INTERNATIONAL AUDIO LABORATORIES ERLANGEN

AUDIO LABS

Lecture

Music Processing

Music Synchronization

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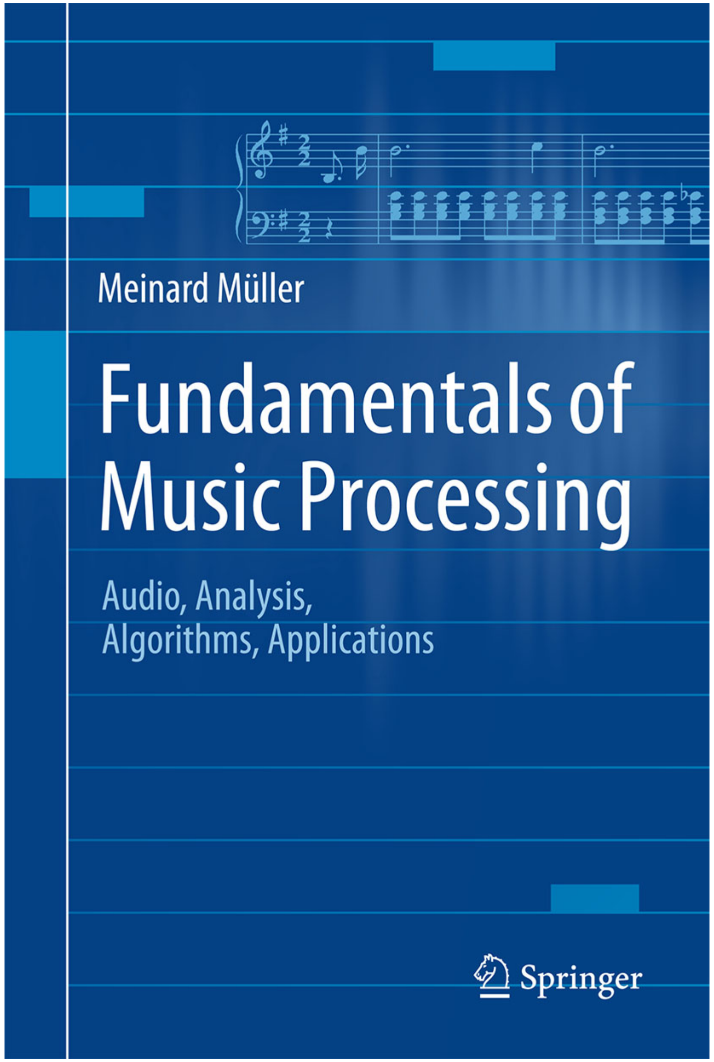
Fraunhofer IIS

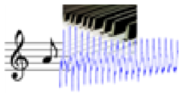
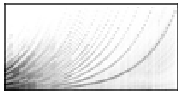
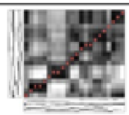
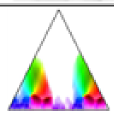
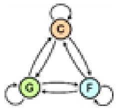
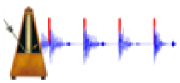
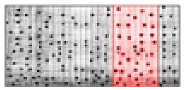
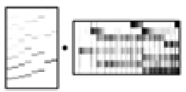
Book: Fundamentals of Music Processing



Meinard Müller
Fundamentals of Music Processing
Audio, Analysis, Algorithms, Applications
483 p., 249 illus., hardcover
ISBN: 978-3-319-21944-8
Springer, 2015

Accompanying website:
www.music-processing.de

Book: Fundamentals of Music Processing

Chapter		Music Processing Scenario
1		Music Representations
2		Fourier Analysis of Signals
3		Music Synchronization
4		Music Structure Analysis
5		Chord Recognition
6		Tempo and Beat Tracking
7		Content-Based Audio Retrieval
8		Musically Informed Audio Decomposition

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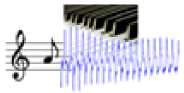
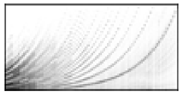
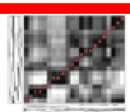
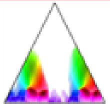
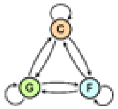
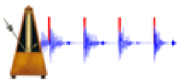
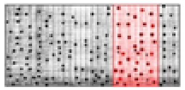
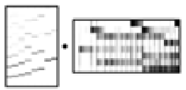
1		**Music Representations**
2		**Fourier Analysis of Signals**
3		**Music Synchronization**
4		**Music Structure Analysis**
5		**Chord Recognition**
6		**Tempo and Beat Tracking**
7		**Content-Based Audio Retrieval**
8		**Musically Informed Audio Decomposition**

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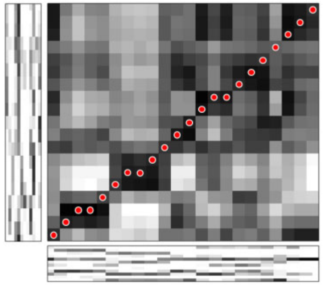
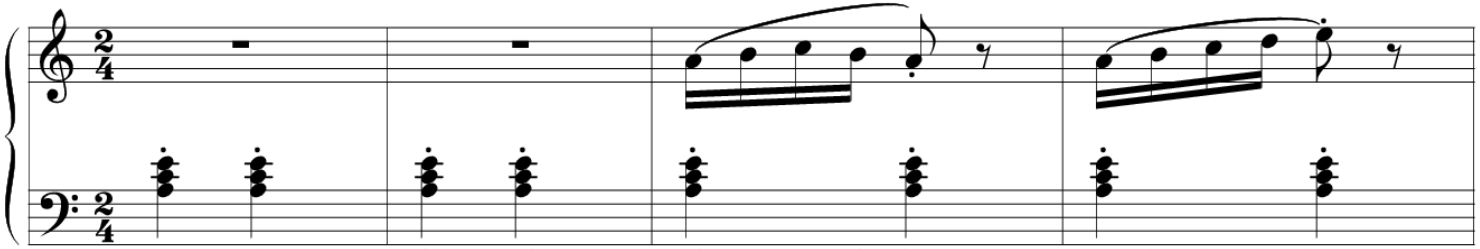
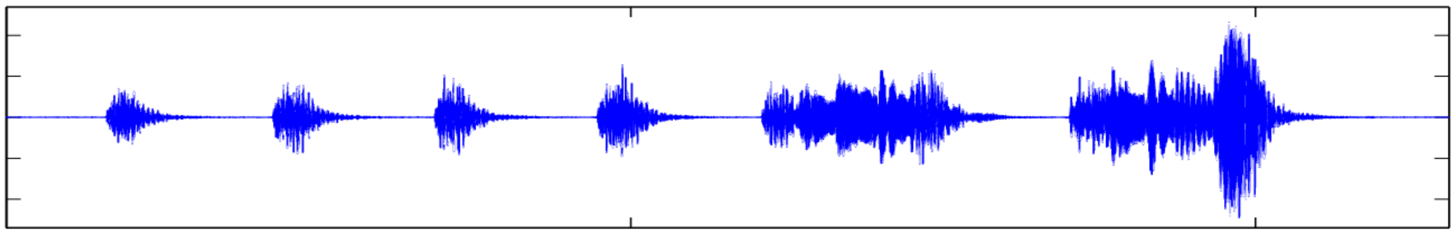
Chapter 3: Music Synchronization

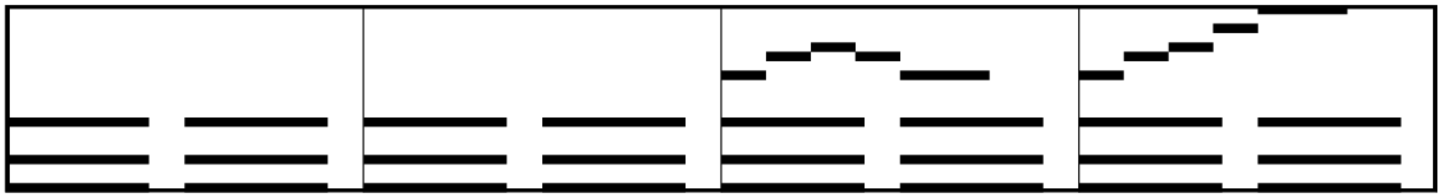


As a first music processing task, we study in Chapter 3 the problem of music synchronization. The objective is to temporally align compatible representations of the same piece of music. Considering this scenario, we explain the need for musically informed audio features. In particular, we introduce the concept of chroma-based music features, which capture properties that are related to harmony and melody. Furthermore, we study an alignment technique known as dynamic time warping (DTW), a concept that is applicable for the analysis of general time series. For its efficient computation, we discuss an algorithm based on dynamic programming—a widely used method for solving a complex problem by breaking it down into a collection of simpler subproblems.

