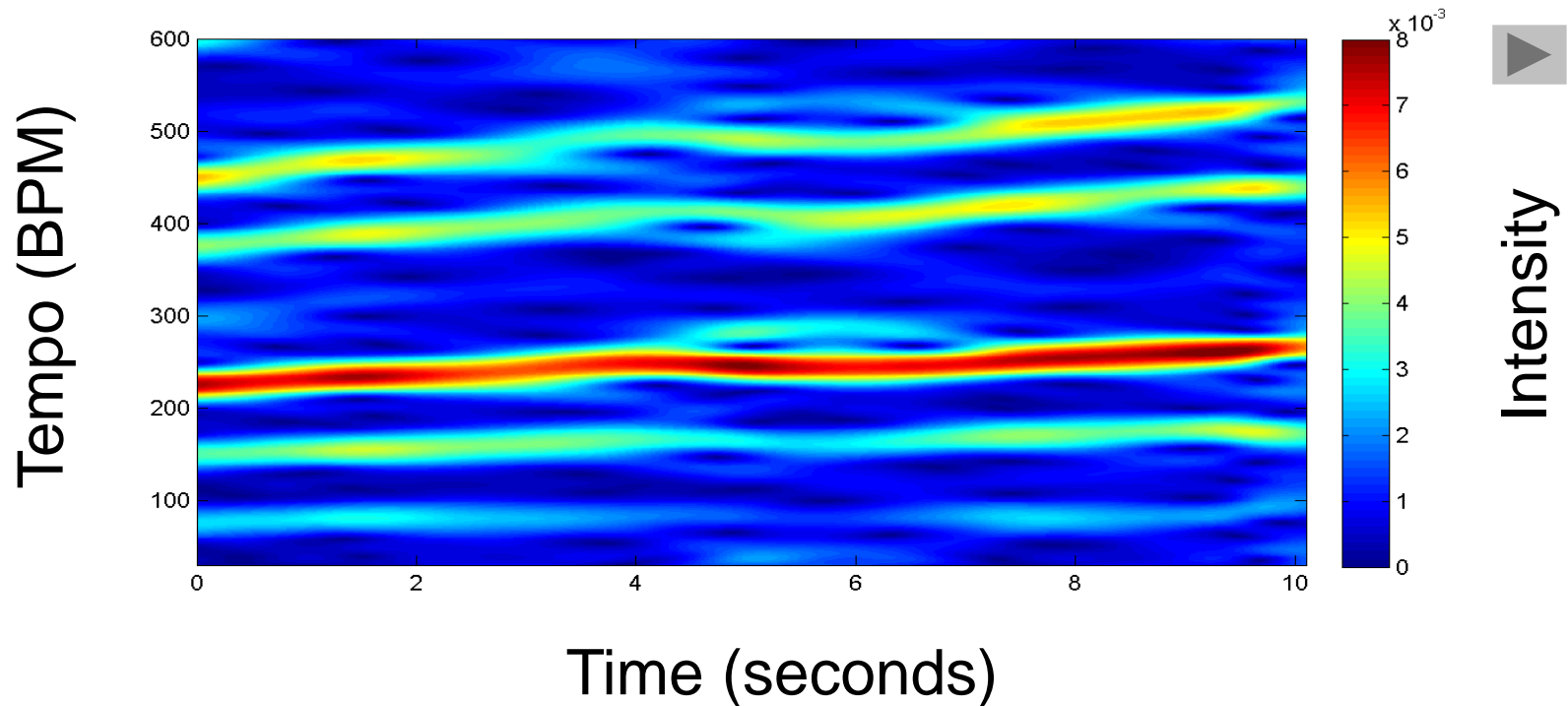
[Peeters, JASP 2007]

[Grosche/Müller, ISMIR 2009]

[Grosche/Müller, IEEE-TASLP 2011]

Tempogram

Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.



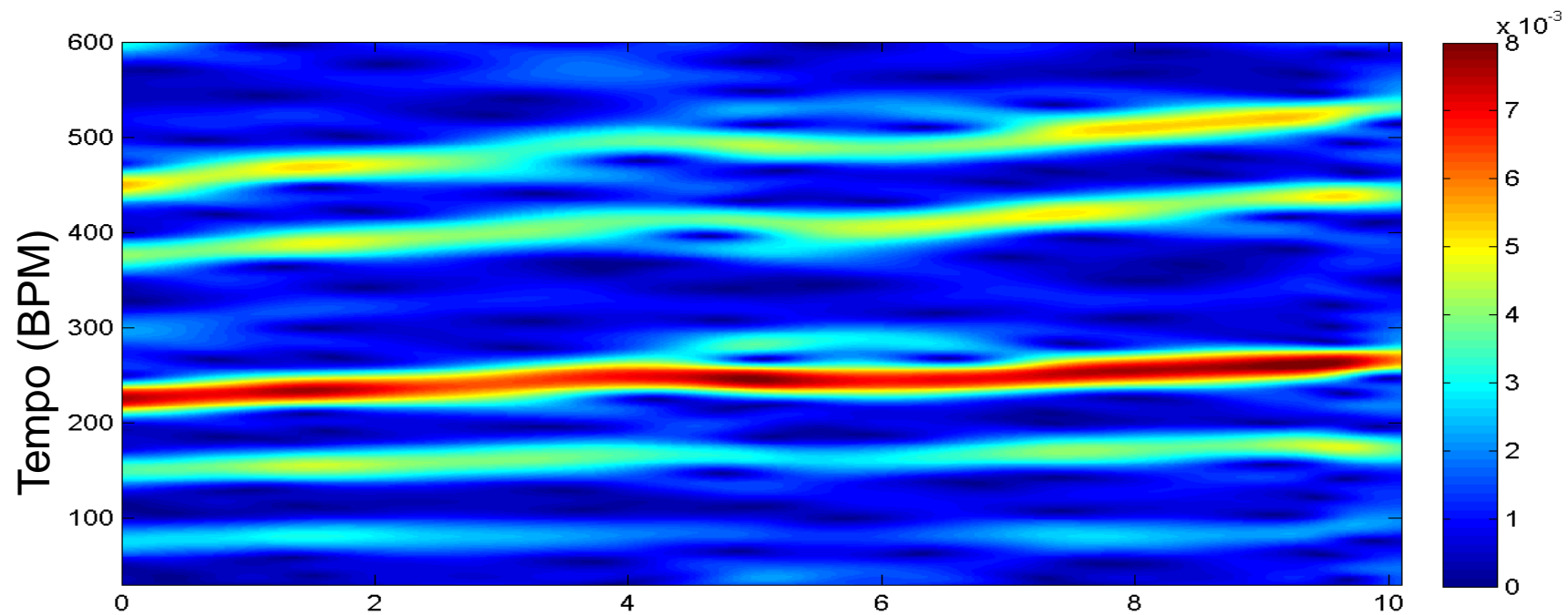
Tempogram (Fourier)

Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.

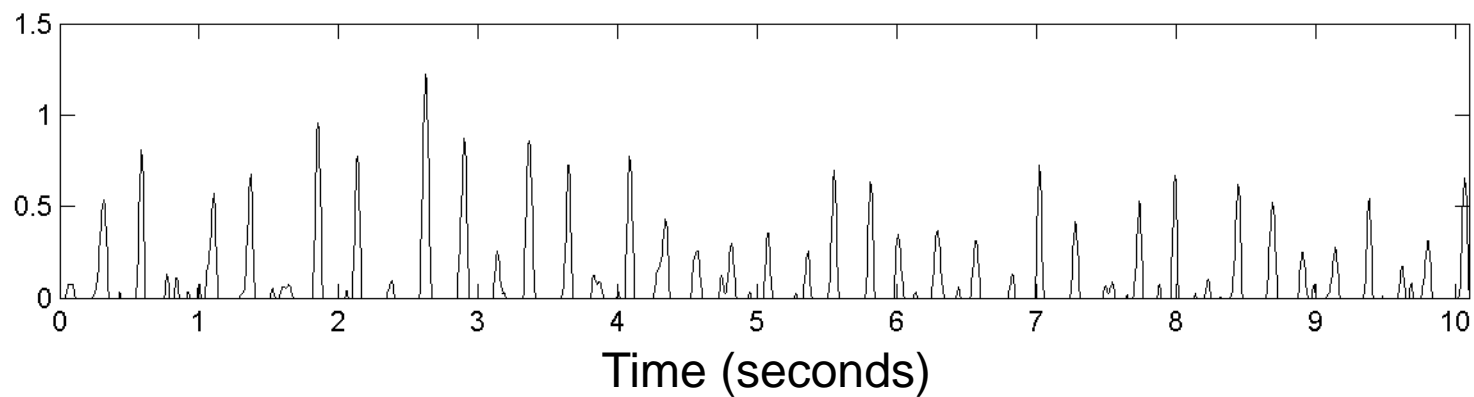
Fourier-based method

- Compute a spectrogram (STFT) of the novelty curve
- Convert frequency axis (given in Hertz) into tempo axis (given in BPM)
- Magnitude spectrogram indicates local tempo

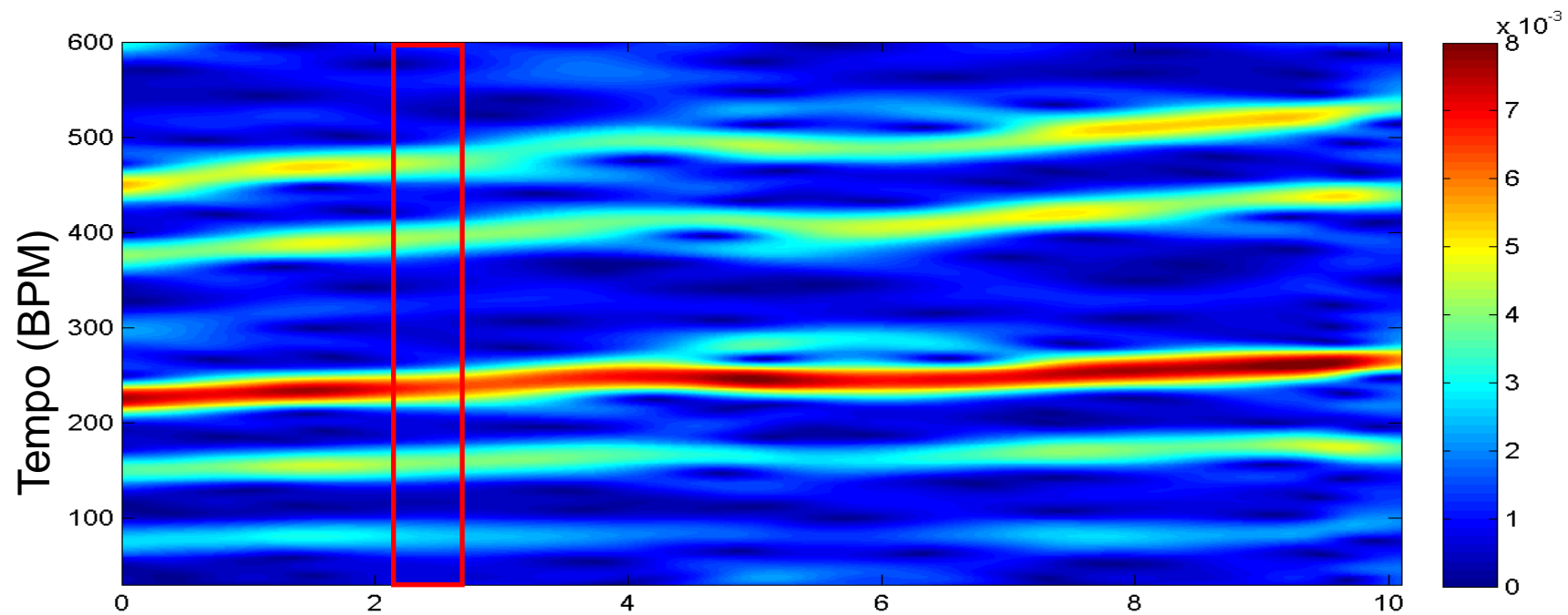
Tempogram (Fourier)



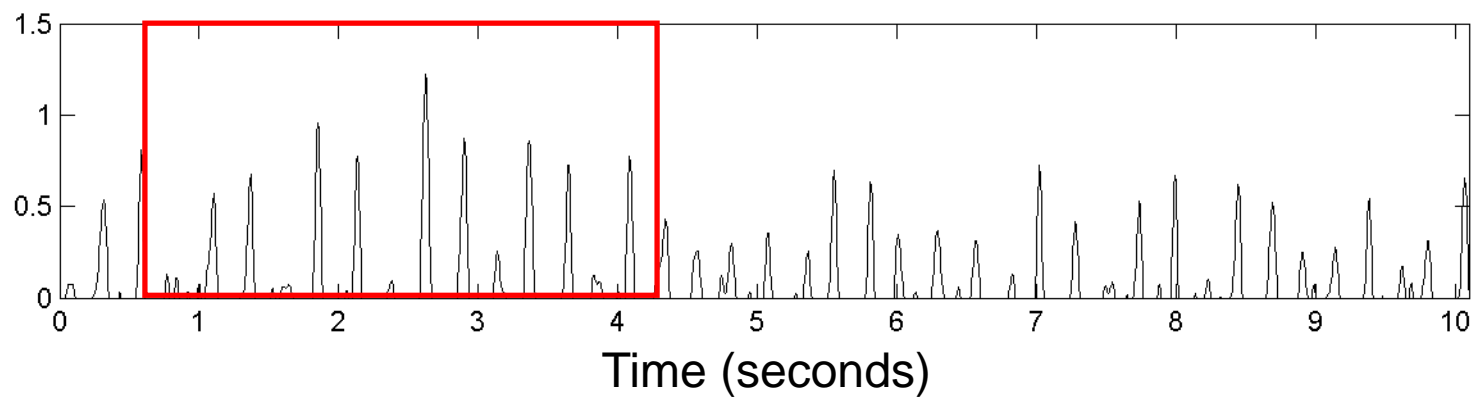
Novelty curve



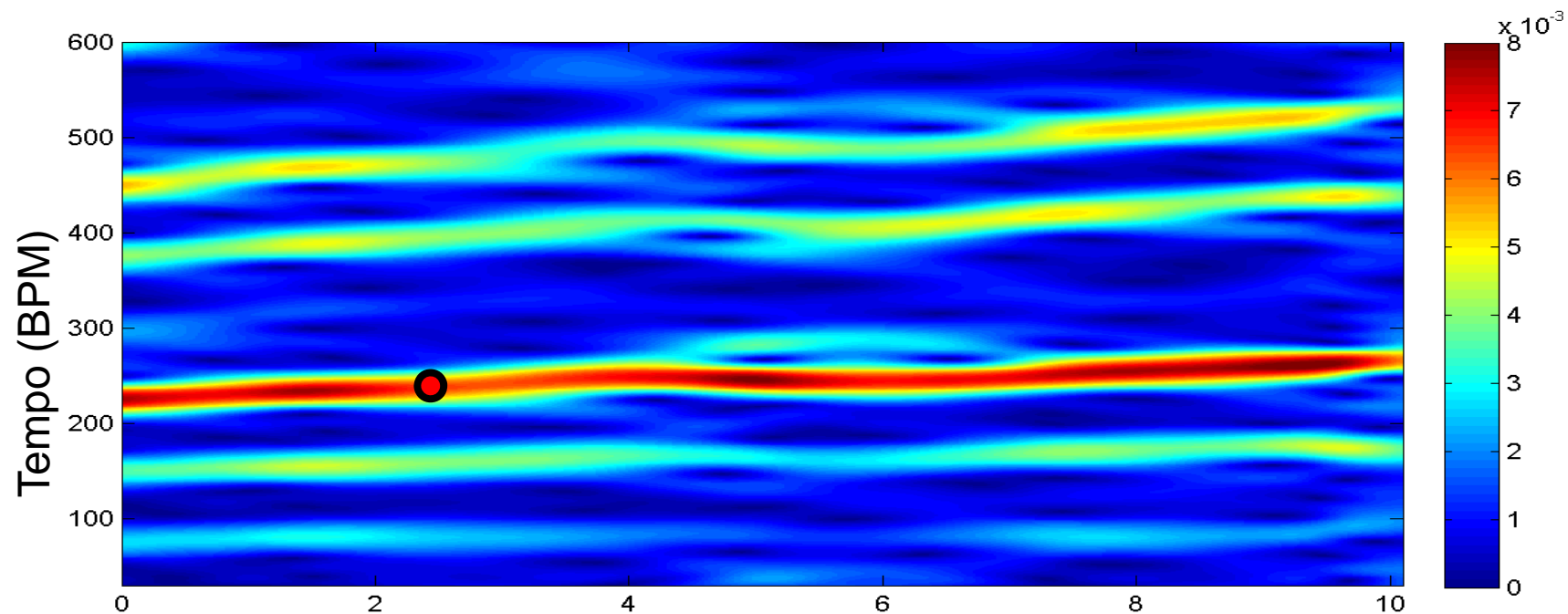
Tempogram (Fourier)



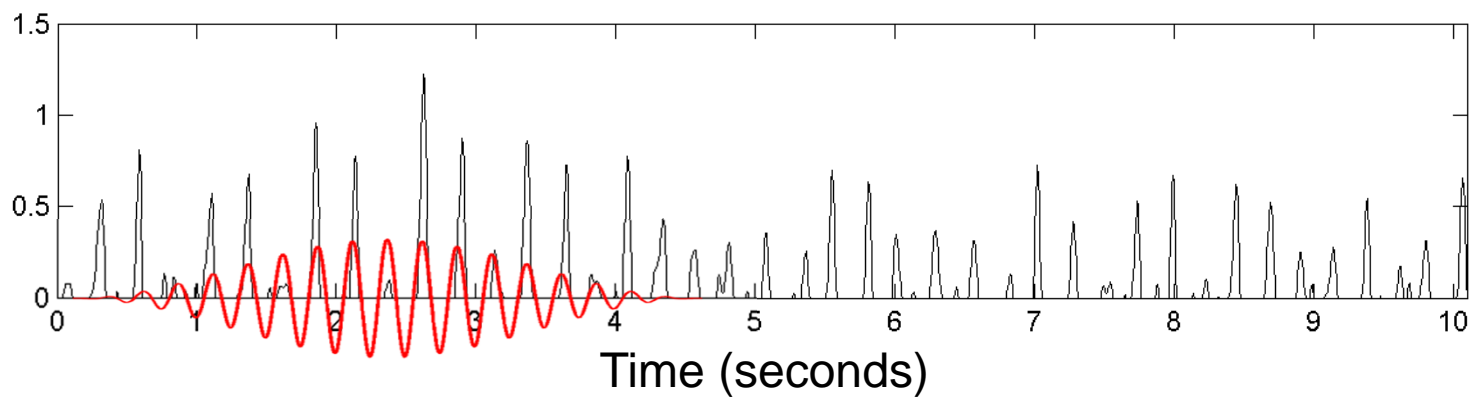
Novelty curve (local section)