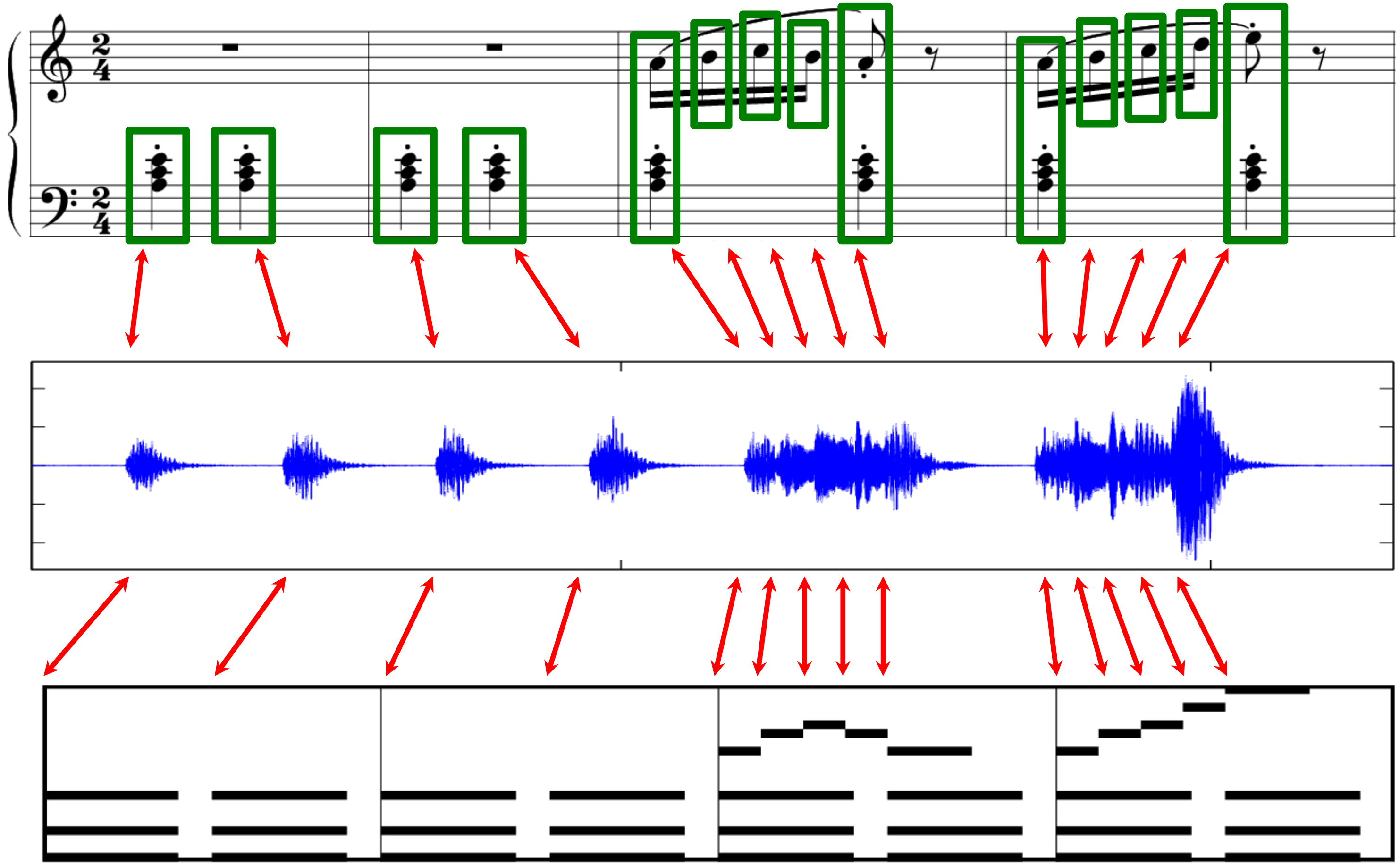
Music Data





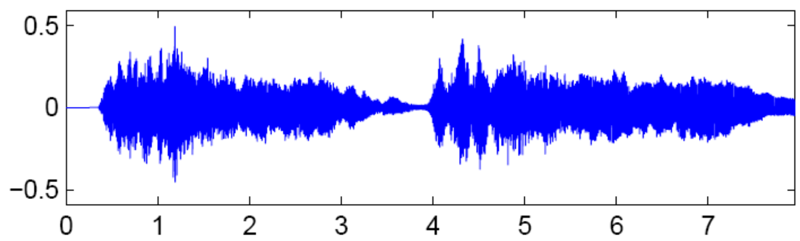


Music Data

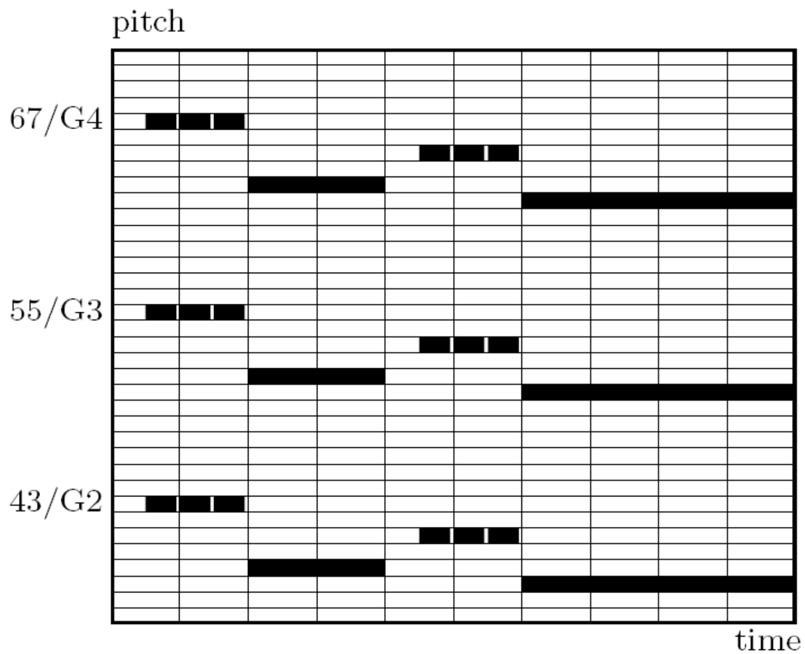


Music Data

Allegro con brio (= 108)
ff
Ped.
Ped.



0.5
0
−0.5
0
1
2
3
4
5
6
7



pitch
67/G4
55/G3
43/G2
time

Music Data

Various interpretations – Beethoven's Fifth

Bernstein	
Karajan	
Gould (piano)	
MIDI (piano)	

Music Synchronization: Audio-Audio

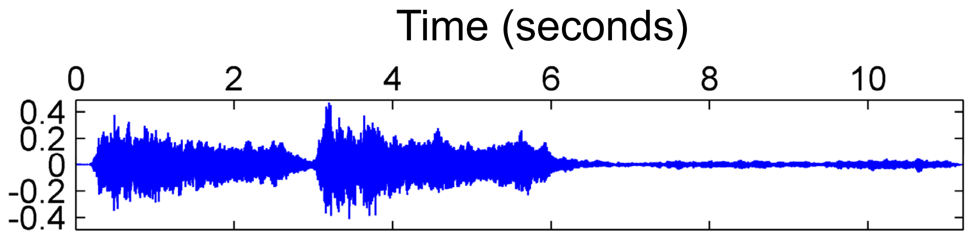
Given: Two different audio recordings of the same underlying piece of music.

Goal: Find for each position in one audio recording the musically corresponding position in the other audio recording.

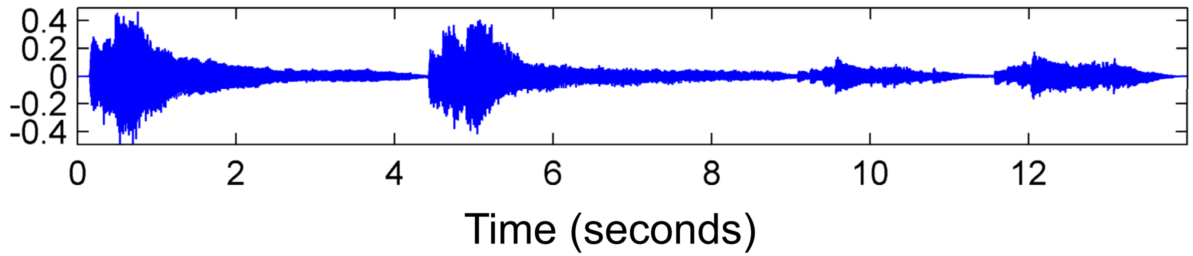
Music Synchronization: Audio-Audio

Beethoven's Fifth

Karajan



Gould

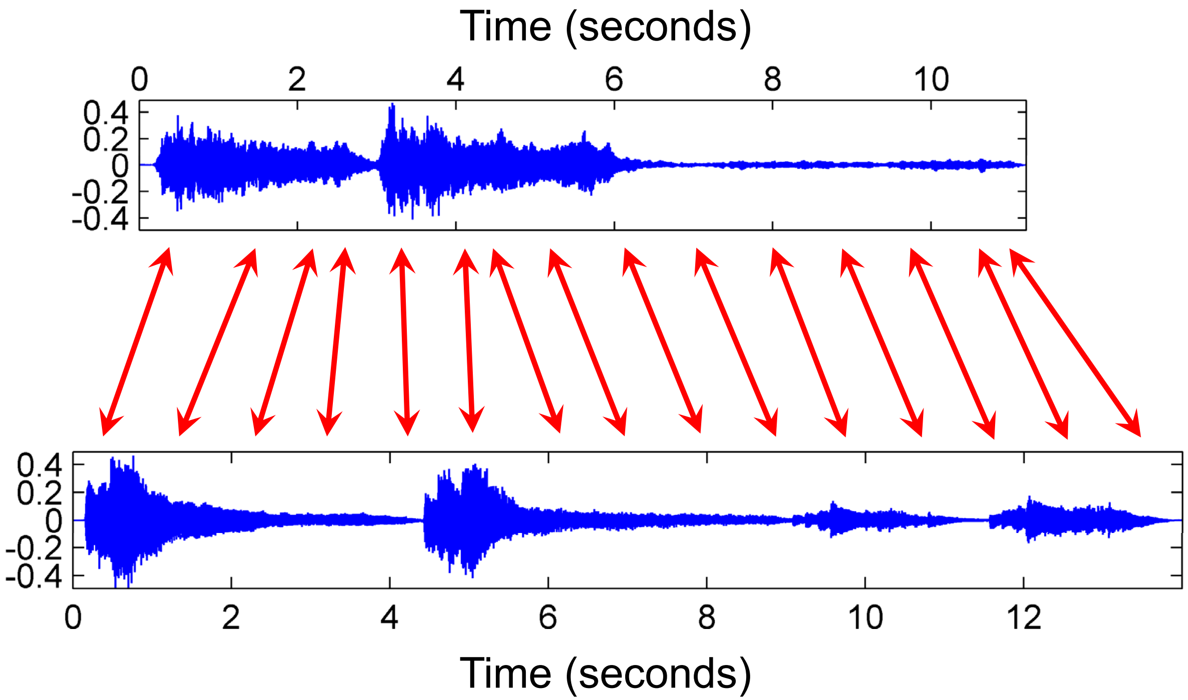


Music Synchronization: Audio-Audio

Beethoven's Fifth

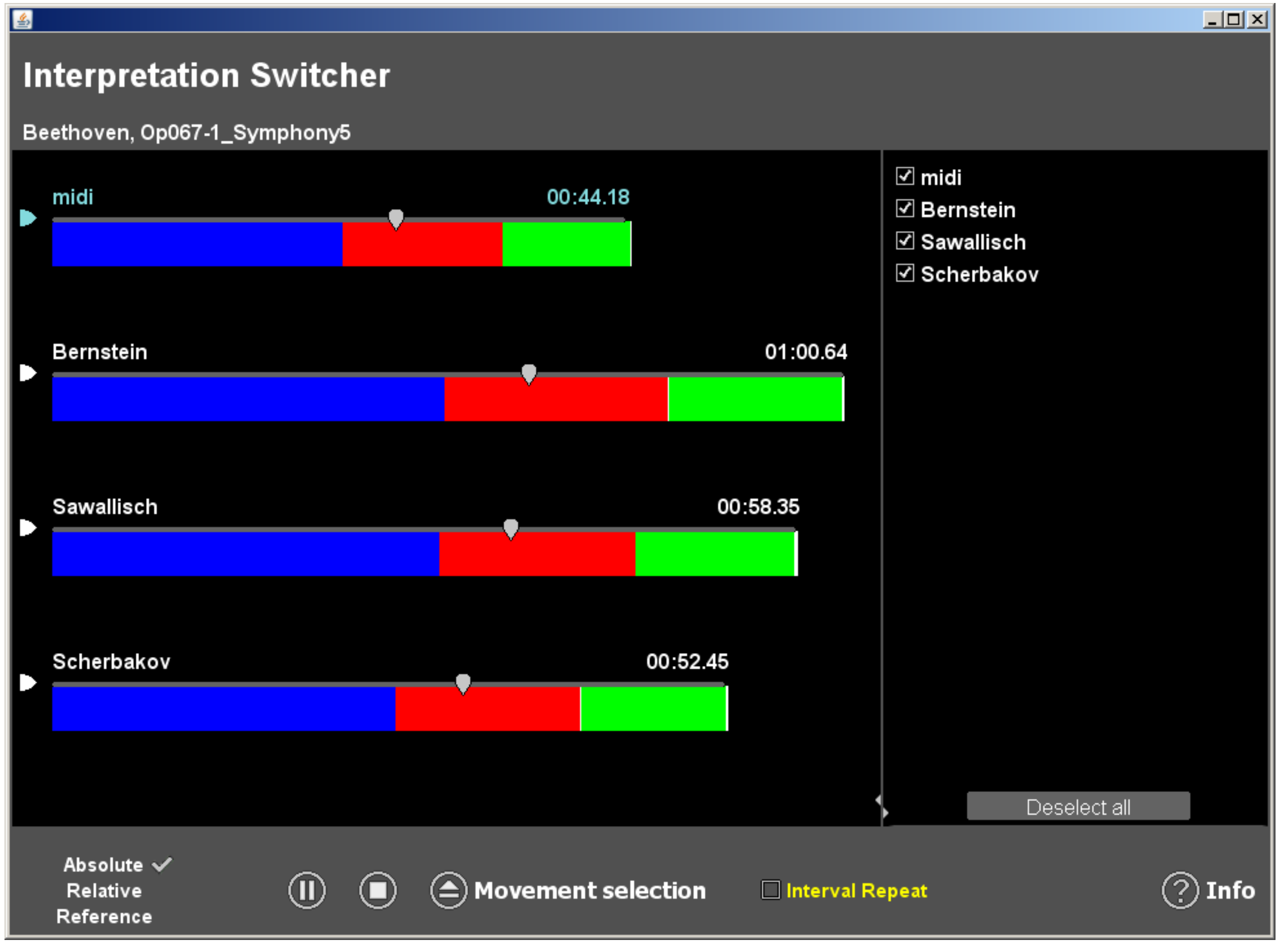
Karajan

Gould



Music Synchronization: Audio-Audio

Application: Interpretation Switcher



Music Synchronization: Audio-Audio

Two main steps:

1.) Audio features

- Robust but discriminative
- Chroma features
- Robust to variations in instrumentation, timbre, dynamics
- Correlate to harmonic progression