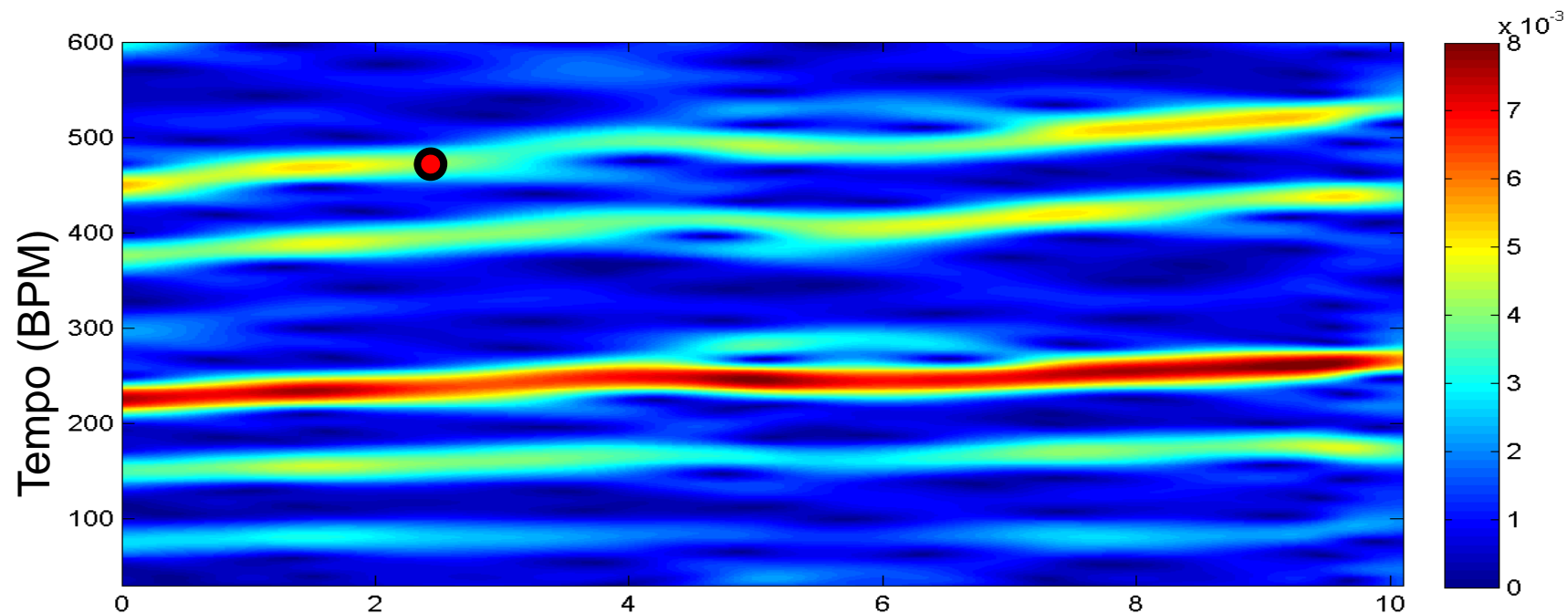
Tempogram (Fourier)



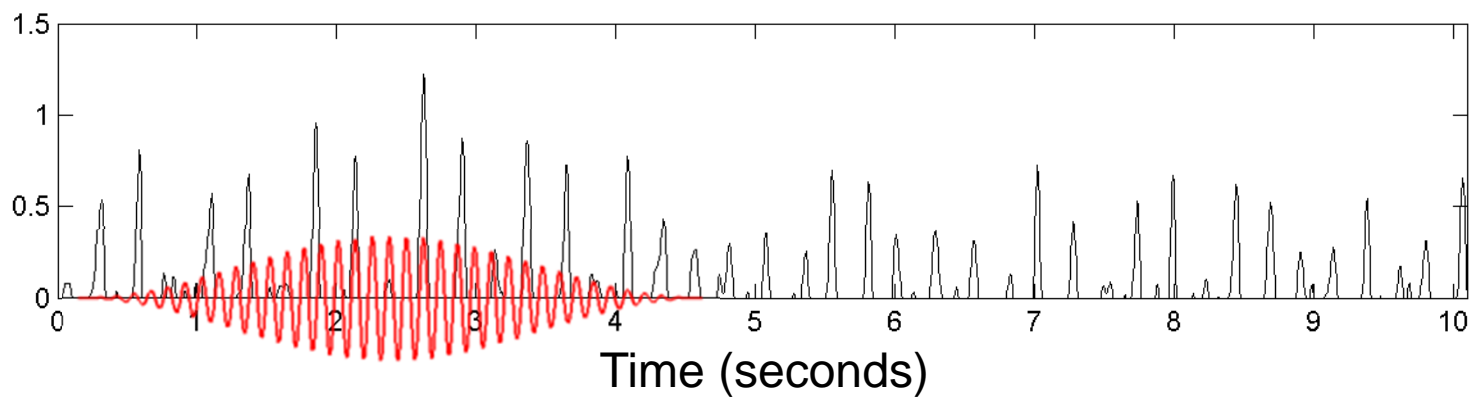
Windowed sinusoidal



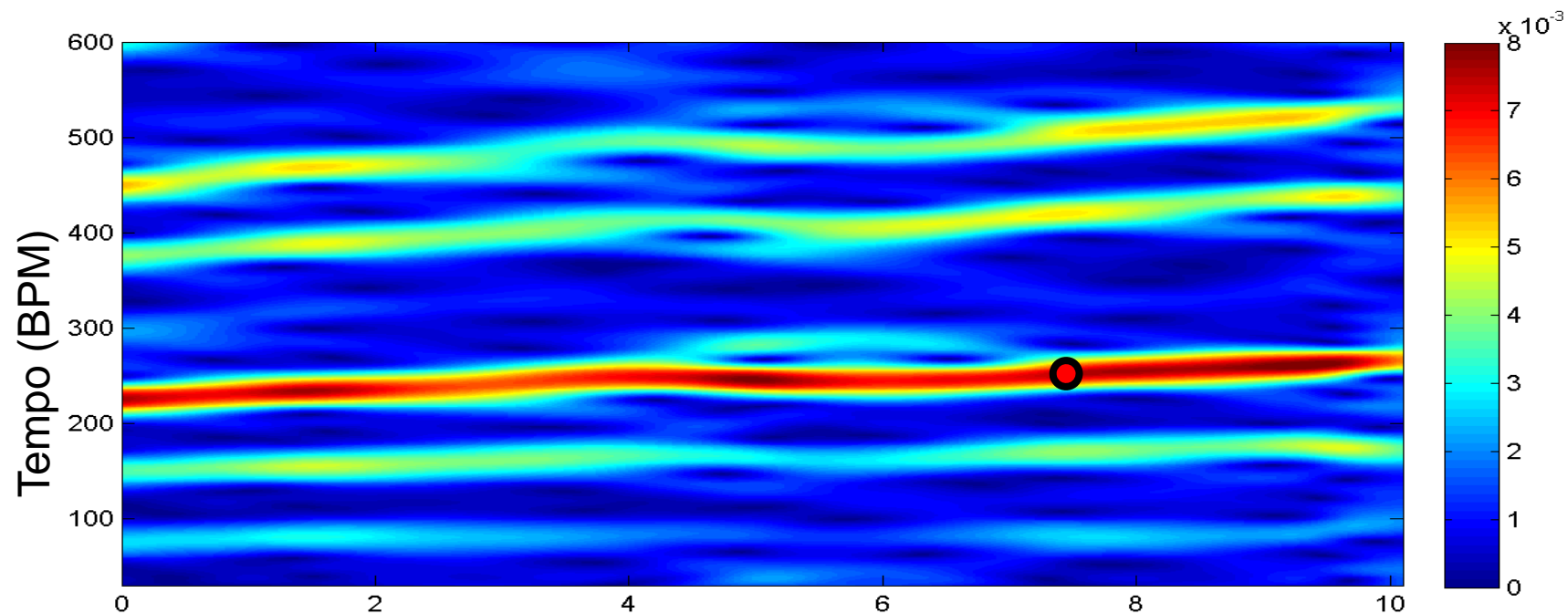
Tempogram (Fourier)



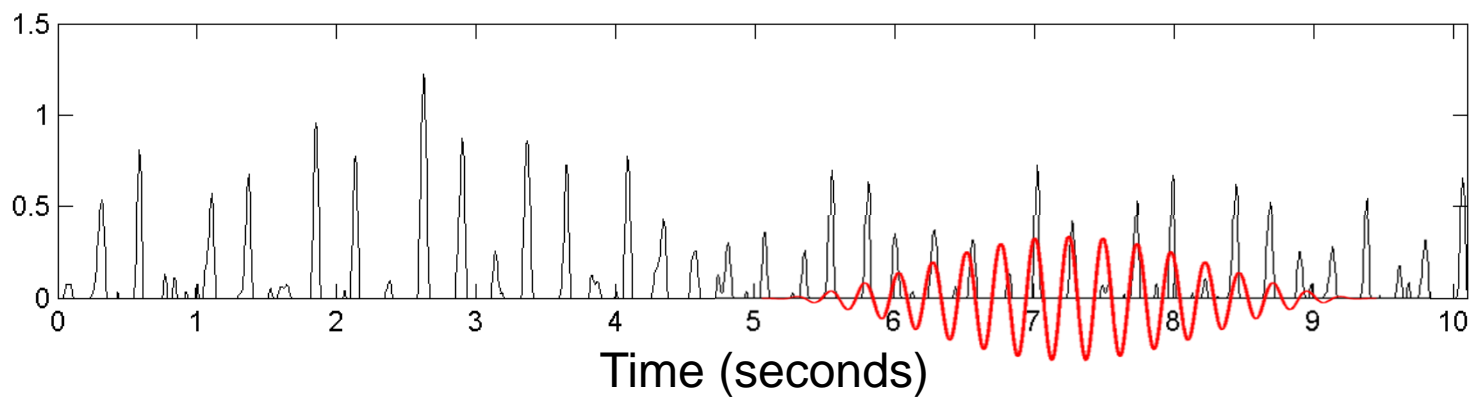
Windowed sinusoidal



Tempogram (Fourier)



Windowed sinusoidal



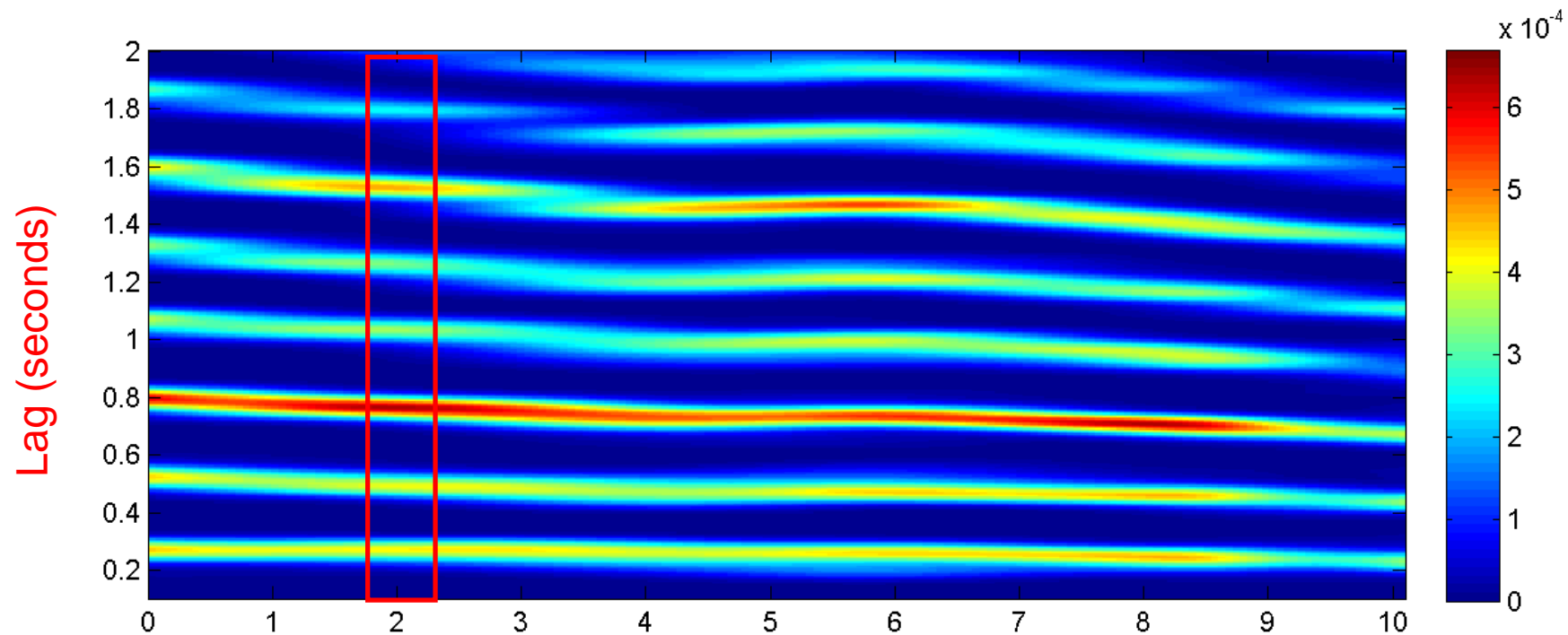
Tempogram (Autocorrelation)

Definition: A **tempogram** is a time-tempo representation that encodes the local tempo of a music signal over time.

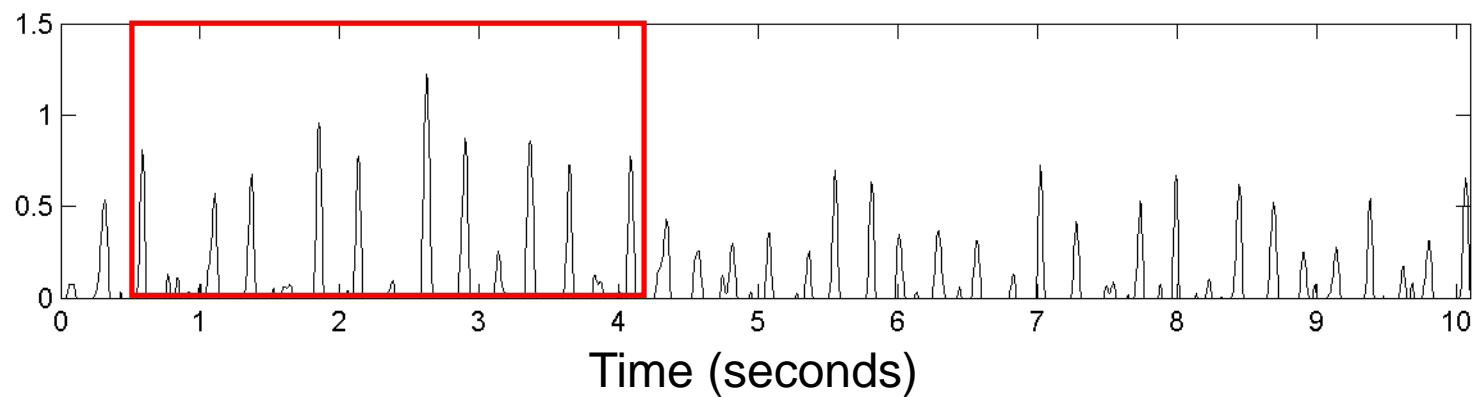
Autocorrelation-based method

- Compare novelty curve with time-lagged local sections of itself
- Convert lag-axis (given in seconds) into tempo axis (given in BPM)
- Autocorrelogram indicates local tempo

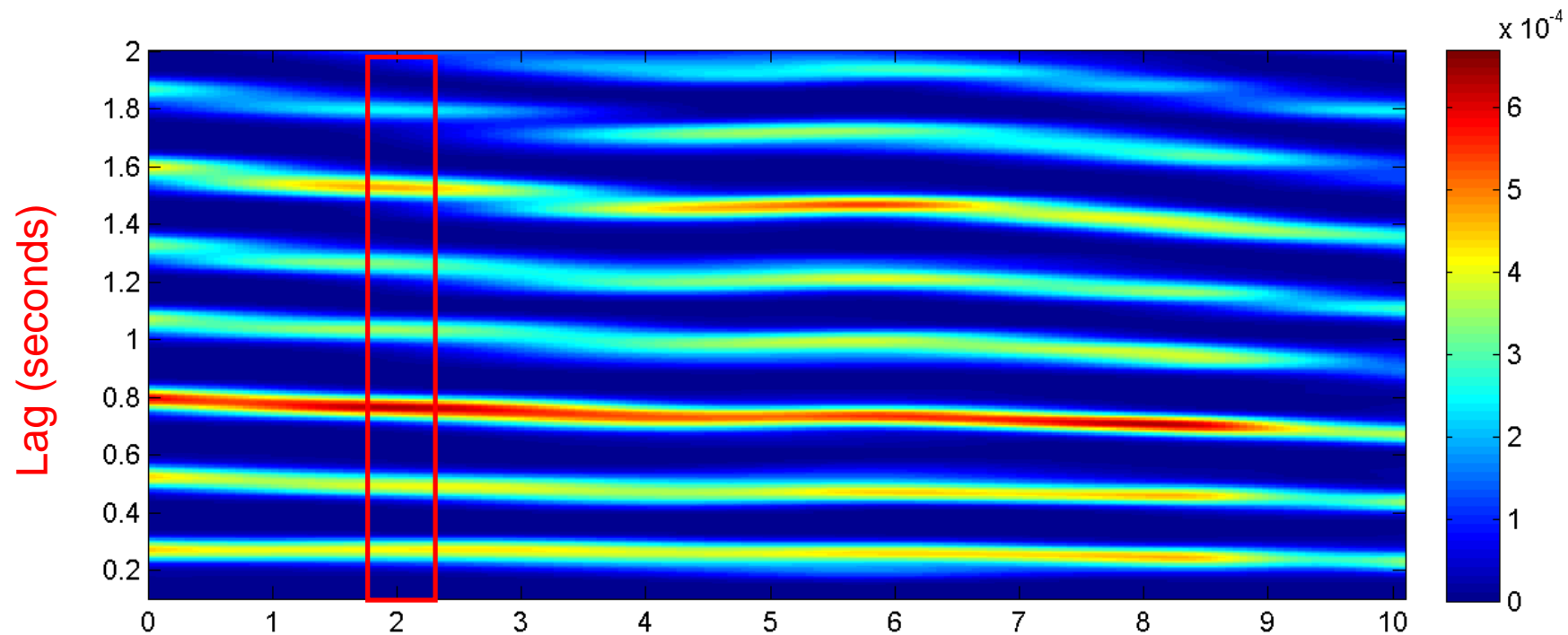
Tempogram (Autocorrelation)



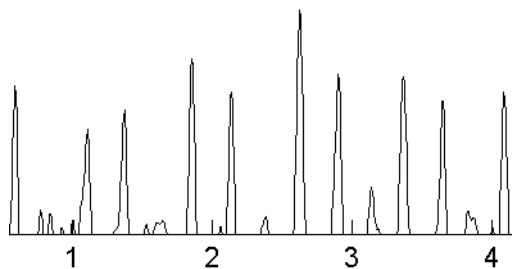
Novelty curve (local section)



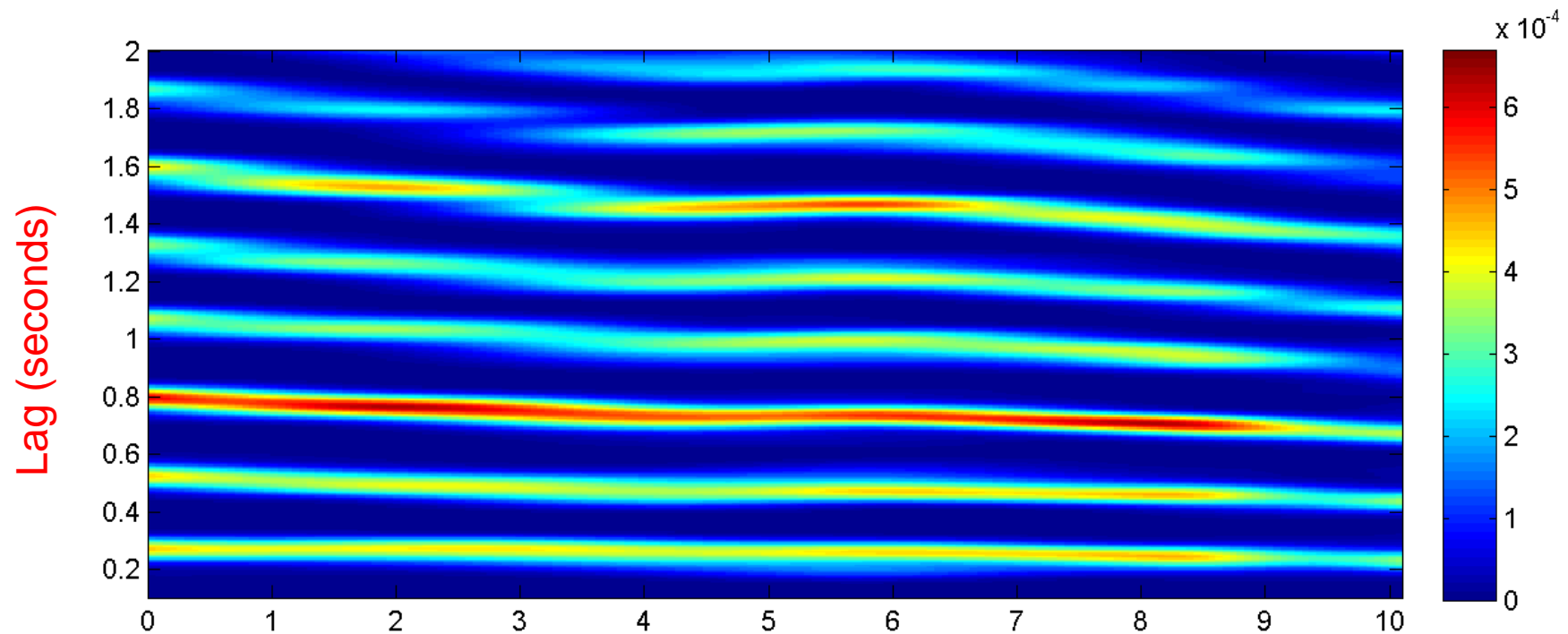
Tempogram (Autocorrelation)



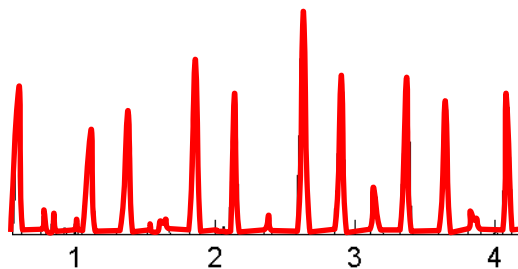
Windowed autocorrelation



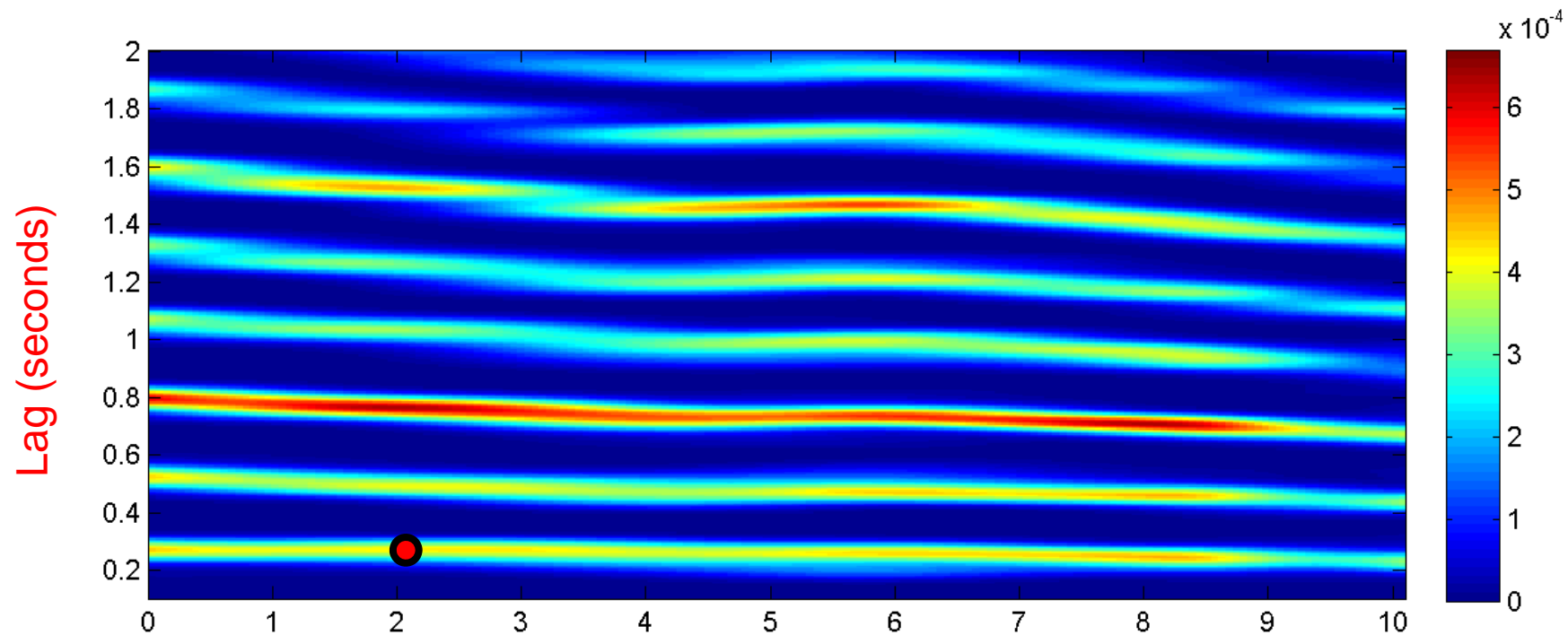
Tempogram (Autocorrelation)



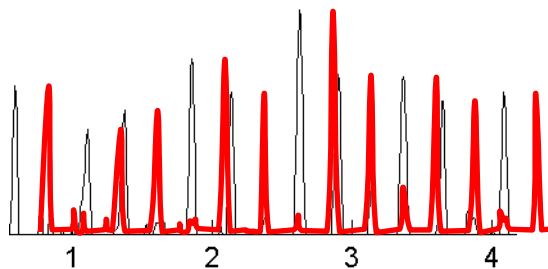
Lag = 0 (seconds)



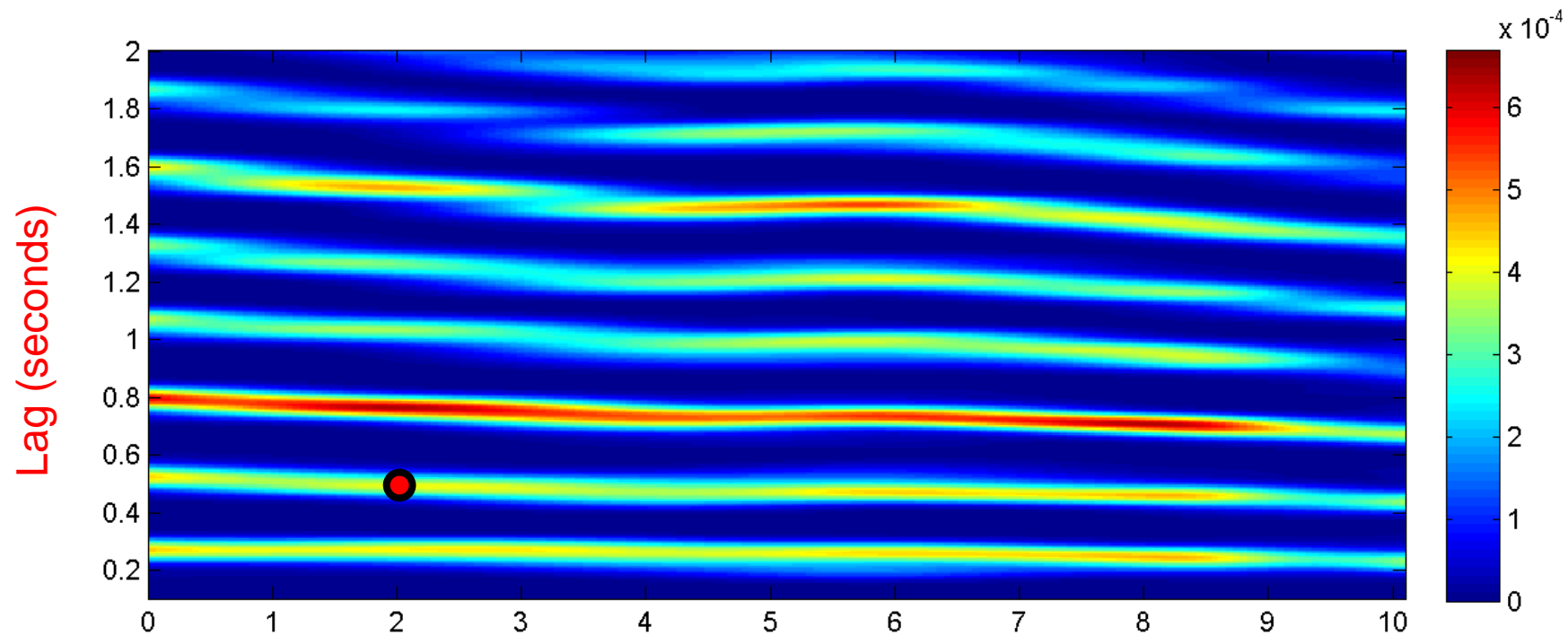
Tempogram (Autocorrelation)



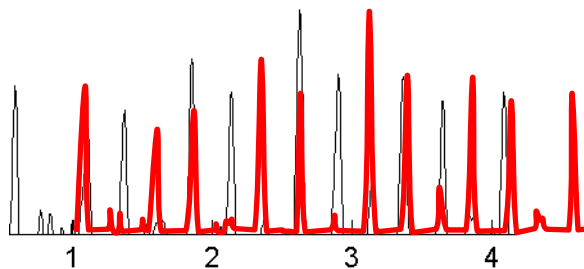
Lag = 0.26 (seconds)



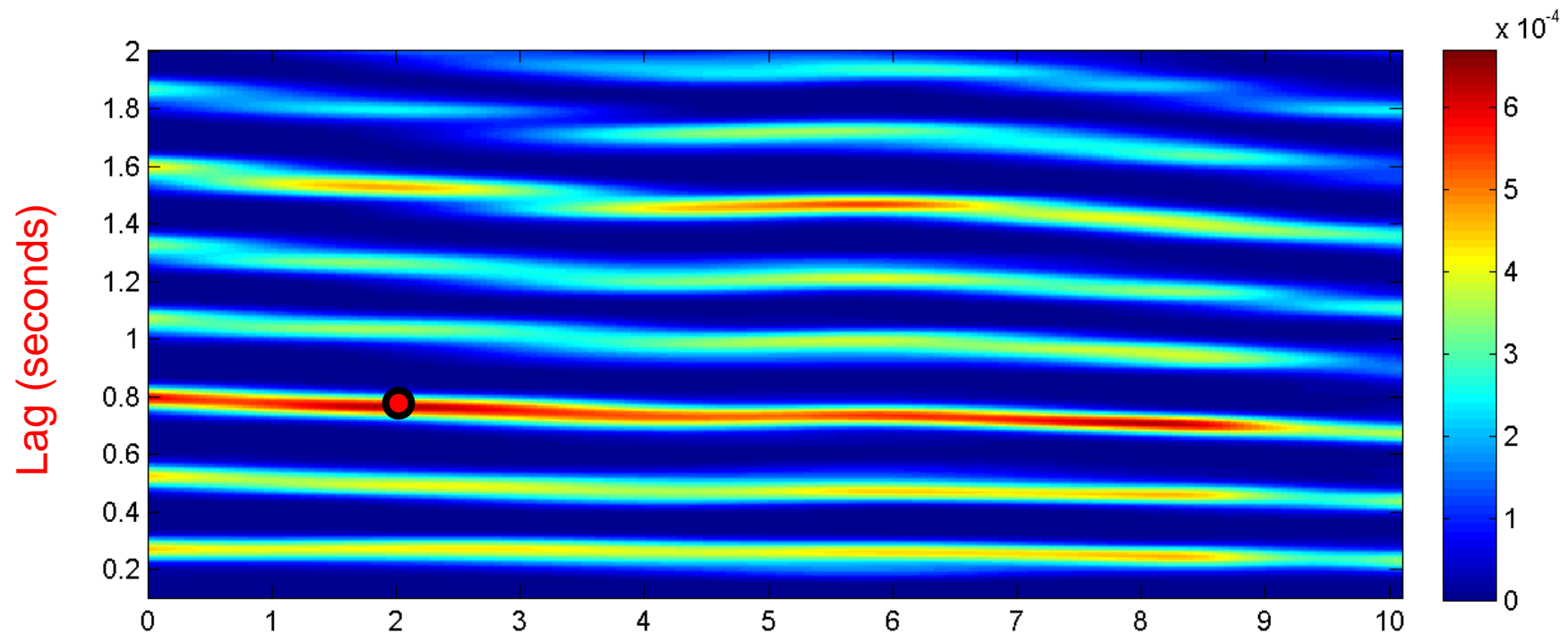
Tempogram (Autocorrelation)



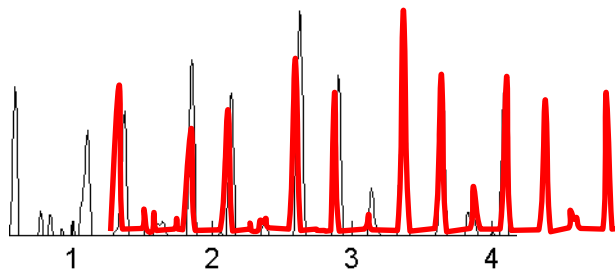
Lag = 0.52 (seconds)



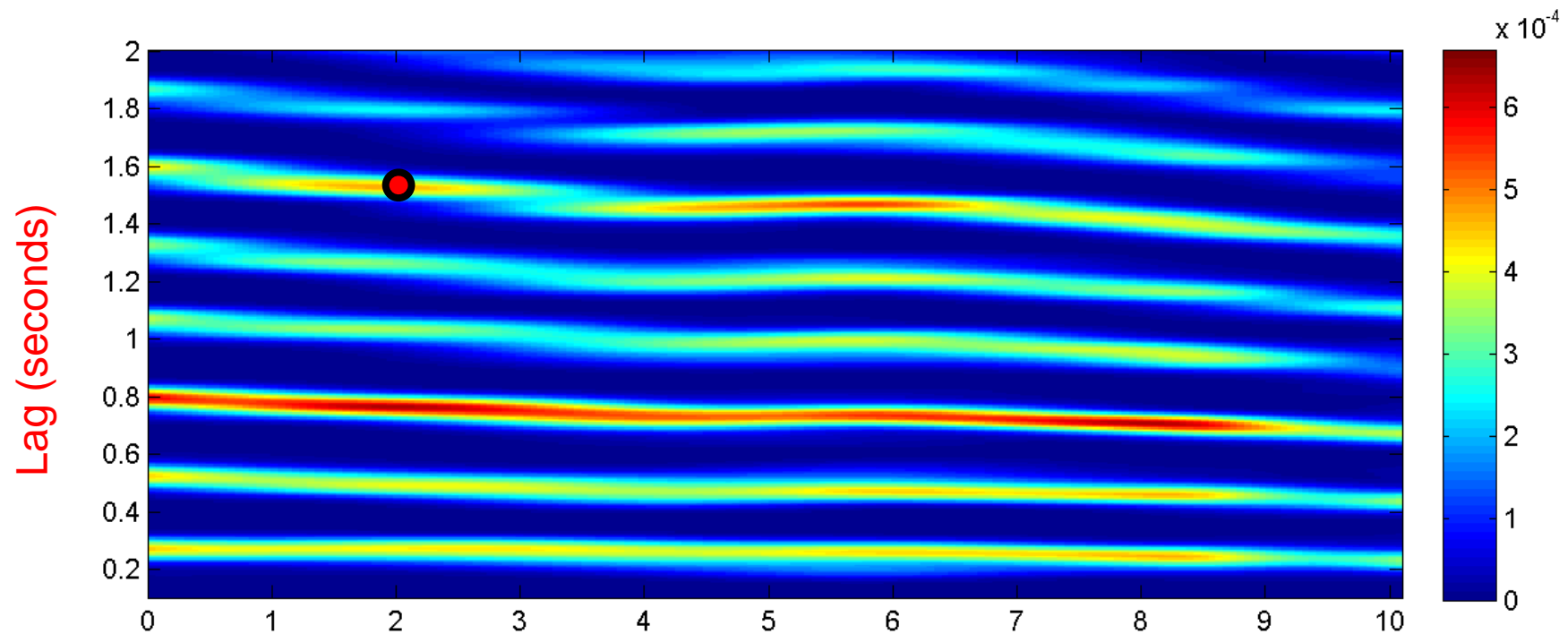
Tempogram (Autocorrelation)



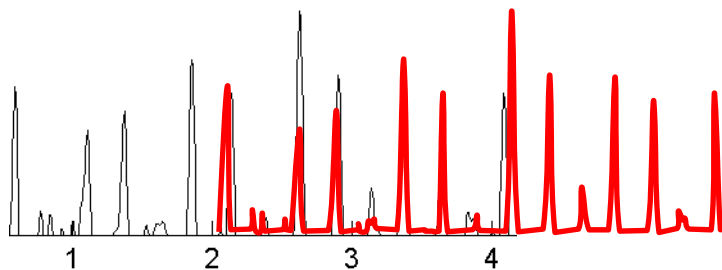
Lag = 0.78 (seconds)