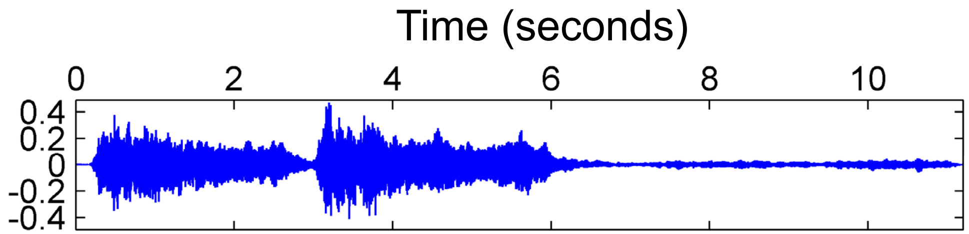
2.) Alignment procedure

- Deals with local and global tempo variations
- Needs to be efficient

Music Synchronization: Audio-Audio

Beethoven's Fifth

Karajan



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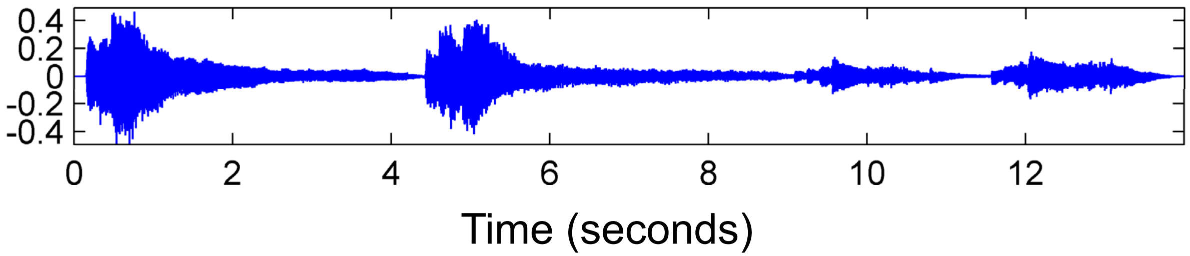
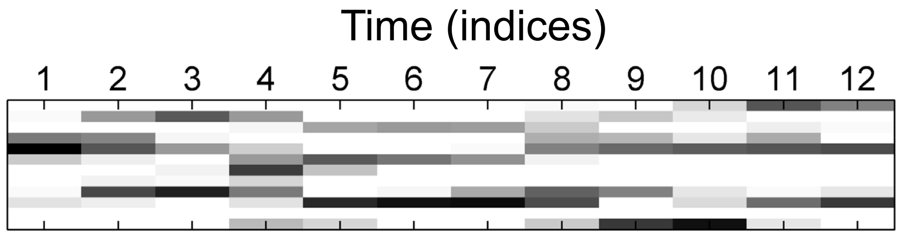


Music Synchronization: Audio-Audio

Beethoven's Fifth

Karajan



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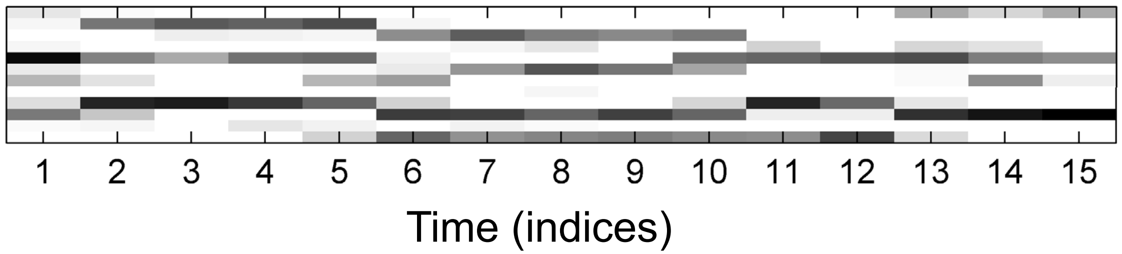
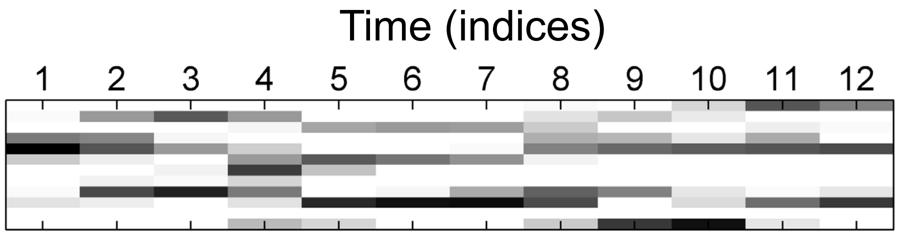


Music Synchronization: Audio-Audio

Beethoven's Fifth

Karajan



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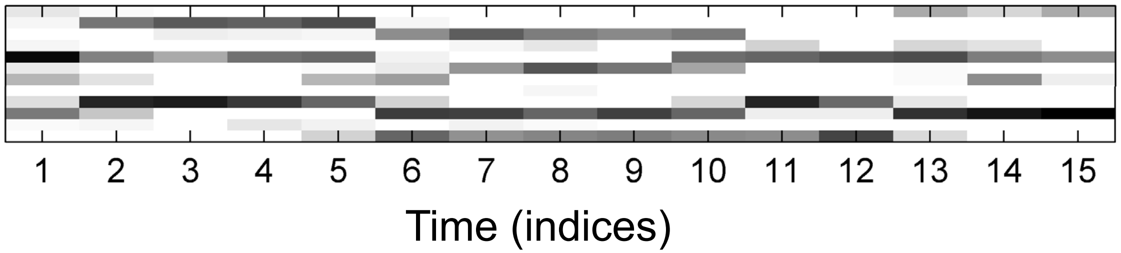
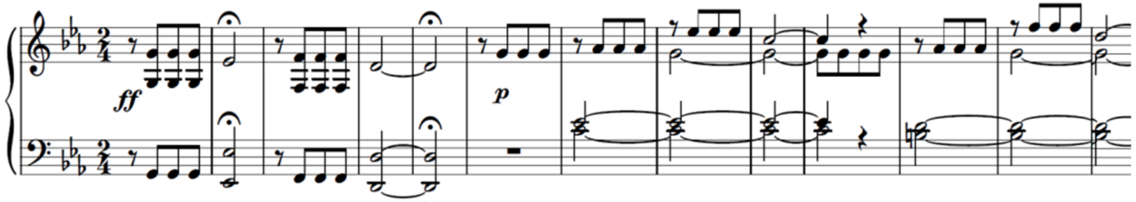
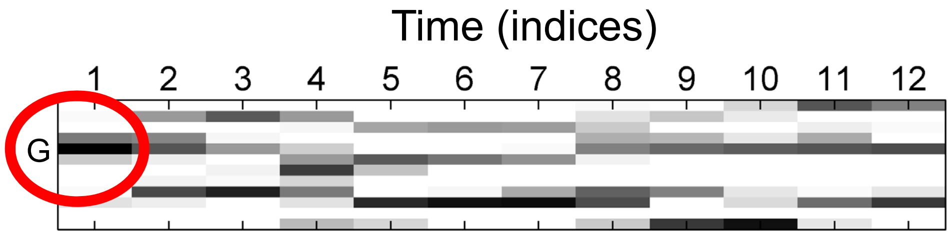