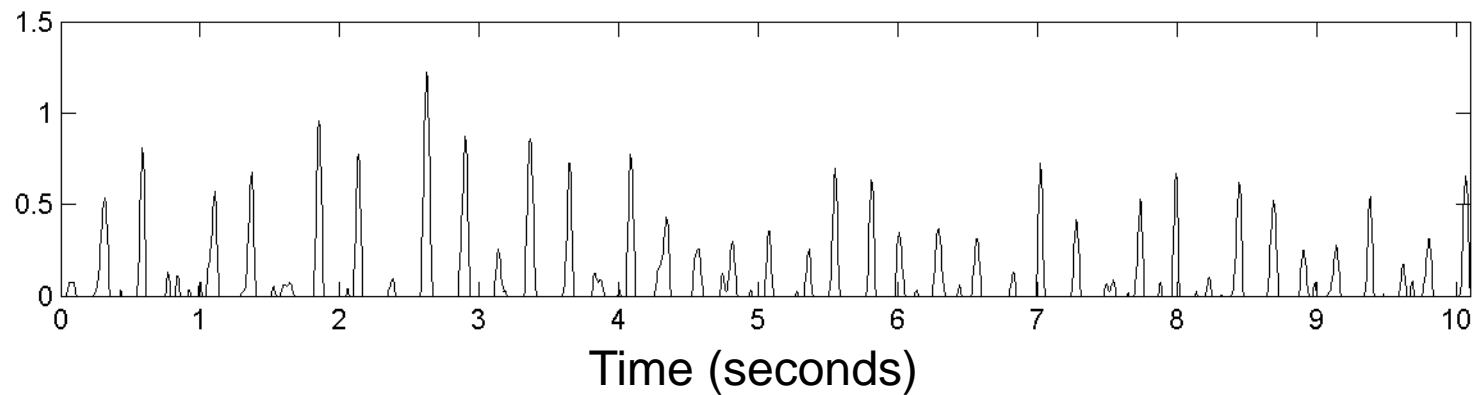
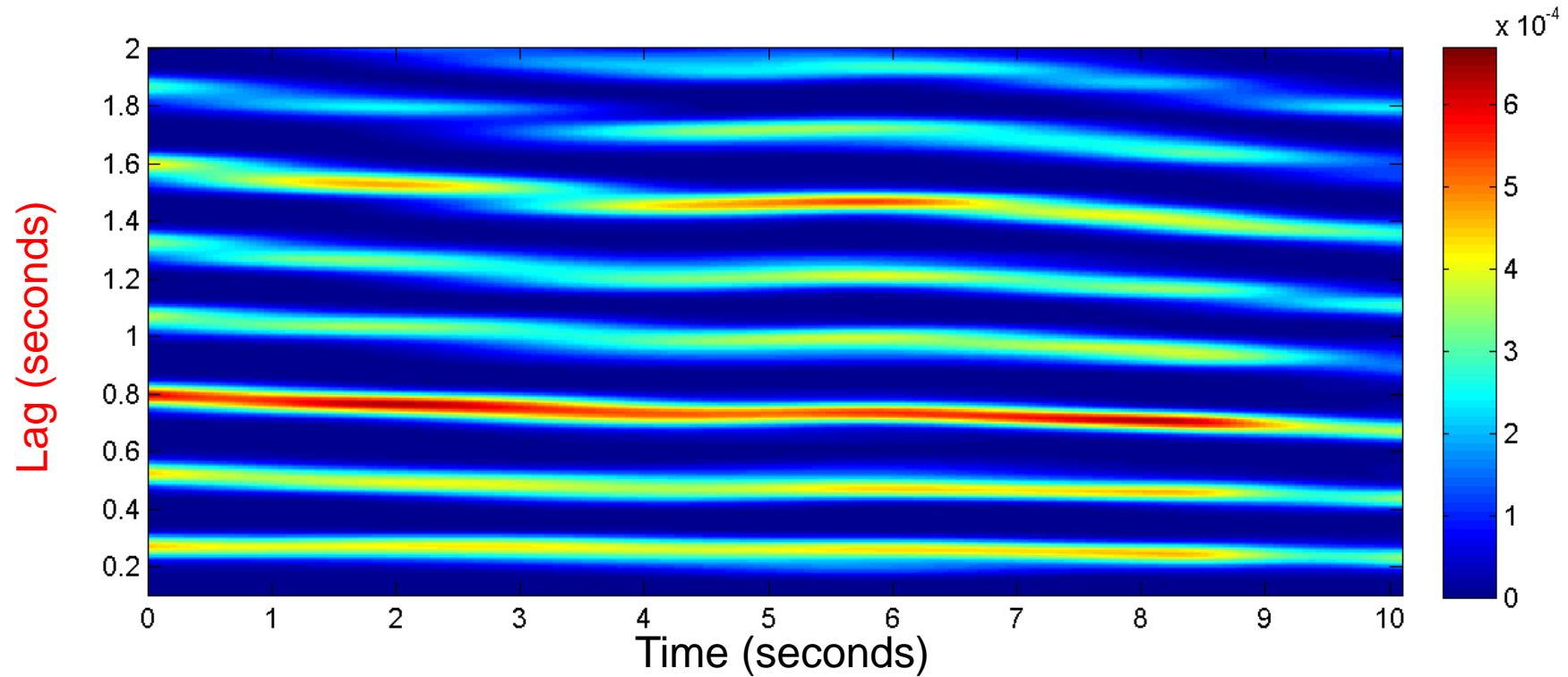
Tempogram (Autocorrelation)



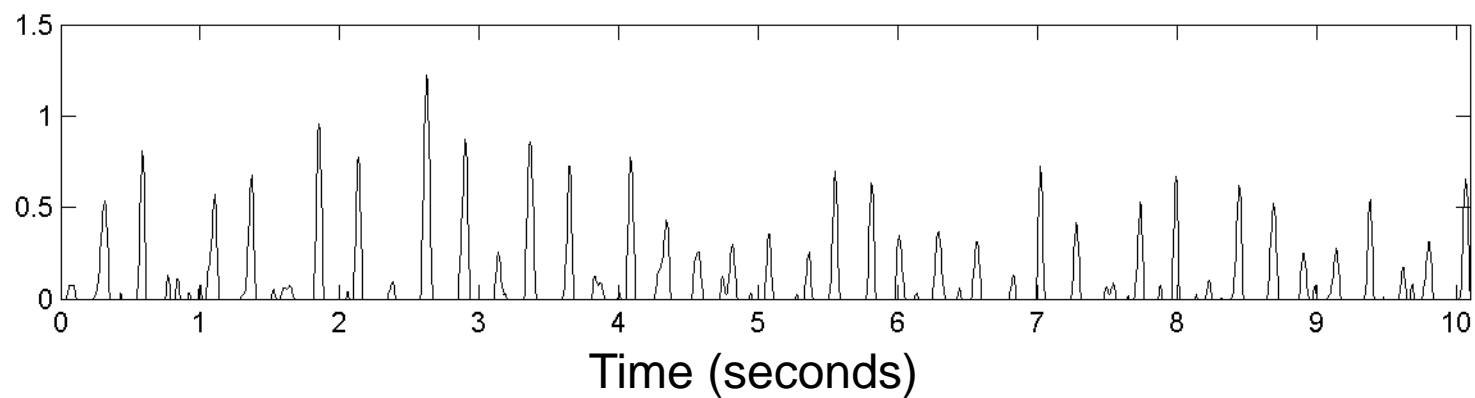
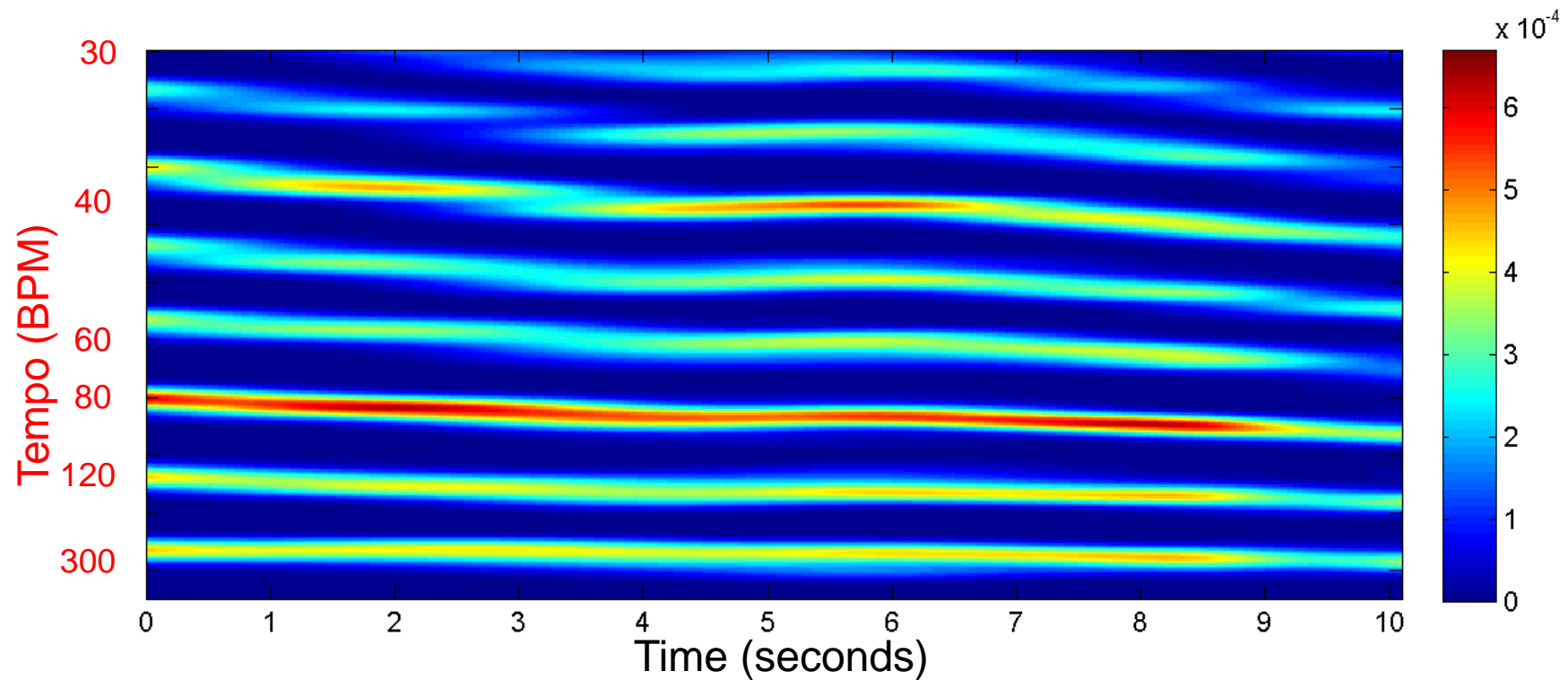
Lag = 1.56 (seconds)



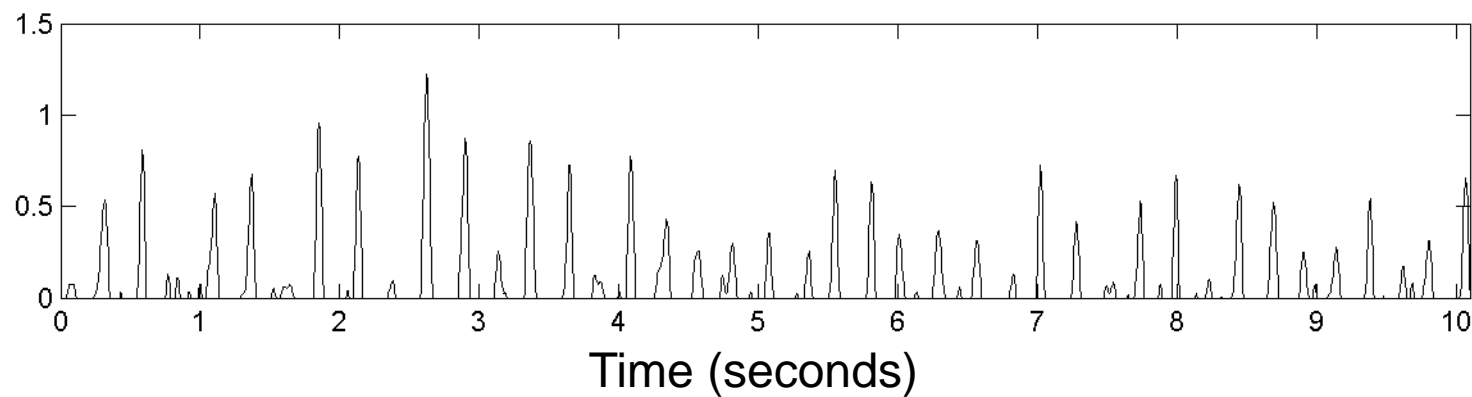
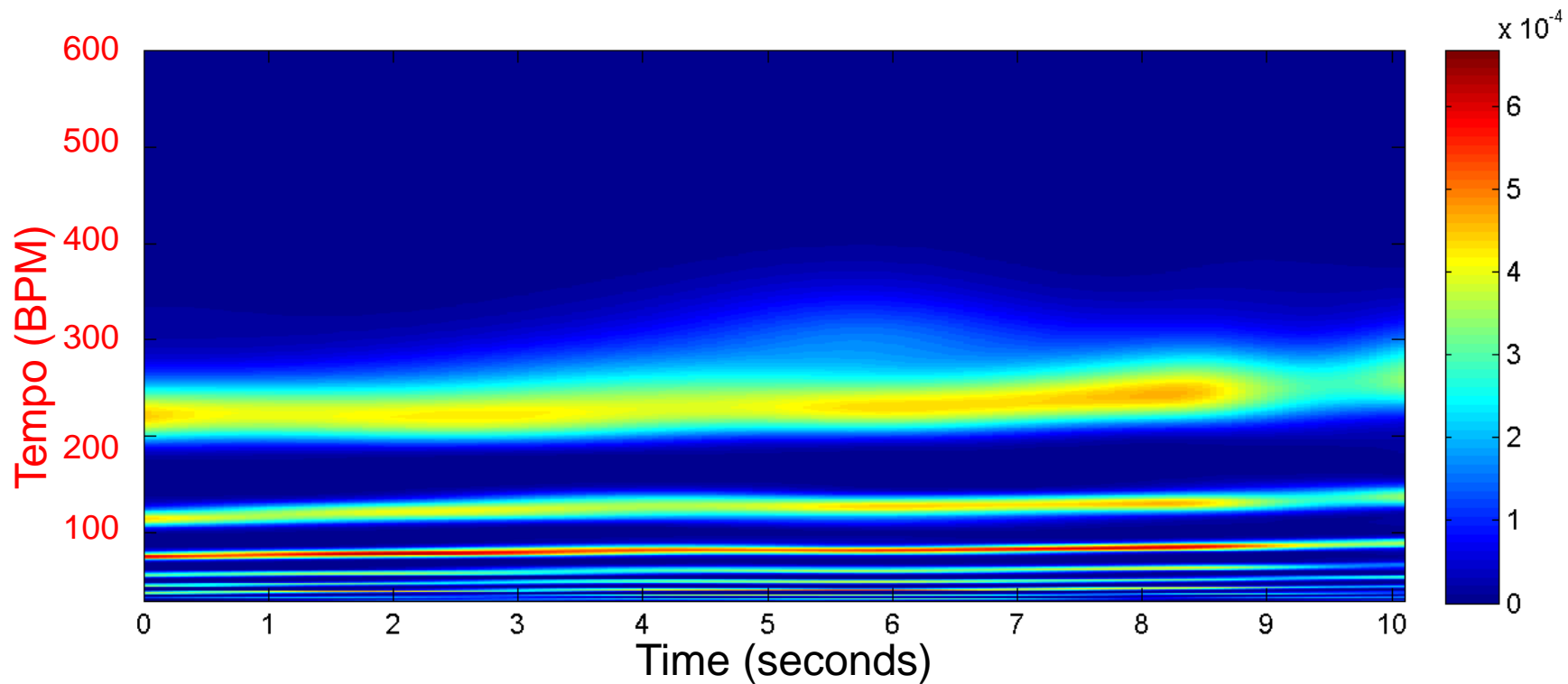
Tempogram (Autocorrelation)



Tempogram (Autocorrelation)

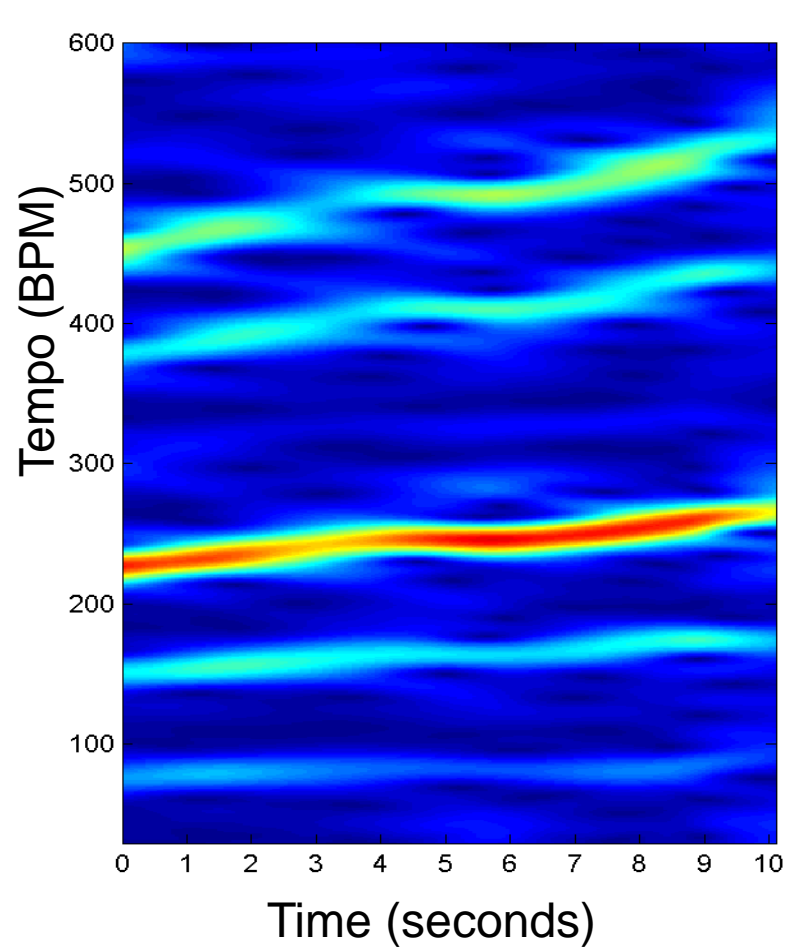


Tempogram (Autocorrelation)

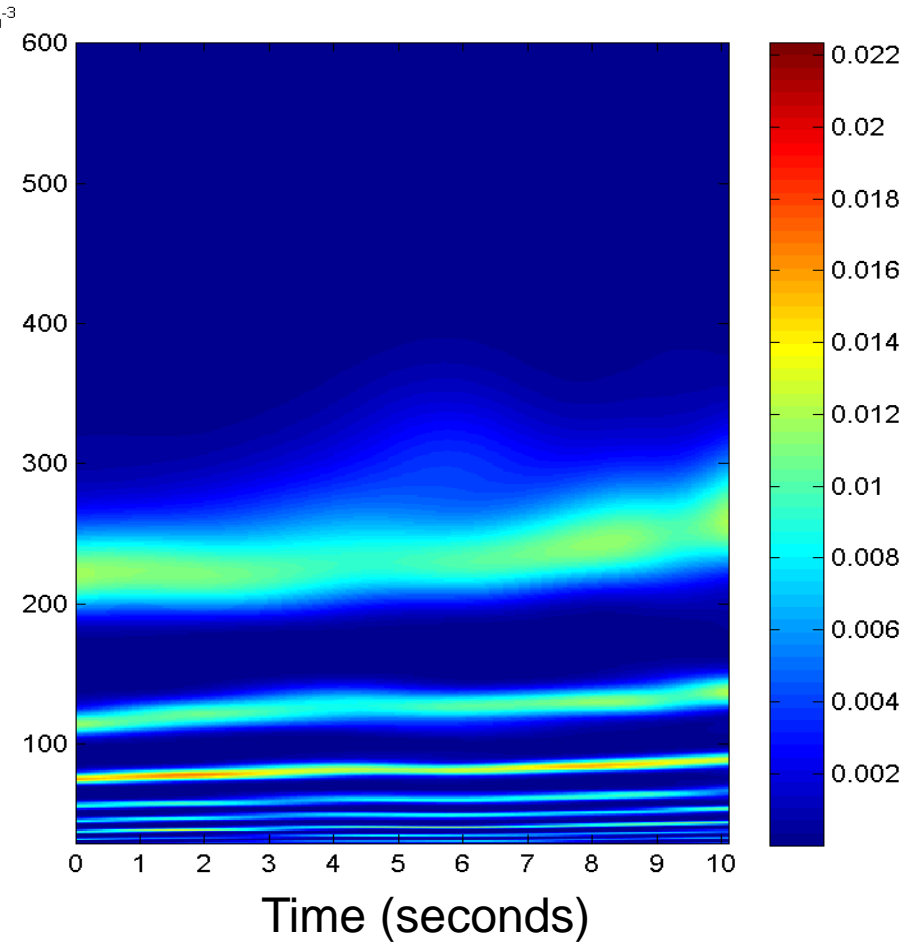


Tempogram

Fourier

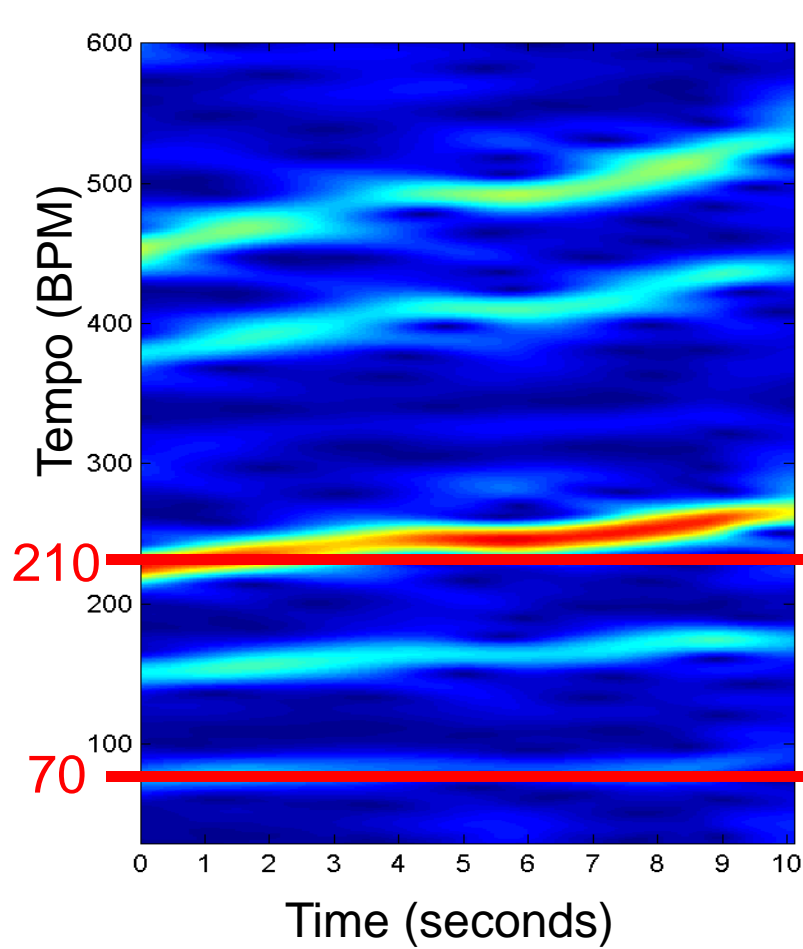


Autocorrelation

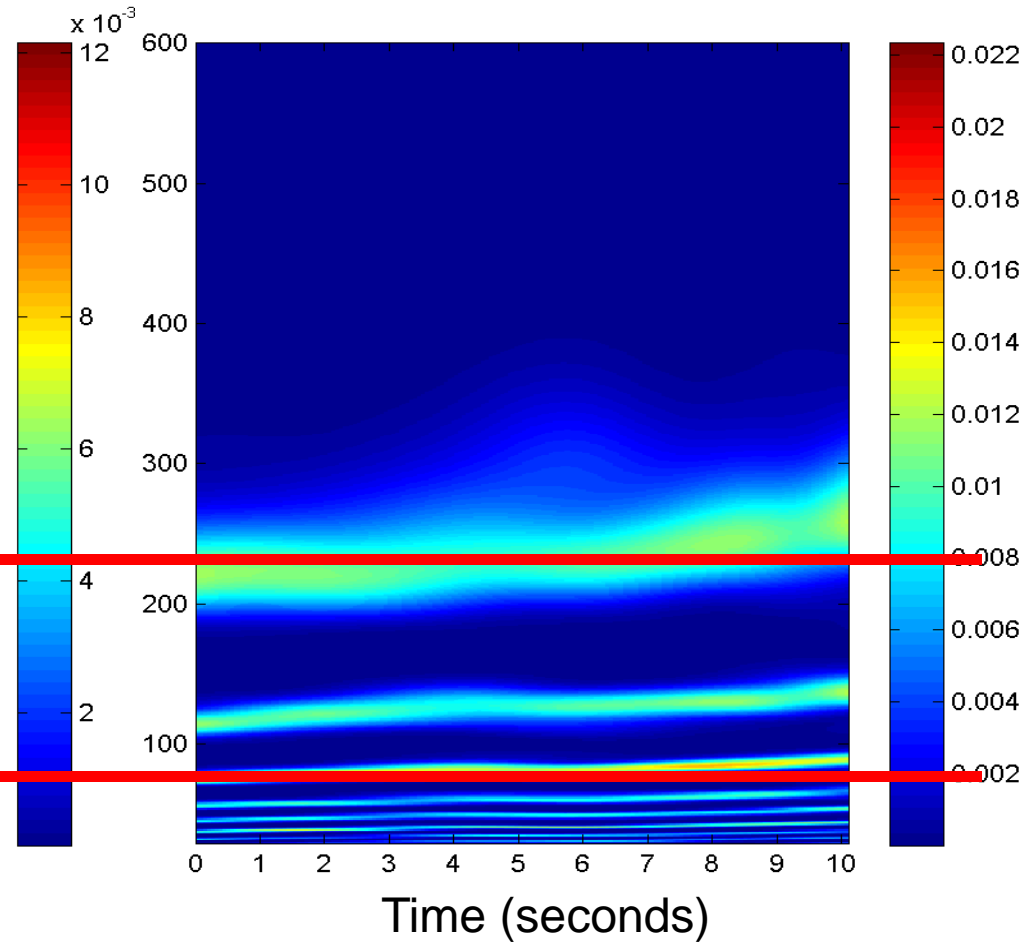


Tempogram

Fourier



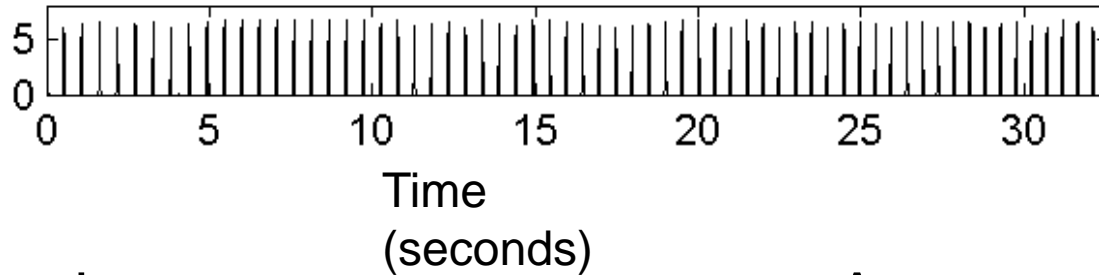
Autocorrelation



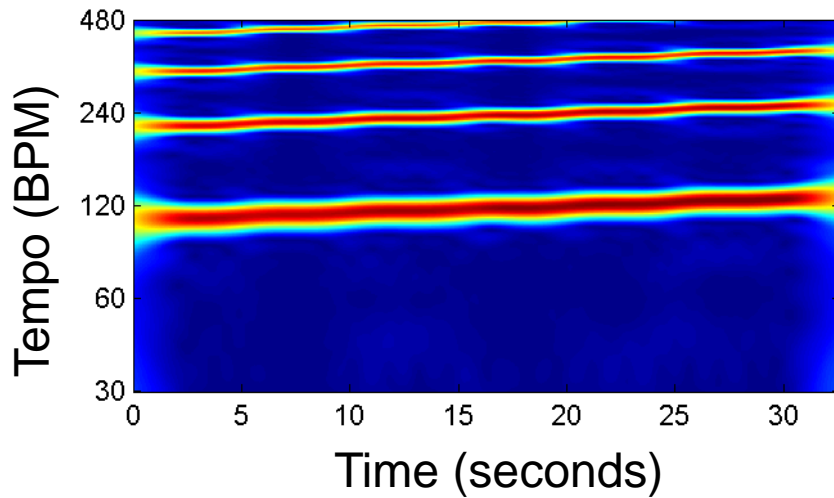
Tempo@Tatum = 210 BPM

Tempo@Measure = 70 BPM

Tempogram

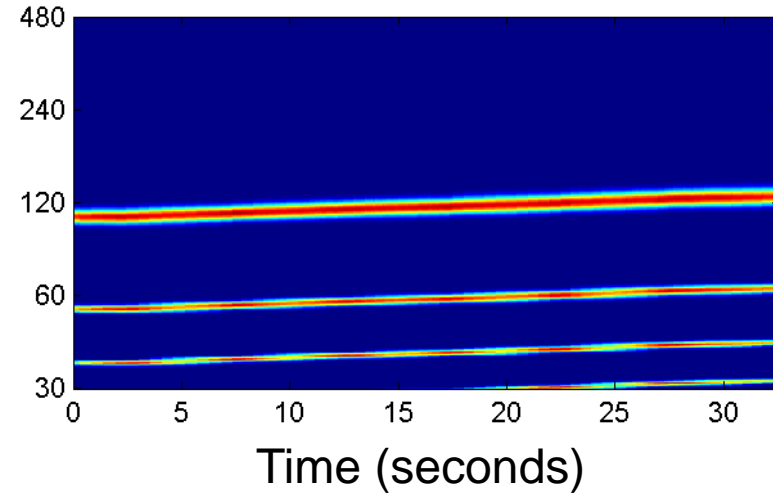


Fourier



Emphasis of tempo harmonics
(integer multiples)

Autocorrelation



Emphasis of tempo subharmonics
(integer fractions)

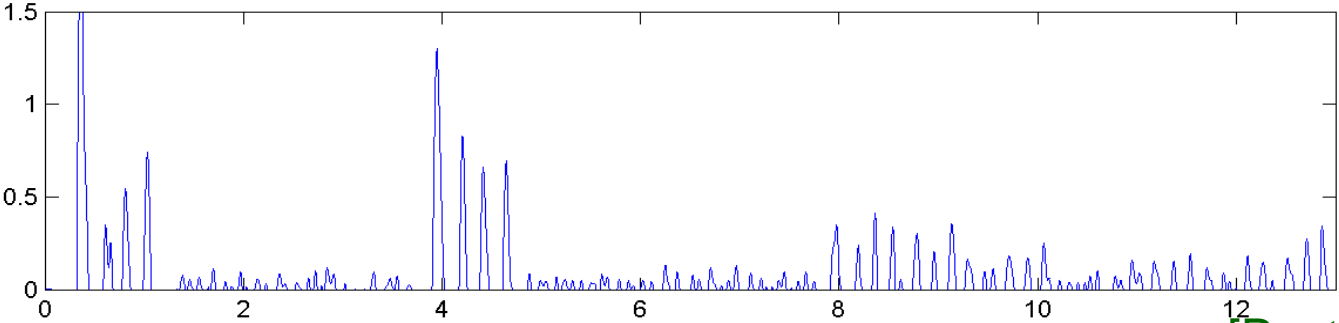
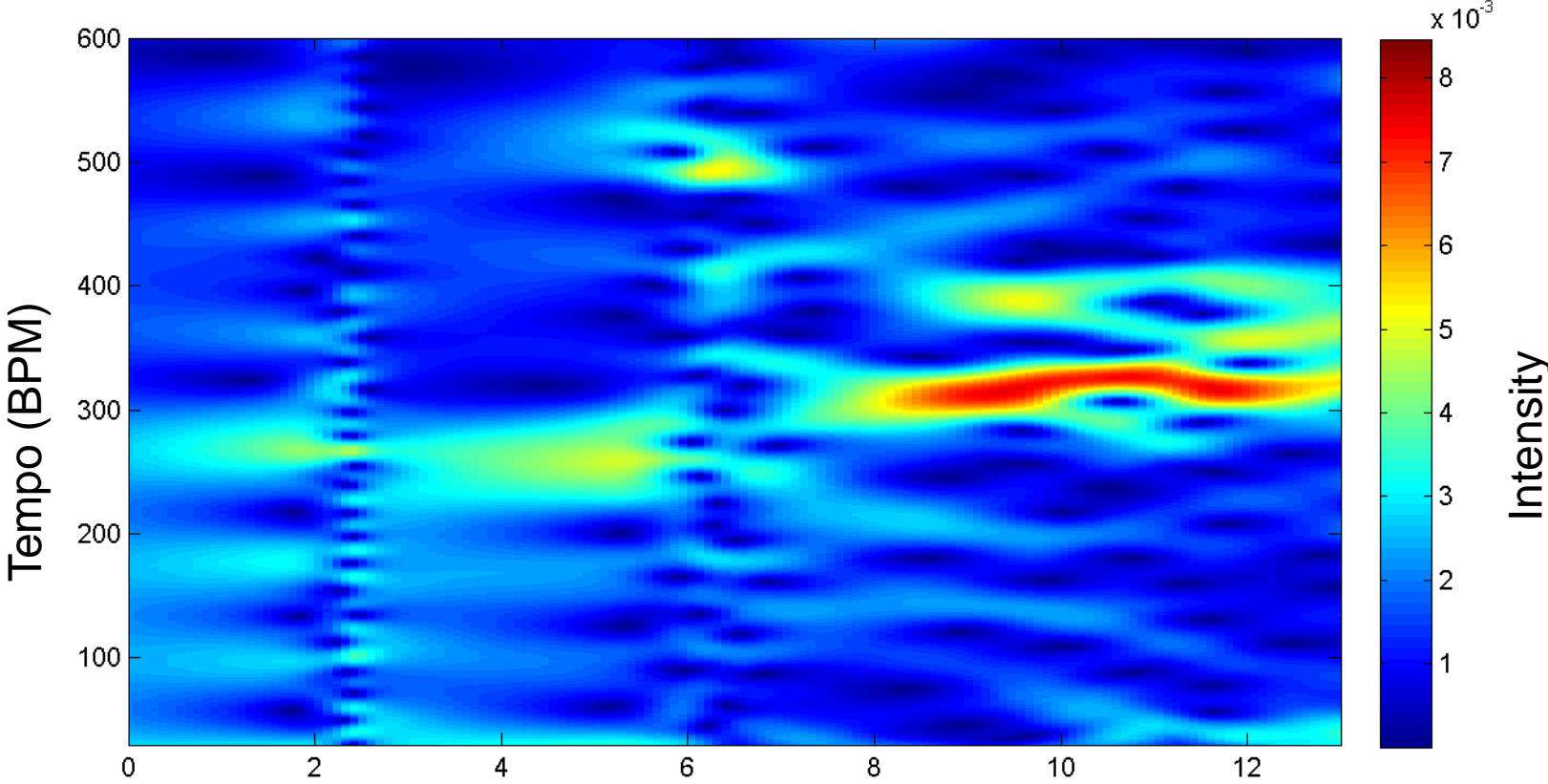
Tempogram (Summary)

Fourier	Autocorrelation
Novelty curve is compared with sinusoidal kernels each representing a specific tempo	Novelty curve is compared with time-lagged local (windowed) sections of itself
Convert frequency (Hertz) into tempo (BPM)	Convert time-lag (seconds) into tempo (BPM)
Reveals novelty periodicities	Reveals novelty self-similarities
Emphasizes harmonics	Emphasizes subharmonics
Suitable to analyze tempo on tatum and tactus level	Suitable to analyze tempo on tactus and measure level