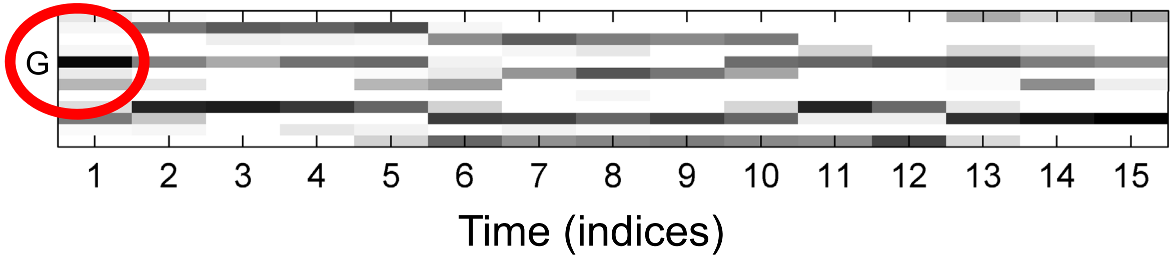
Music Synchronization: Audio-Audio

Beethoven's Fifth

Karajan



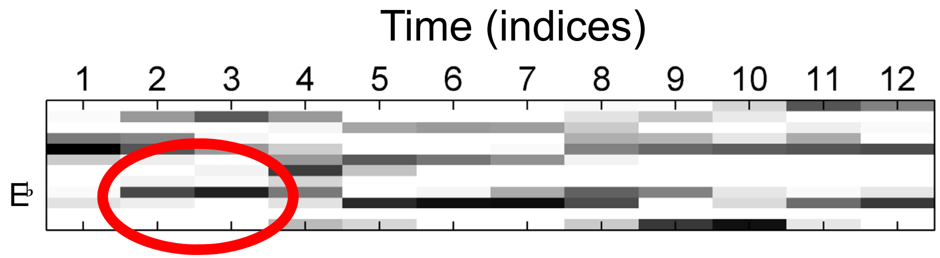
Gould



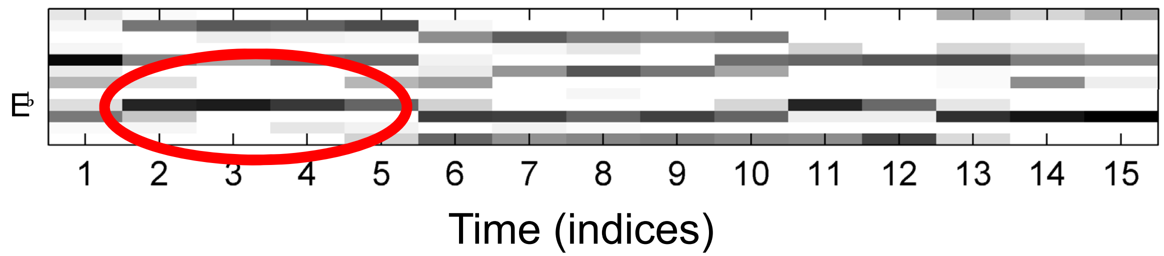
Music Synchronization: Audio-Audio

Beethoven's Fifth

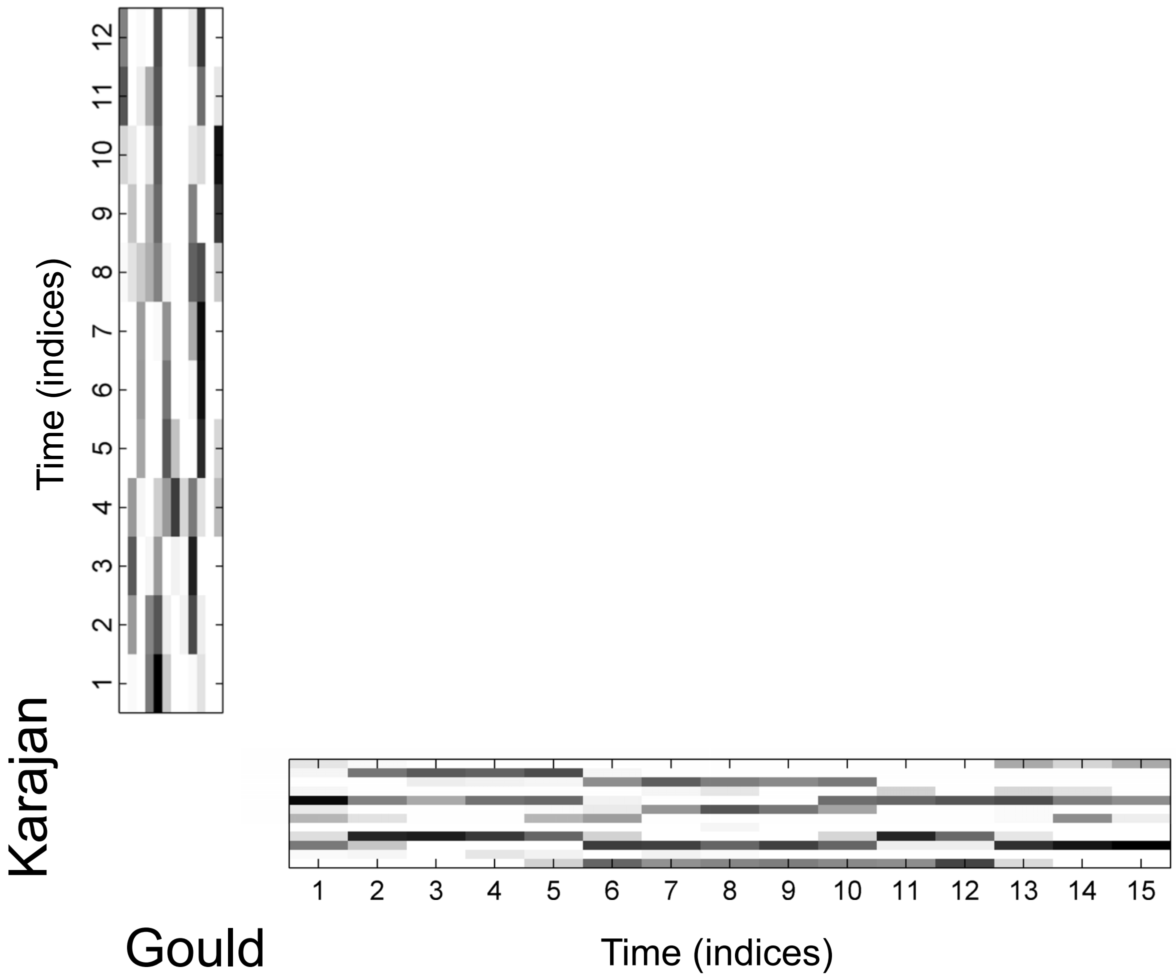
Karajan



Gould