Beat Tracking

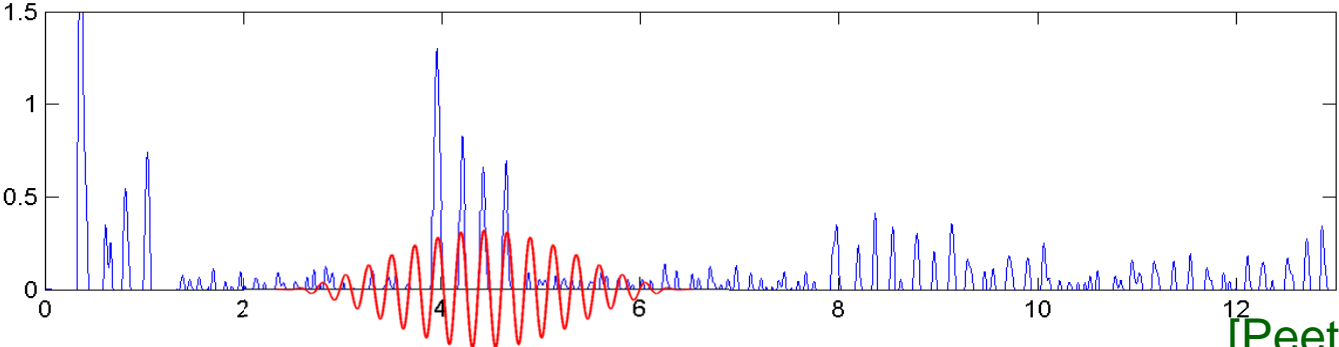
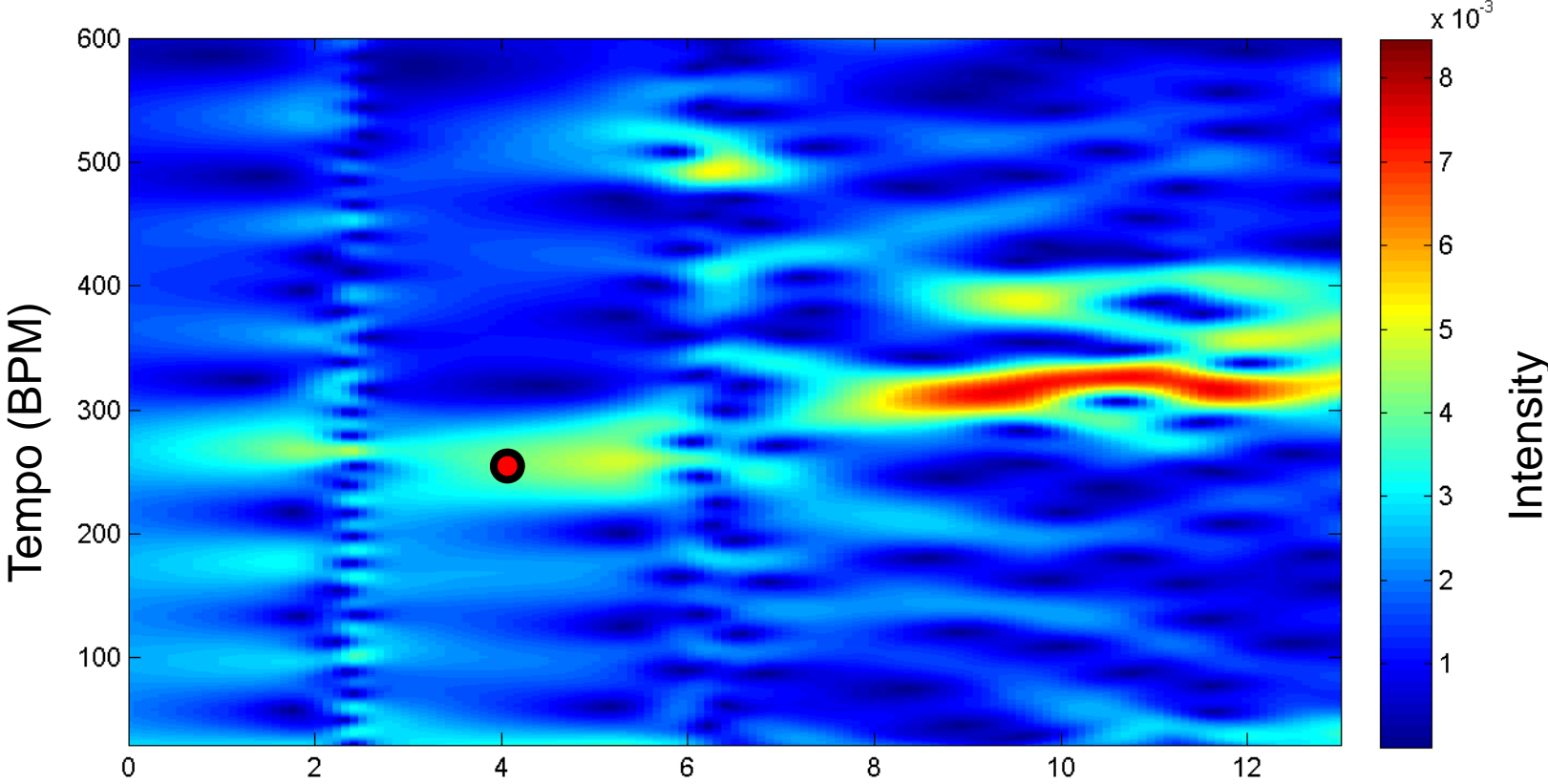
- Given the tempo, find the best sequence of beats
- Complex Fourier tempogram contains **magnitude** and **phase** information
- The **magnitude** encodes how well the novelty curve resonates with a sinusoidal kernel of a specific tempo
- The **phase** optimally aligns the sinusoidal kernel with the peaks of the novelty curve

Beat Tracking



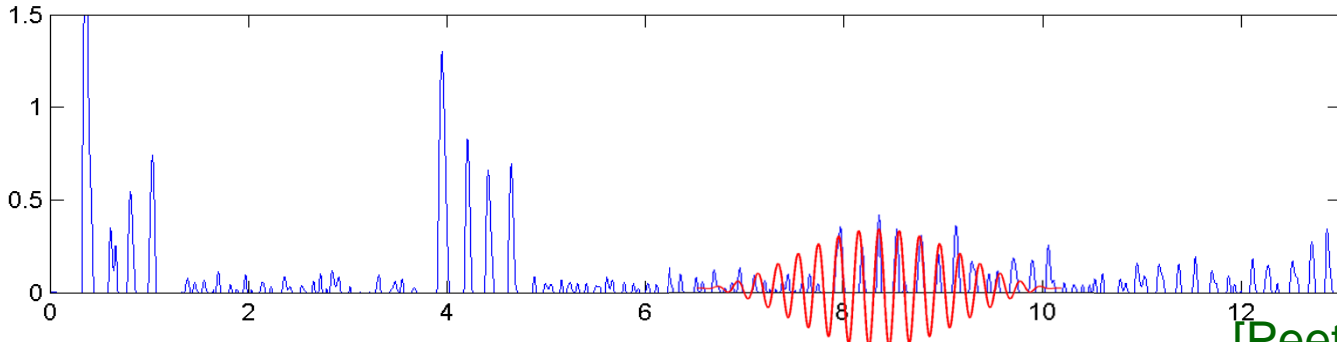
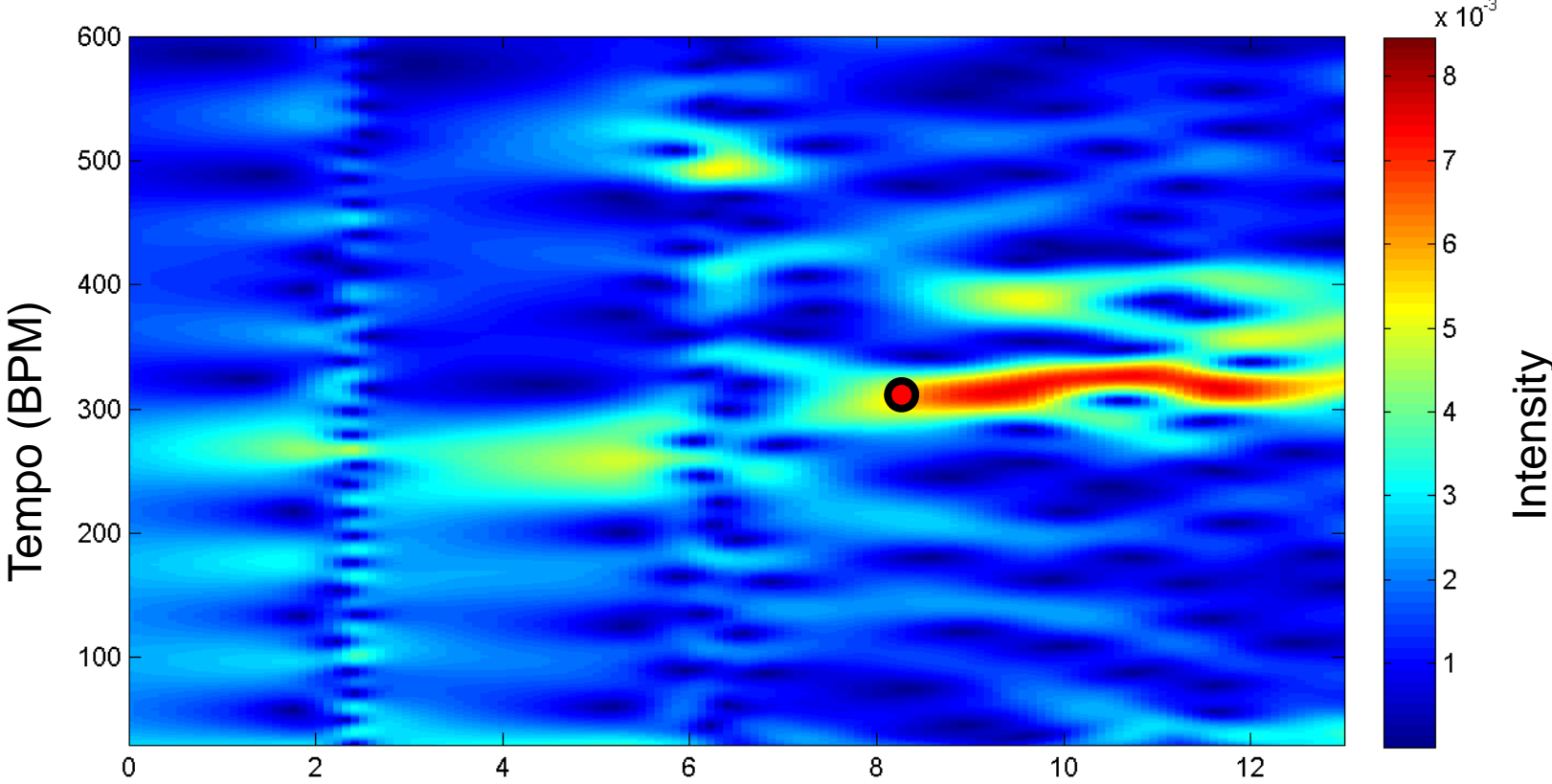
[Peeters, JASP 2005]

Beat Tracking



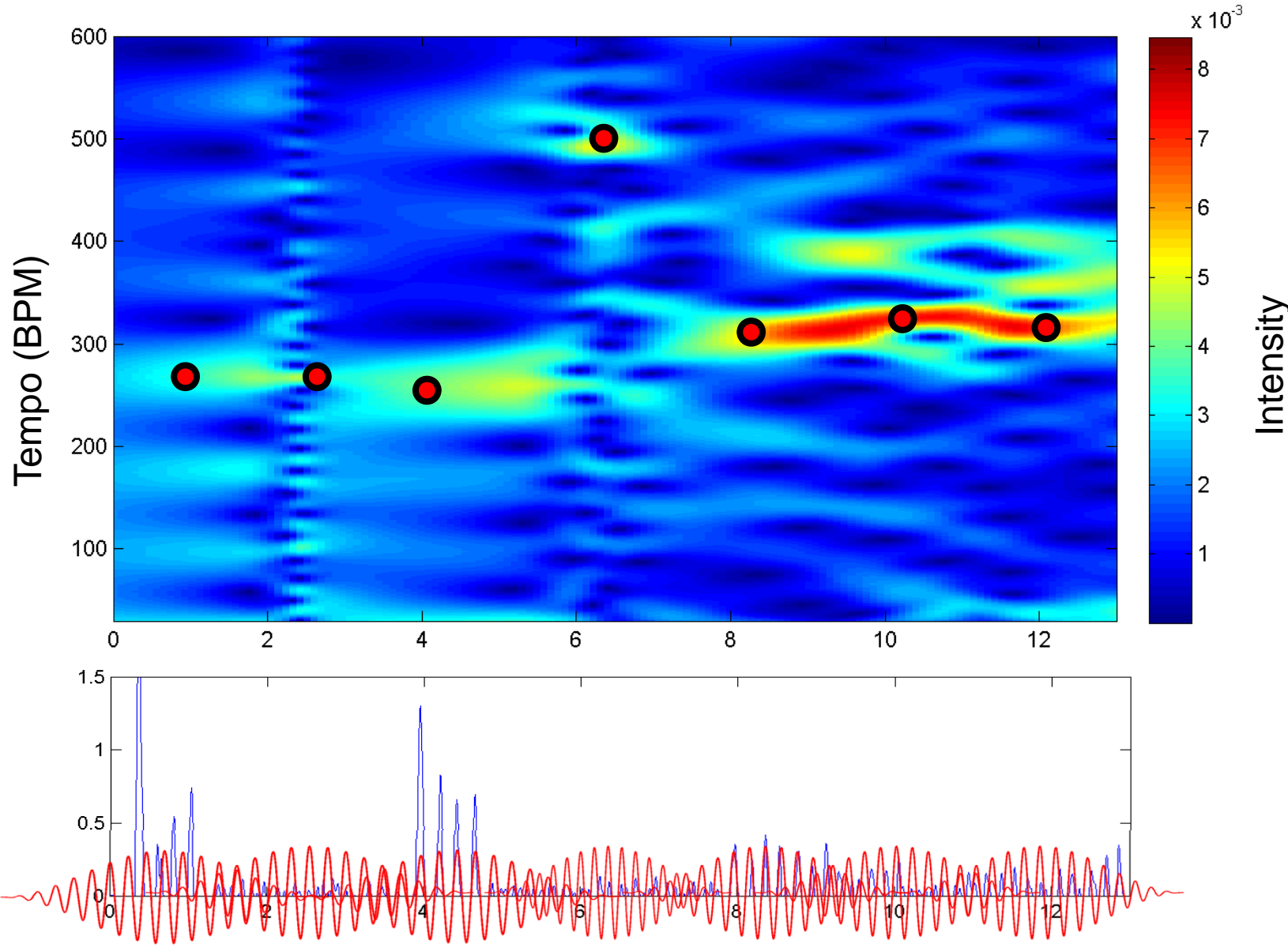
[Peeters, JASP 2005]

Beat Tracking

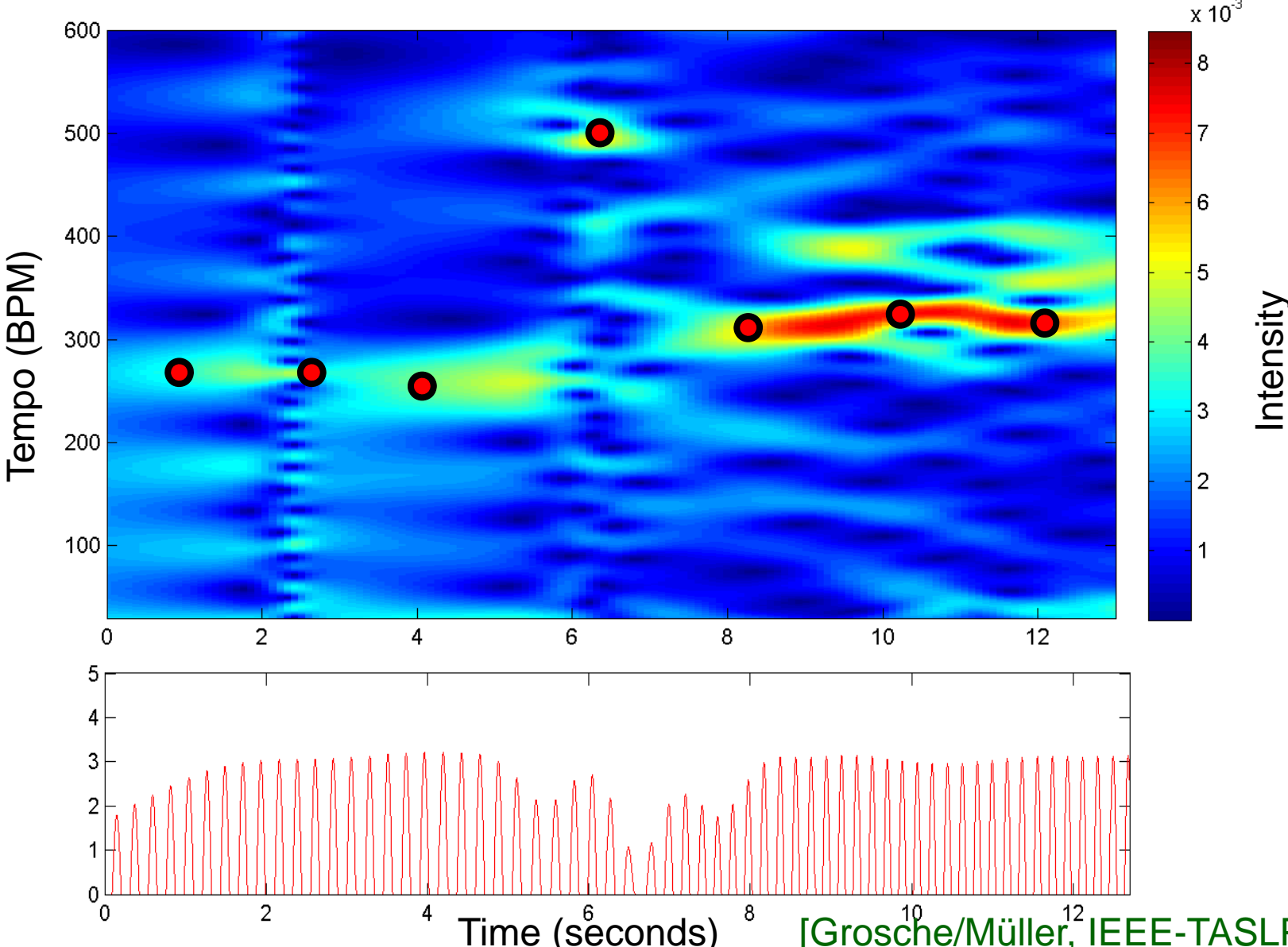


[Peeters, JASP 2005]