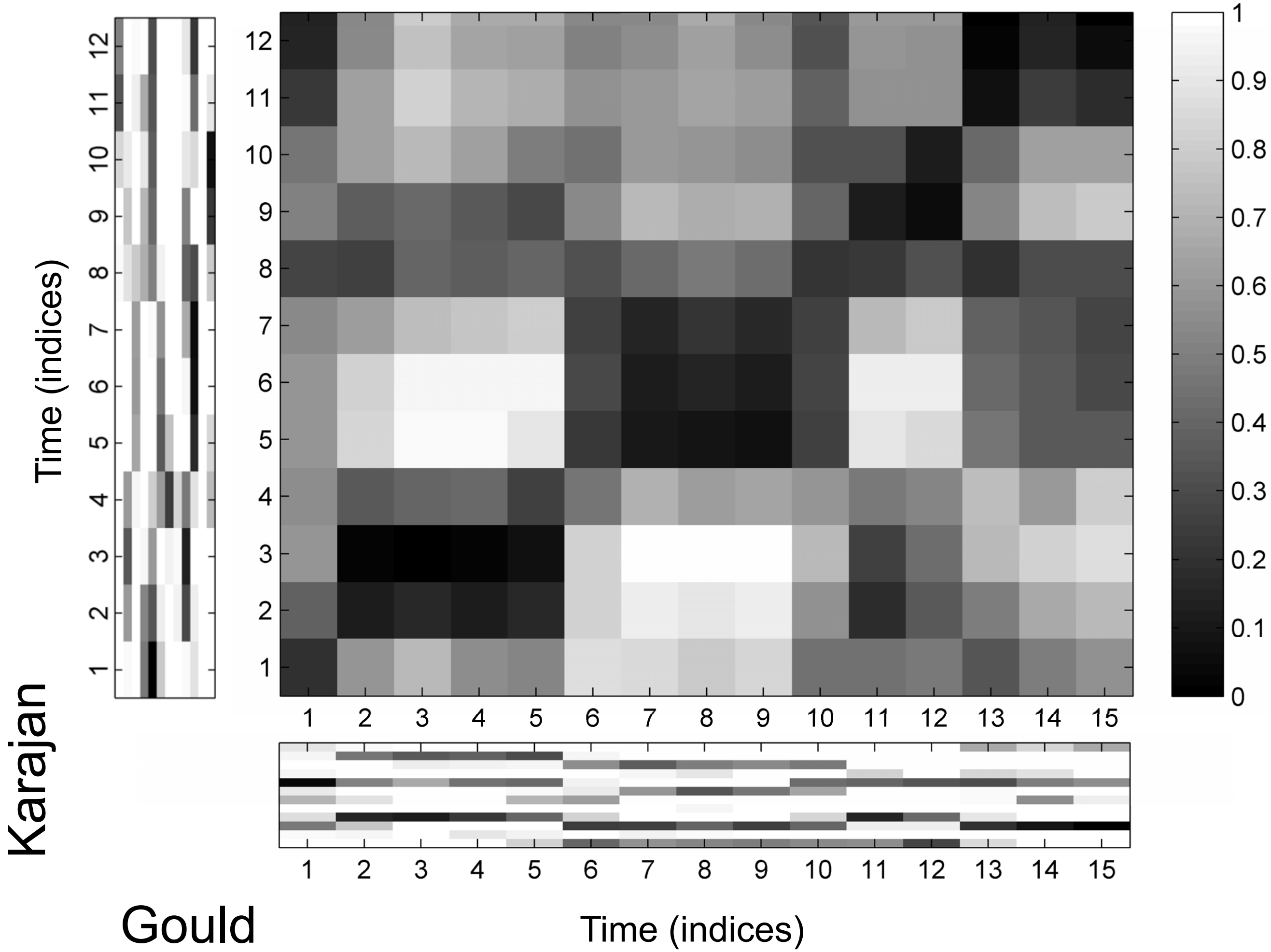


Music Synchronization: Audio-Audio



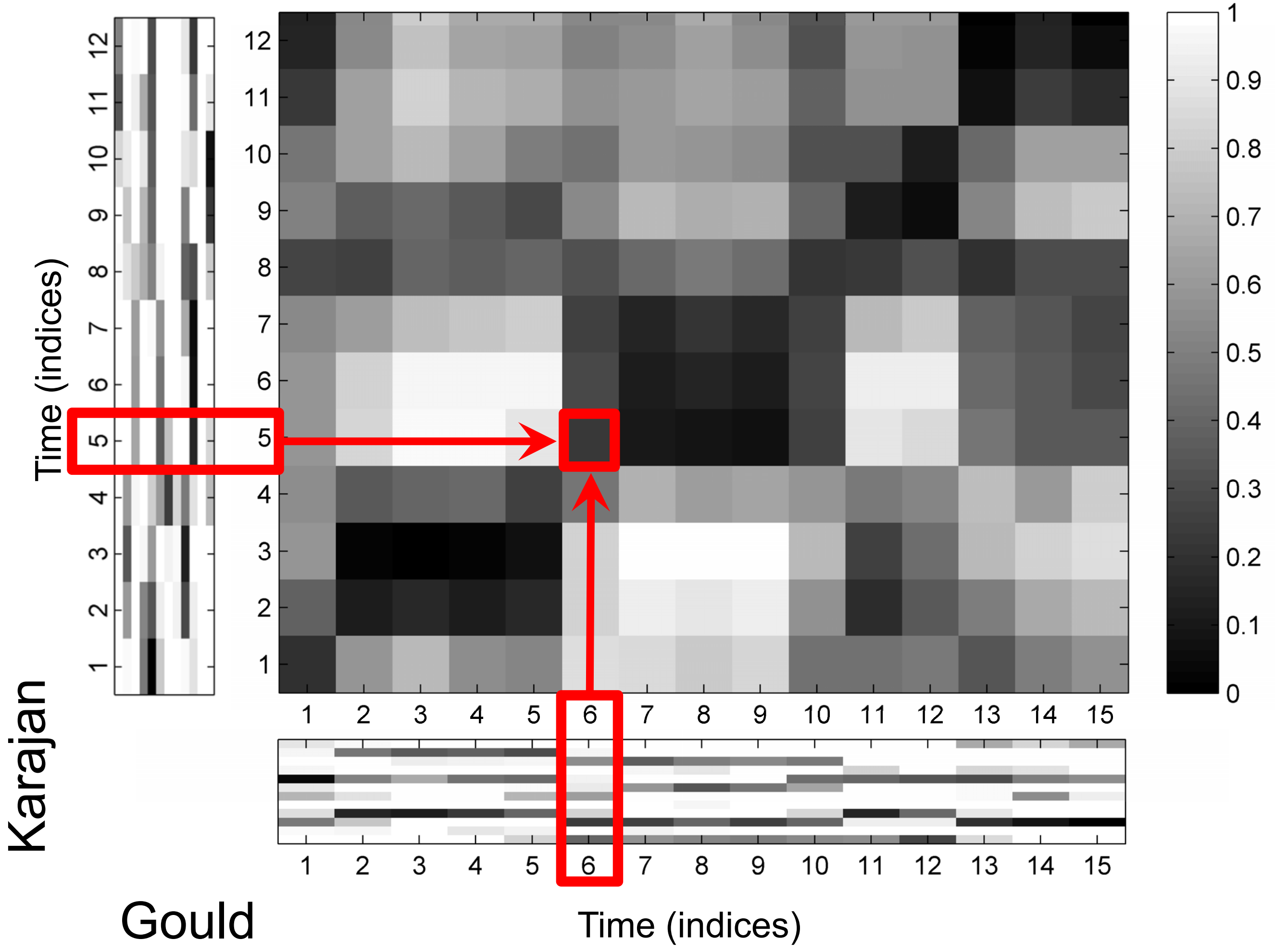
Music Synchronization: Audio-Audio

Cost matrix



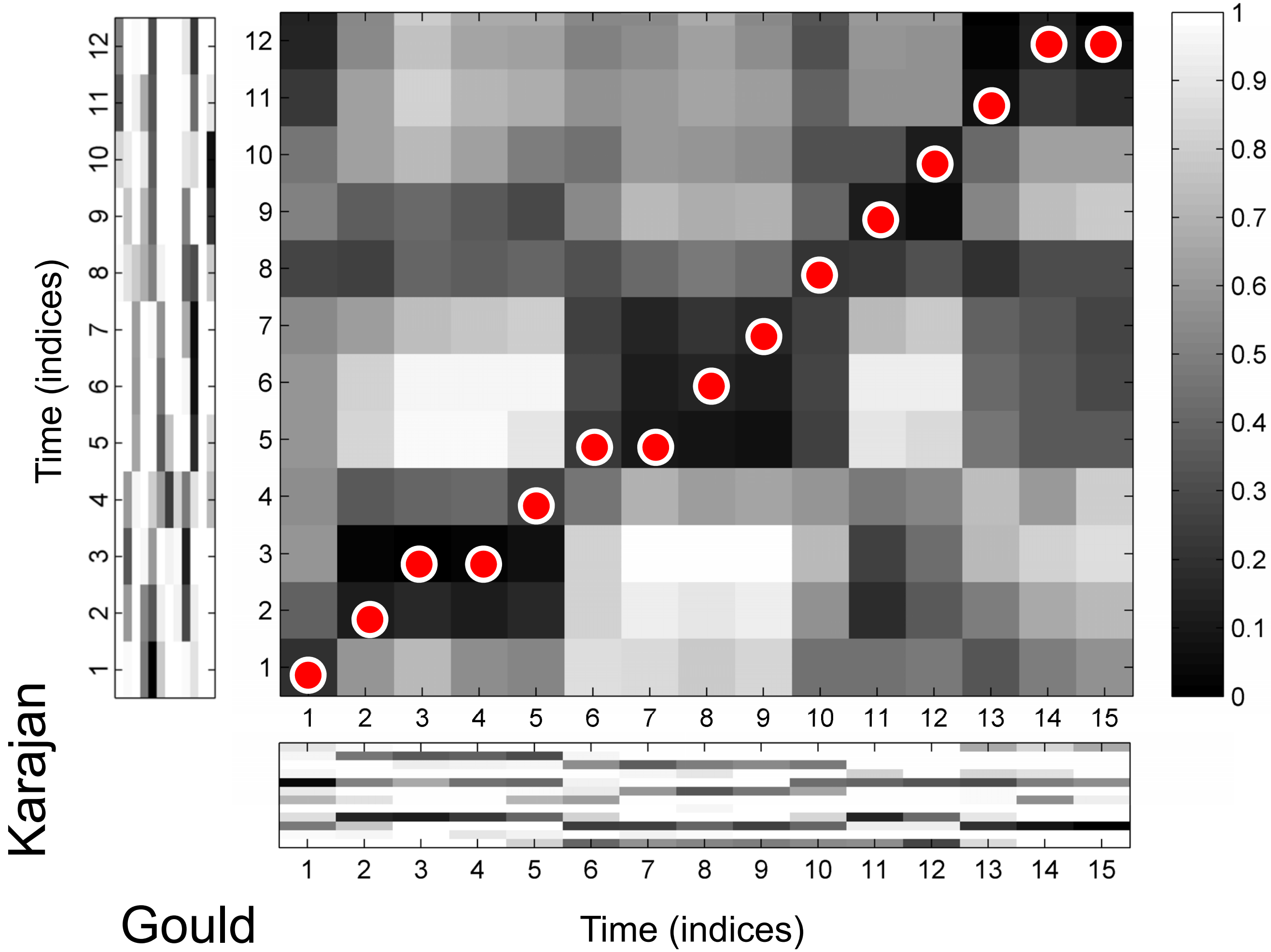
Music Synchronization: Audio-Audio

Cost matrix



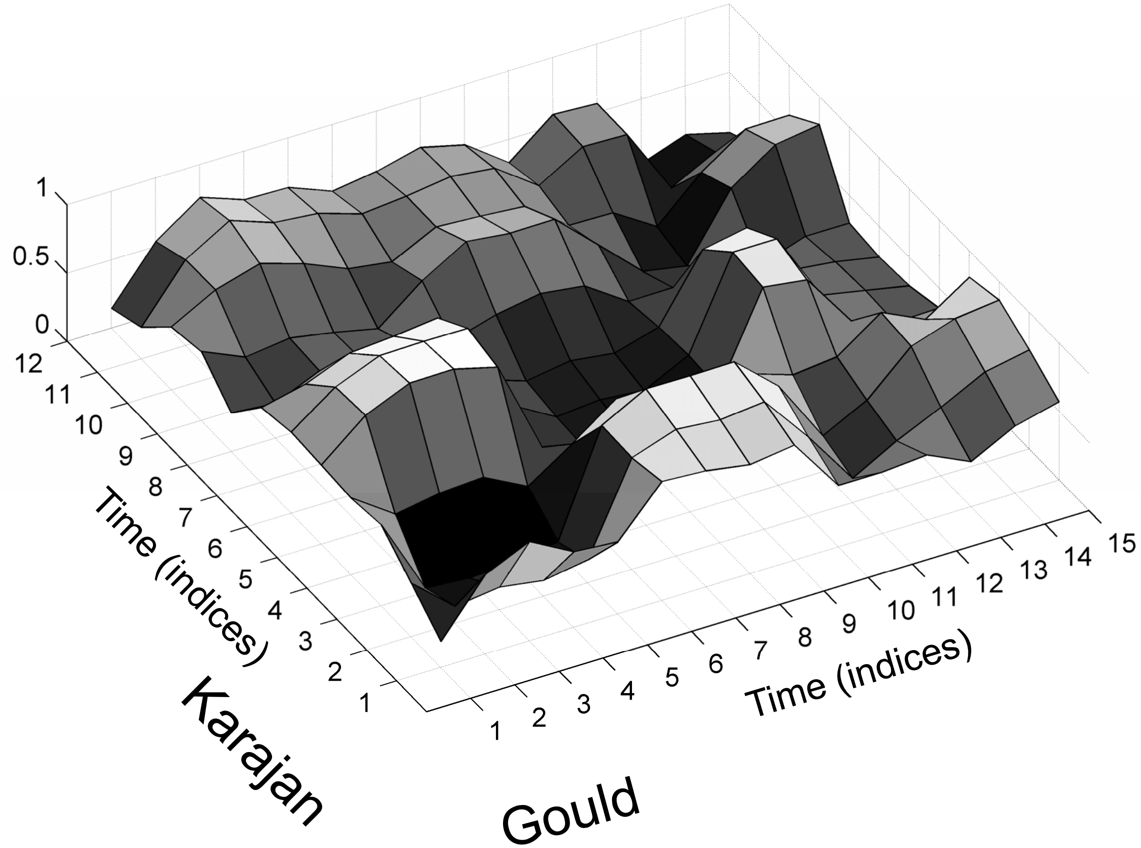
Music Synchronization: Audio-Audio

Optimal alignment (cost-minimizing warping path)



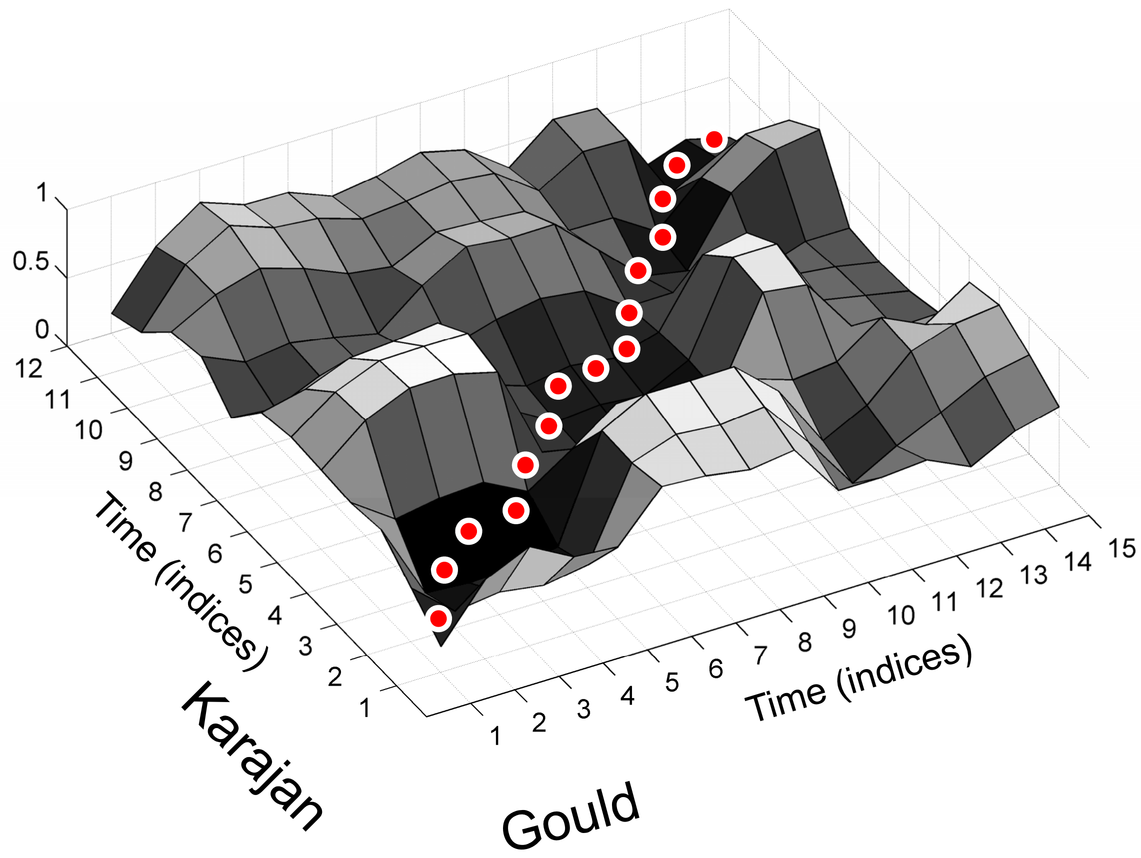
Music Synchronization: Audio-Audio

Cost matrix



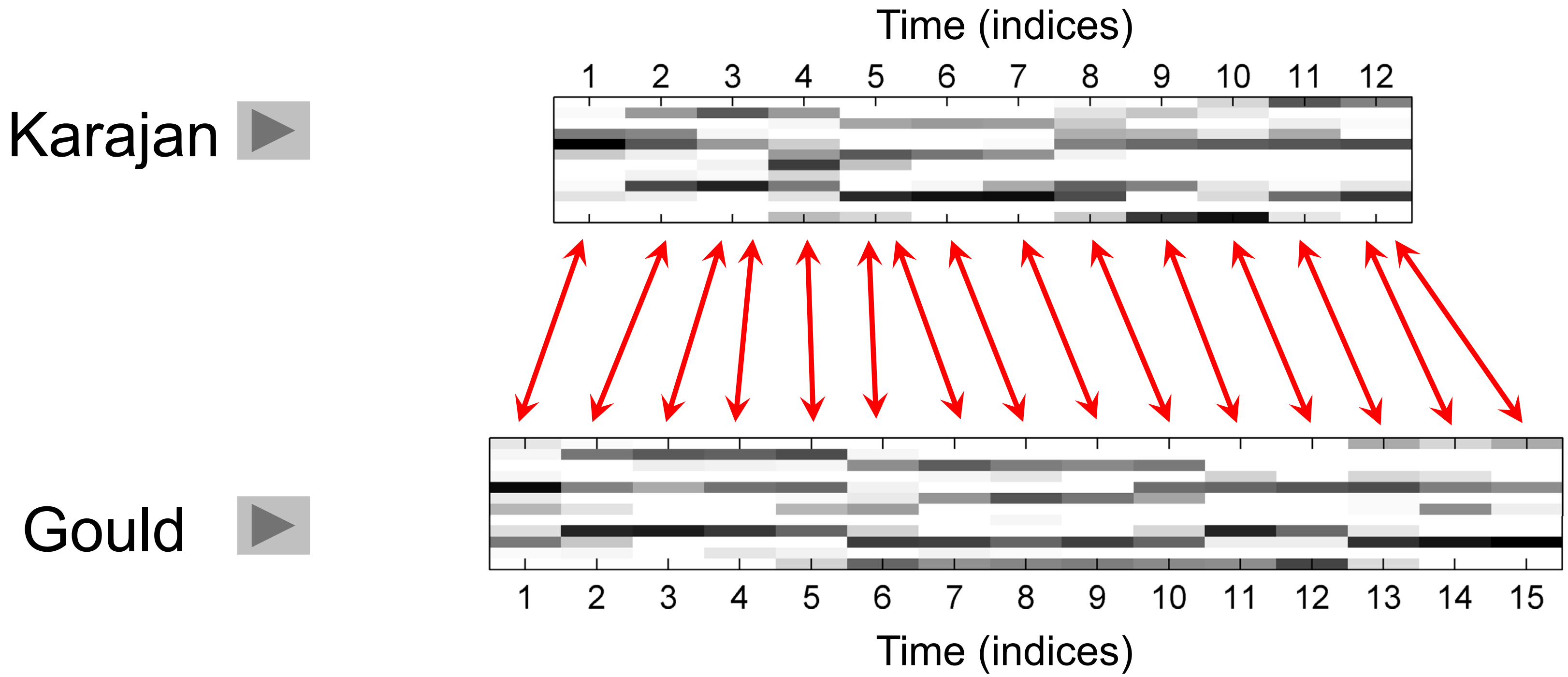
Music Synchronization: Audio-Audio

Optimal alignment (cost-minimizing warping path)