Beat Tracking

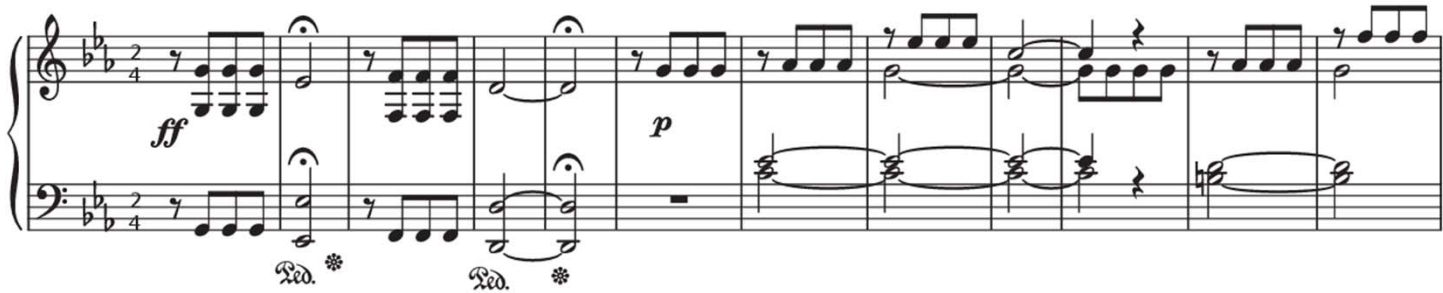


Beat Tracking

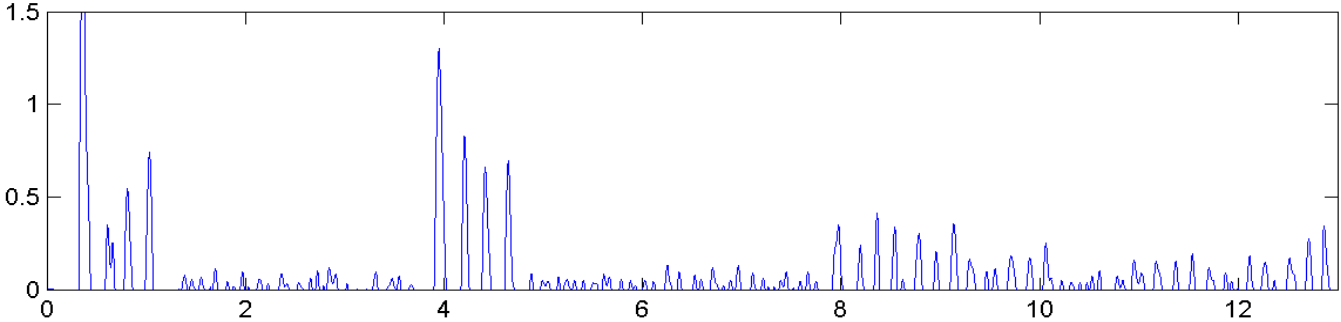


[Grosche/Müller, IEEE-TASLP 2011]

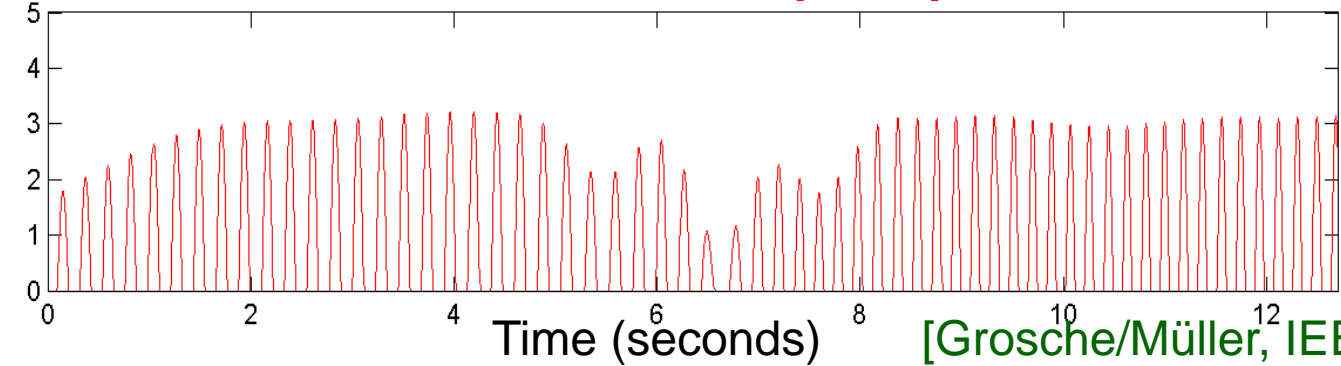
Beat Tracking



Novelty Curve



Predominant Local Pulse (PLP)



[Grosche/Müller, IEEE-TASLP 2011]

Beat Tracking

Novelty Curve

- Indicates note onset candidates
- Extraction errors in particular for soft onsets
- Simple peak-picking problematic



Predominant Local Pulse (PLP)

- Periodicity enhancement of novelty curve
- Accumulation introduces error robustness
- Locality of kernels handles tempo variations



Beat Tracking

- Local tempo at time t : $\tau_t \in \Theta$ $\Theta = [60:240]$ BPM

- Phase $\varphi_t := \frac{1}{2\pi} \arccos \left(\frac{\text{Re}(\mathcal{T}(t, \tau_t))}{|\mathcal{T}(t, \tau_t)|} \right)$

- Sinusoidal kernel $\kappa_t : \mathbb{Z} \rightarrow \mathbb{R}$

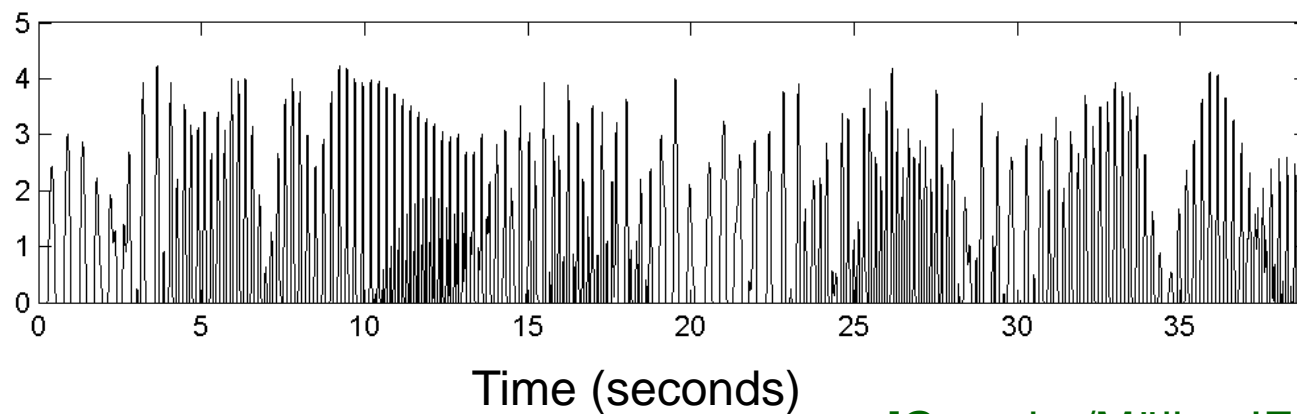
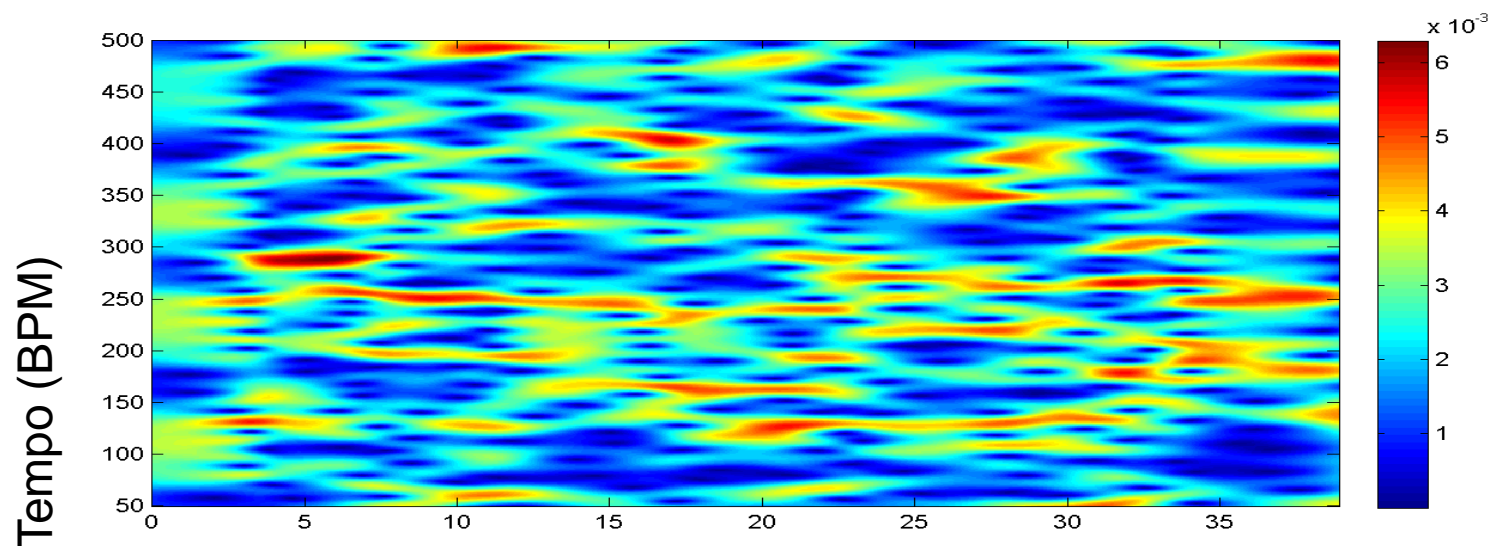
$$\kappa_t(n) := W(n - t) \cos(2\pi(\tau_t/60 \cdot n - \varphi_t)) \quad n \in \mathbb{Z}$$

- Periodicity curve $\Gamma : [1 : T] \rightarrow \mathbb{R}_{\geq 0}$

$$\Gamma(n) = \left| \sum_{t \in [1:T]} \kappa_t(n) \right|_{\geq 0} \quad n \in [1 : T]$$

Beat Tracking

Borodin – String Quartet No. 2

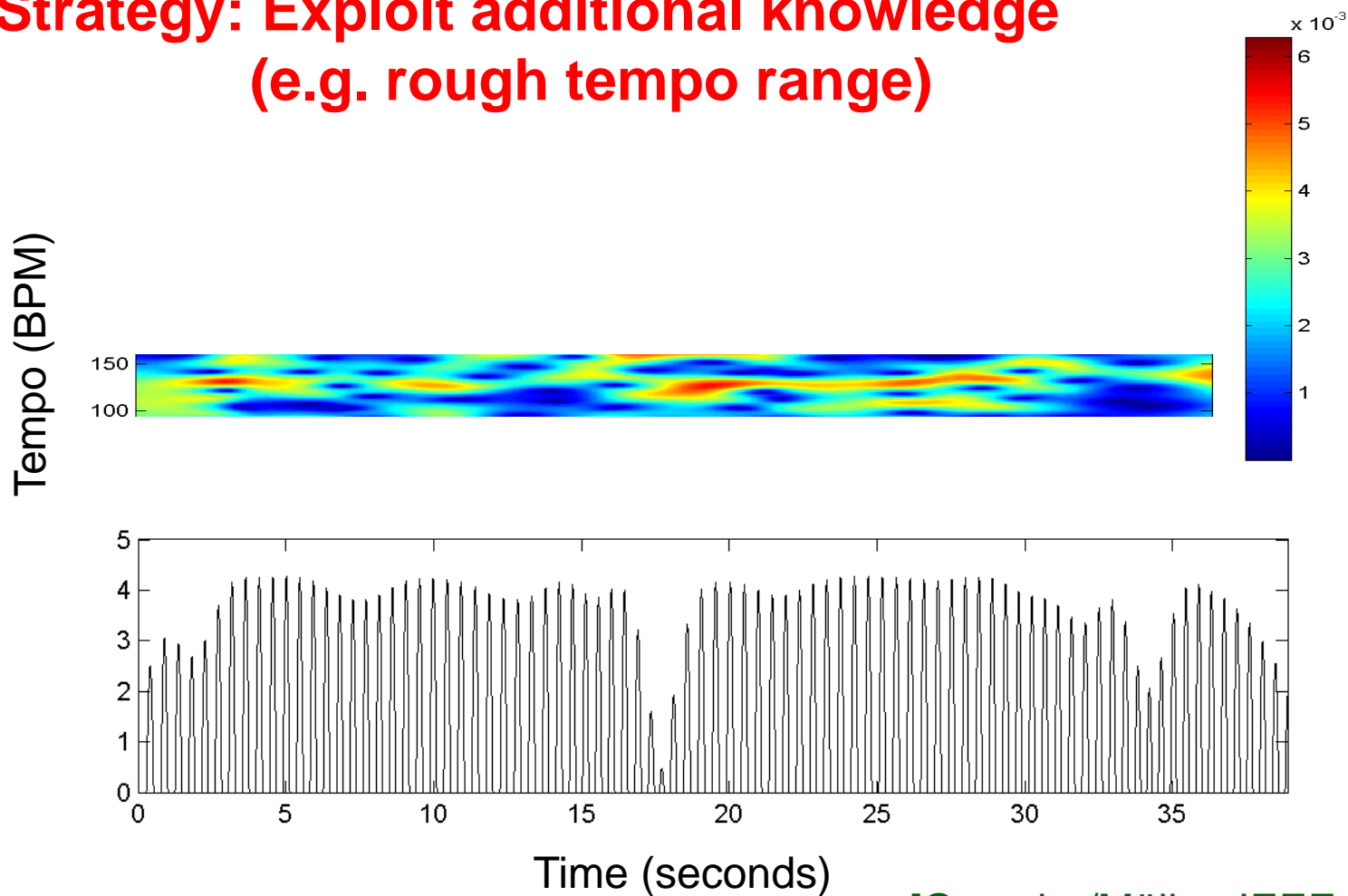


[Grosche/Müller, IEEE-TASLP 2011]

Beat Tracking

Borodin – String Quartet No. 2

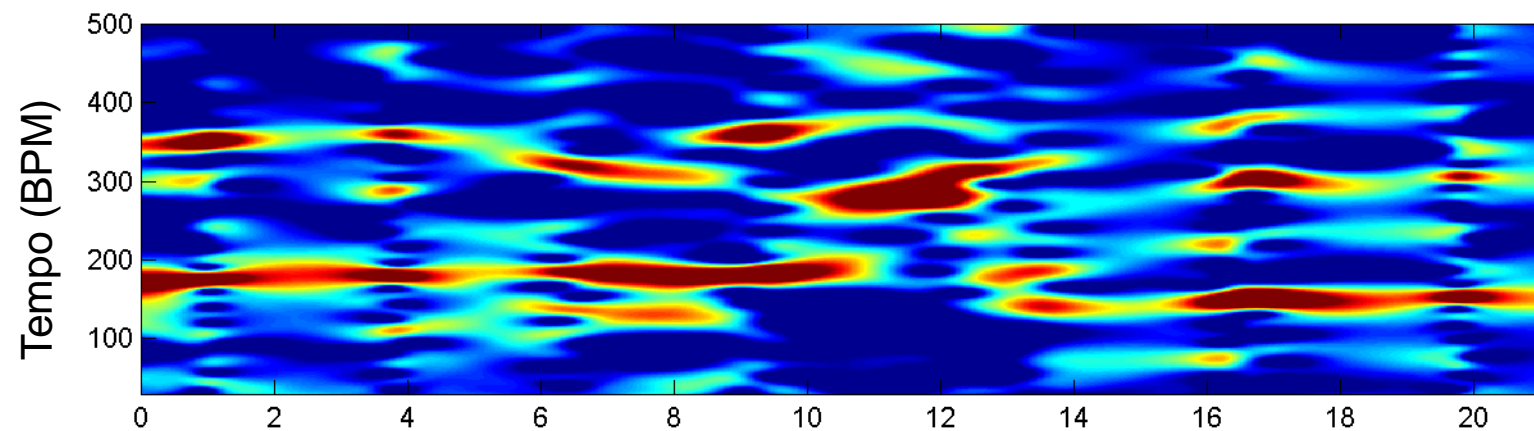
**Strategy: Exploit additional knowledge
(e.g. rough tempo range)**



[Grosche/Müller, IEEE-TASLP 2011]

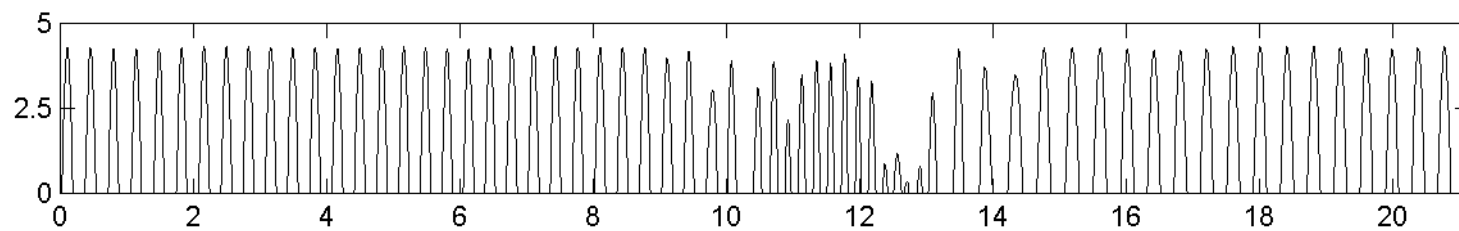
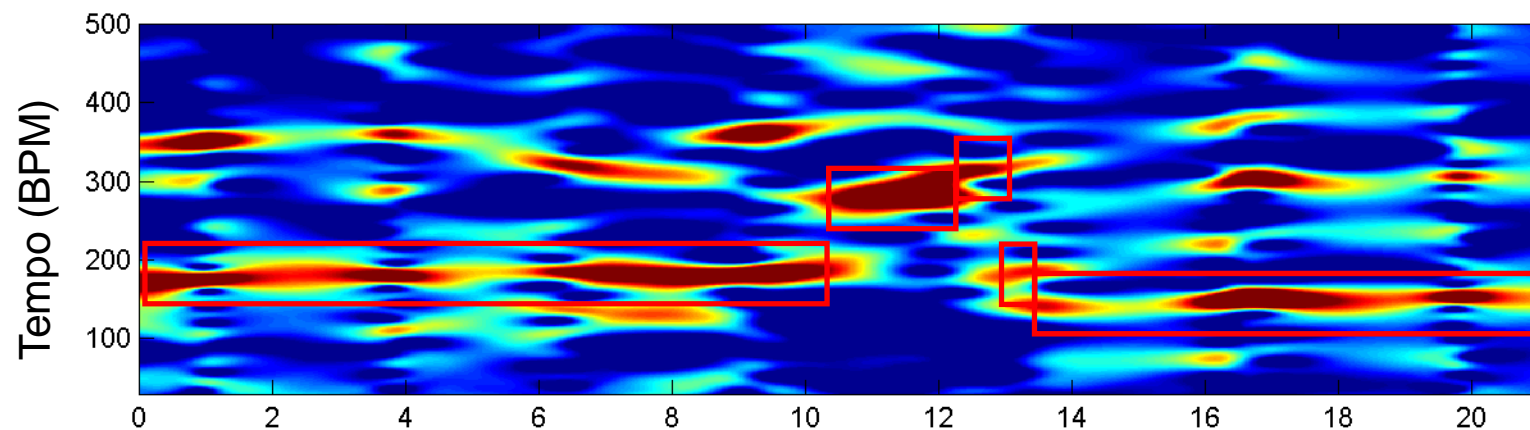
Beat Tracking

Brahms Hungarian Dance No. 5



Beat Tracking

Brahms Hungarian Dance No. 5



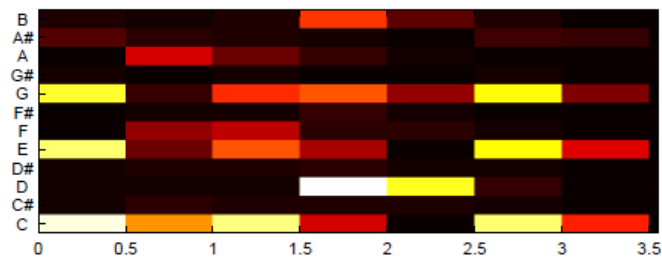
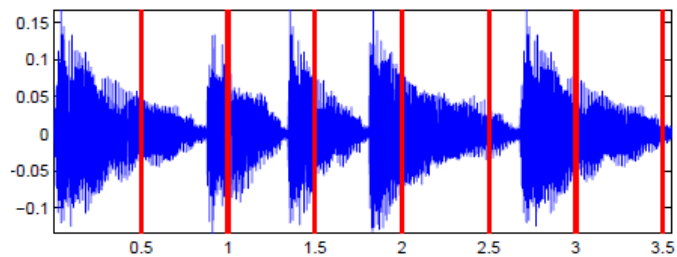
Time (seconds)



Applications

- Feature design
(beat-synchronous features, adaptive windowing)
- Digital DJ / audio editing
(mixing and blending of audio material)
- Music classification
- Music recommendation
- Performance analysis
(extraction of tempo curves)

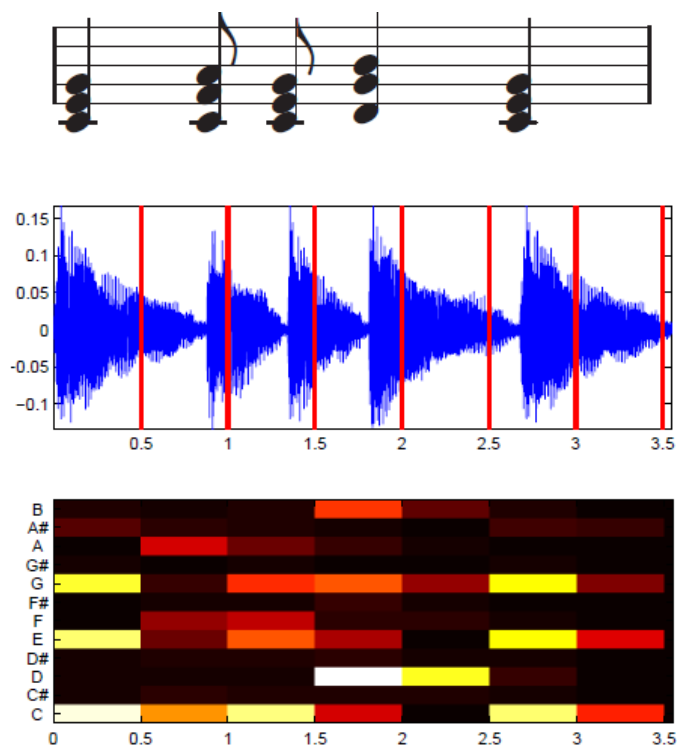
Application: Feature Design



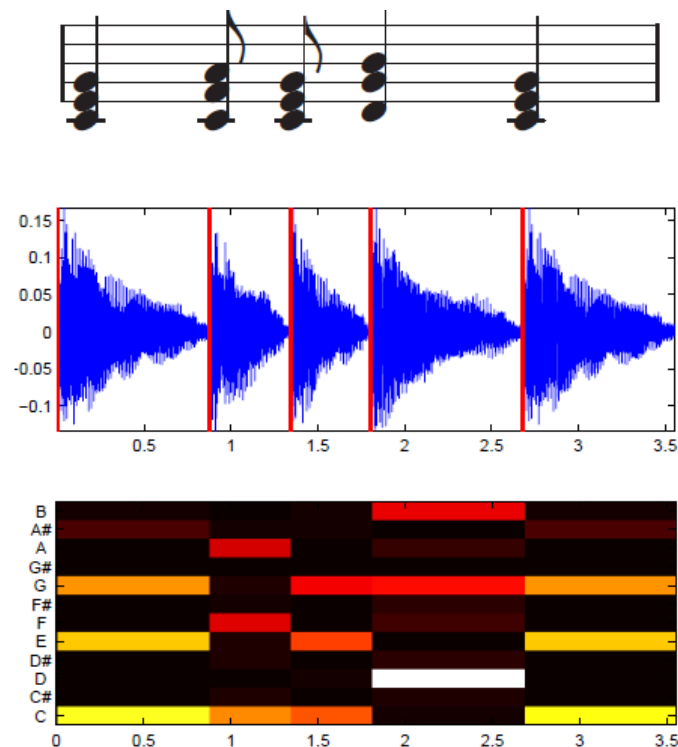
Fixed window size

[Ellis et al., ICASSP 2008] [Bello/Pickens, ISMIR 2005]

Application: Feature Design

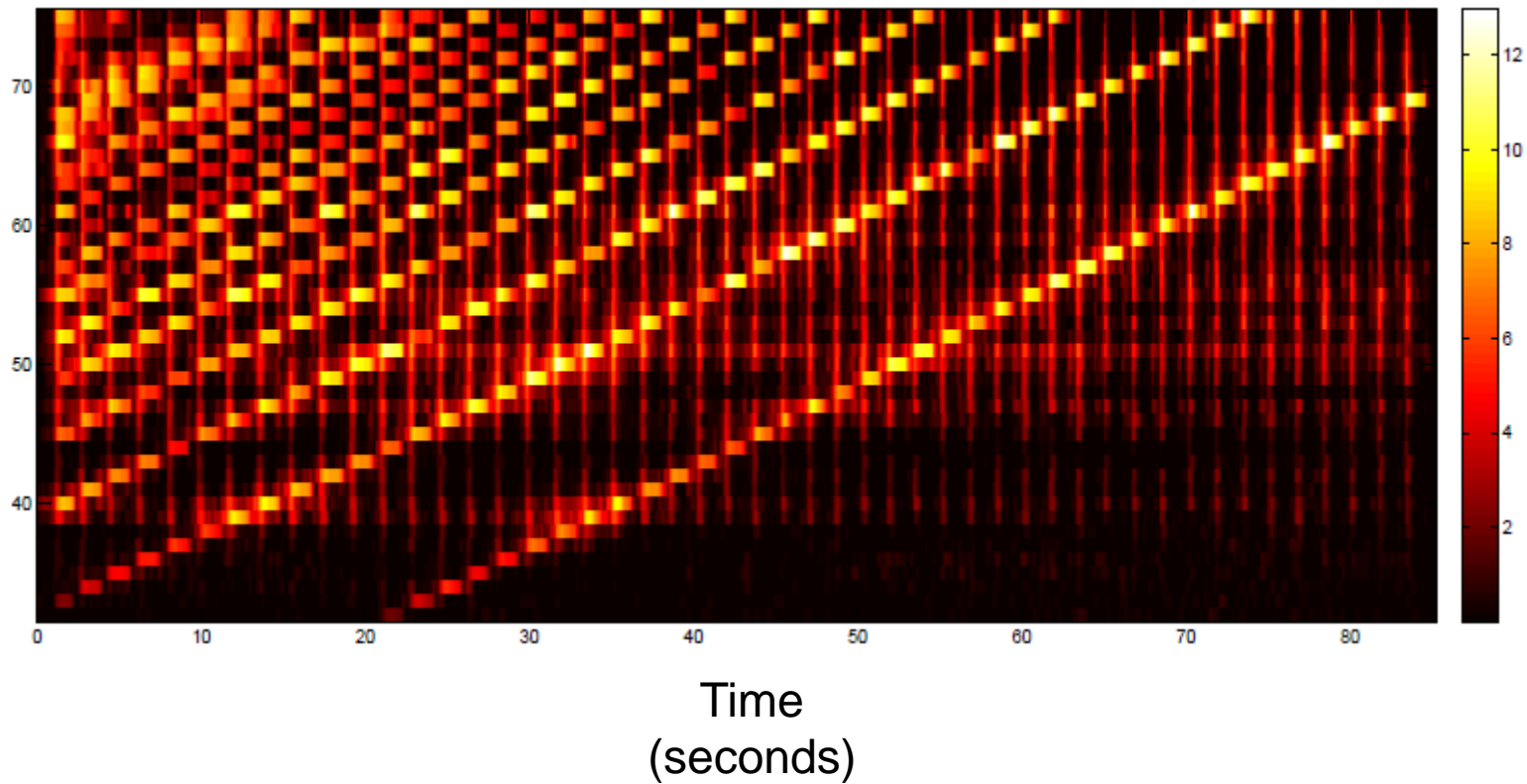


Fixed window size



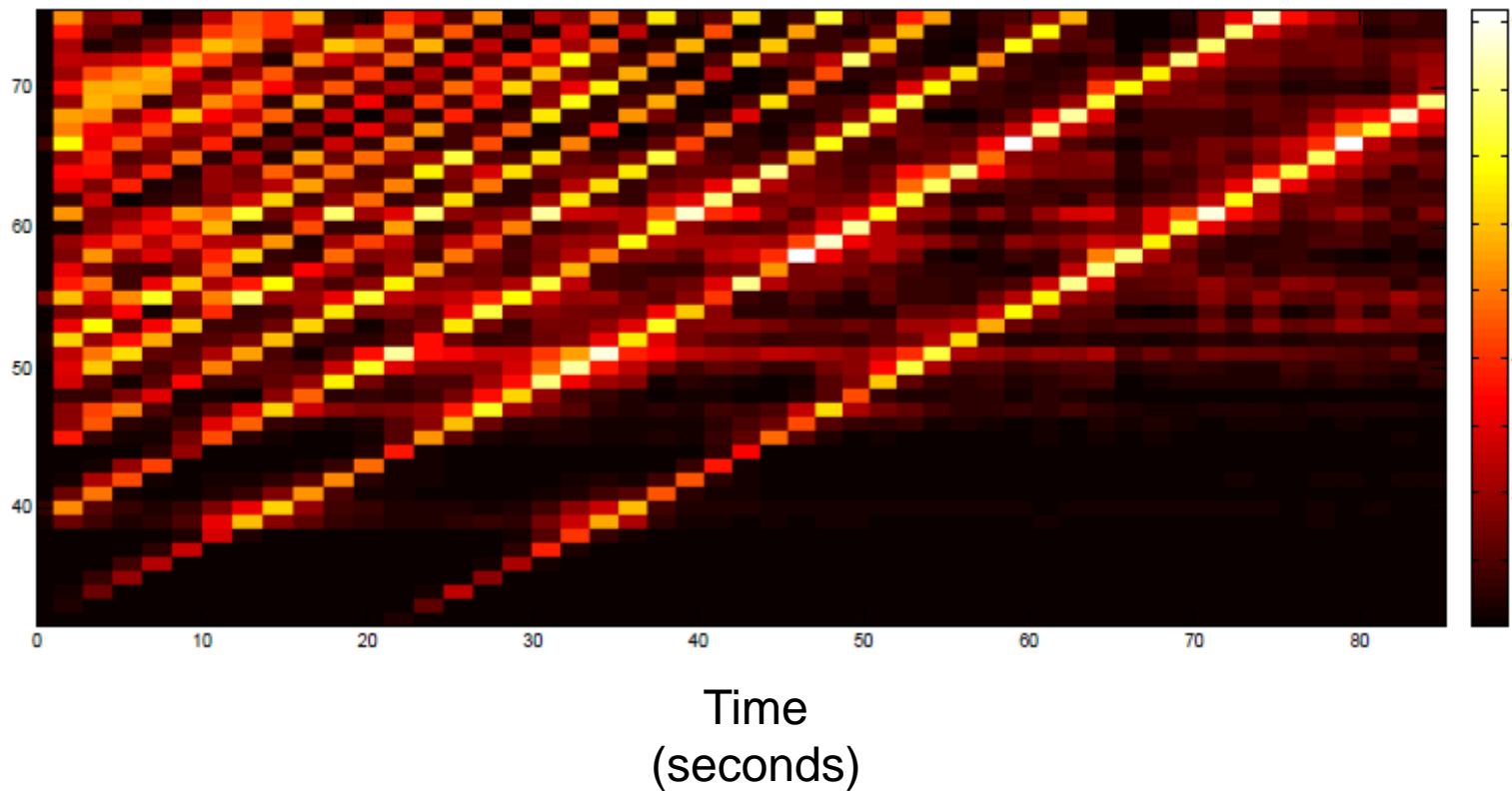
Adaptive window size

Application: Feature Design



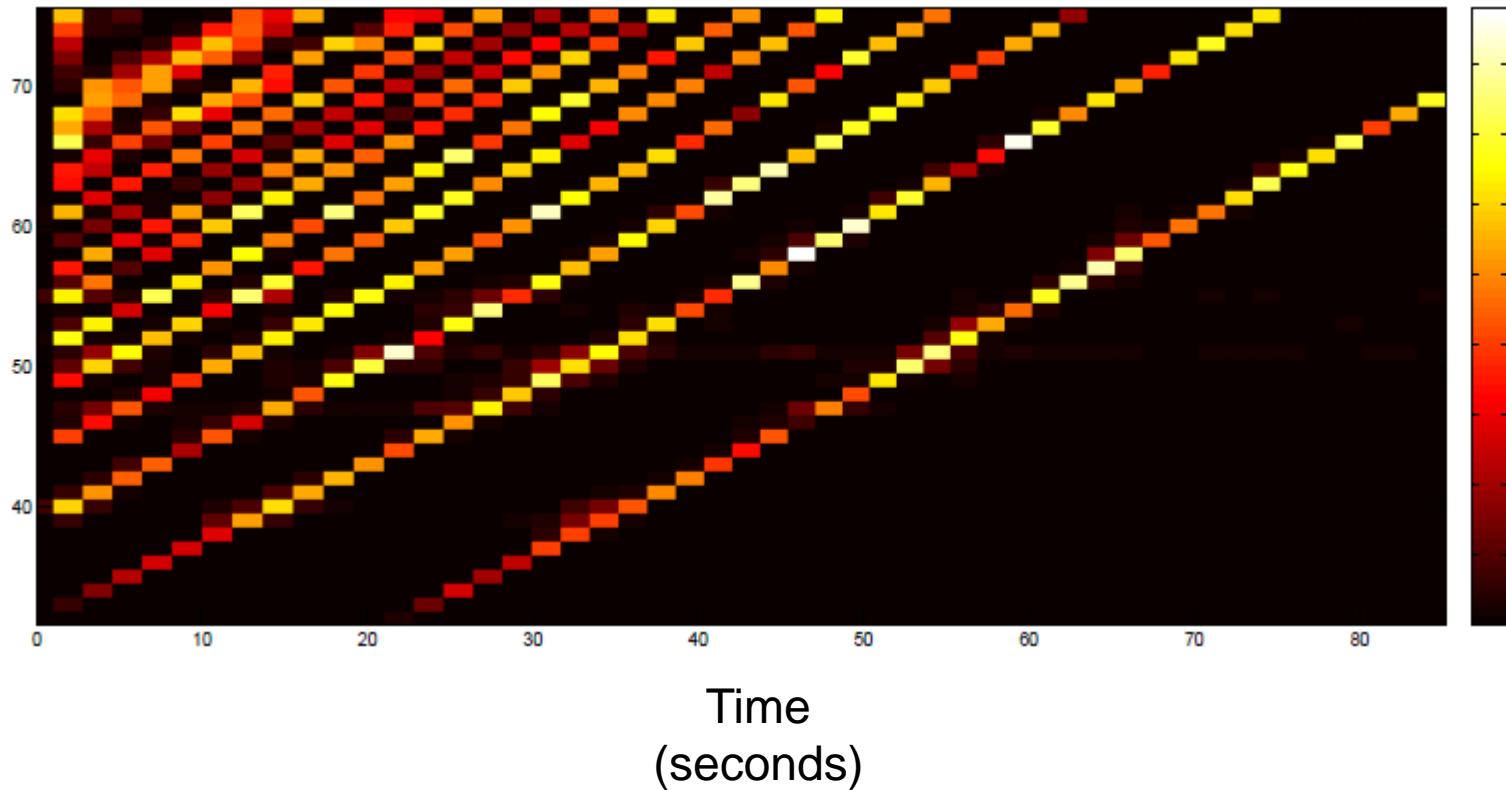
Fixed window size (100 ms)

Application: Feature Design



Adaptive window size (roughly 1200 ms)
Note onset positions define boundaries

Application: Feature Design



Adaptive window size (roughly 1200 ms)
Note onset positions define boundaries

Denoising by excluding boundary neighborhoods

Application: Audio Editing (Digital DJ)

The screenshot displays the Mixxx 1.7.0 software interface. At the top, the title bar reads "Mixxx 1.7.0" and the menu bar includes "File", "Library", "Options", and "Help".

CHANNEL 1: Displays the track "Alex Metric, Deadly On A Mission (Dub)". The waveform is green. The BPM is 123.98, and the current time is 00:24 with a total duration of 06:18. The volume level is -1.64. Controls include CUE, REV, SYNC, and a volume slider.

CHANNEL 2: Displays the track "Junior Boys, No Kinda Man (Chloé Remix)". The waveform is green. The BPM is 123.98, and the current time is 07:55 with a total duration of 08:54. The volume level is -0.00. Controls include CUE, REV, SYNC, and a volume slider.

Library: A table of tracks is visible below the channels. The track "Junior Boys - No Kinda Man (Chloé Remix)" is highlighted in red.

Artist	Title	Type	Length	kbit	BPM	Comment
Danger	11h30 - Original Mix	mp3	3:40	320	132.3	
Danger	19h11 - Original Mix	mp3	5:54	320	122.9	
Danger	7h46	mp3	5:25	160	118.0	
Evolve	Safe To Dream Thrillseekers Re	mp3	7:32	0	139.5	
Futurecop!	Class of 1984 (Anoraak Remix)	mp3	7:28	0	120.0	
Global Deejays feat. Techno...	Get Up (Before The Night Is Over) (General Elektric ...	mp3	6:35	0	128.2	
Hardfloor	Murano	mp3	8:22	0	126.6	
lio	Rapture	mp3	3:27	128	125.5	
Junior Boys	No Kinda Man (Chloé Remix)	mp3	8:54	0	124.0	
Justice	D.A.N.C.E.	mp3	4:02	0	113.0	
Justice	Newjack	mp3	3:36	0	115.1	
Justice	Waters of Nazareth	mp3	?	0	0.0	
Kavinsky	Wayfarer	mp3	4:29	128	125.4	
Kavinsky	Testarossa SebAstian Remix	mp3	4:58	0	130.0	

Application: Beat-Synchronous Light Effects



Summary

1. Onset Detection

- Novelty curve (*something is changing*)
- Indicates note onset candidates
- Hard task for non-percussive instruments (strings)

2. Tempo Estimation

- Fourier tempogram
- Autocorrelation tempogram
- Musical knowledge (tempo range, continuity)

3. Beat tracking

- Find most likely beat positions
- Exploiting phase information from Fourier tempogram