Music Synchronization: Audio-Audio

Optimal alignment (cost-minimizing warping path)



Music Synchronization: Audio-Audio

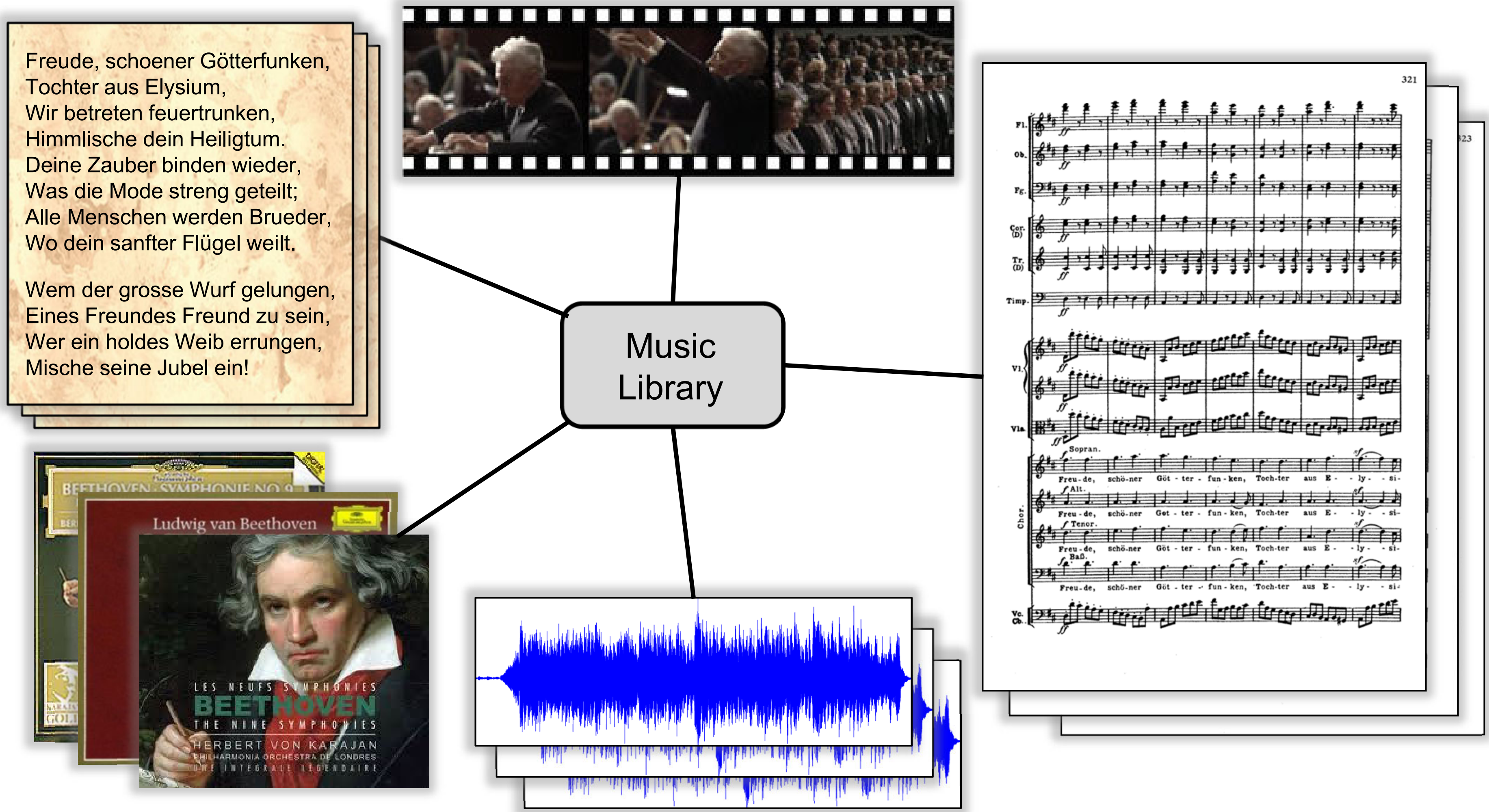
How to compute the alignment?

$\Rightarrow$ Cost matrices

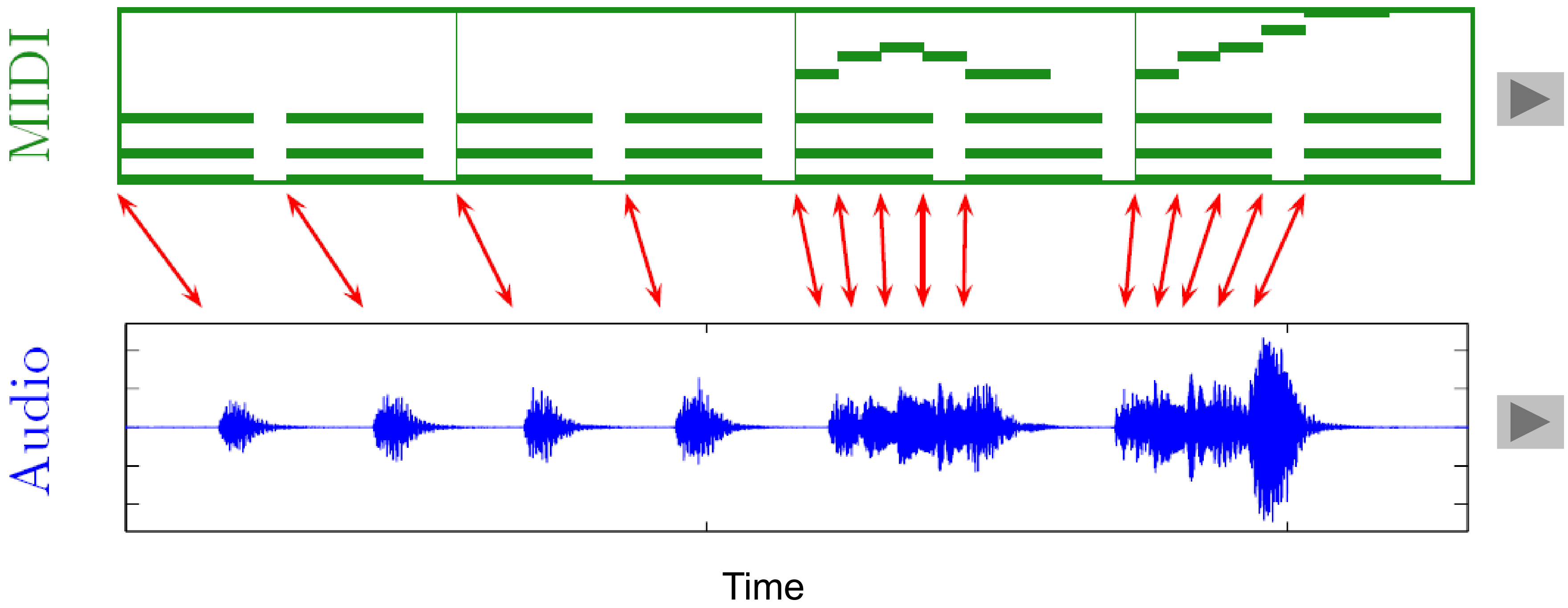
$\Rightarrow$ Dynamic programming

$\Rightarrow$ Dynamic Time Warping (DTW)

Applications



Music Synchronization: MIDI-Audio



Music Synchronization: MIDI-Audio

MIDI = meta data

Automated annotation

Audio recording

Sonification of annotations

Music Synchronization: MIDI-Audio

- Automated audio annotation
- Accurate audio access after MIDI-based retrieval
- Automated tracking of MIDI note parameters during audio playback
- Performance analysis

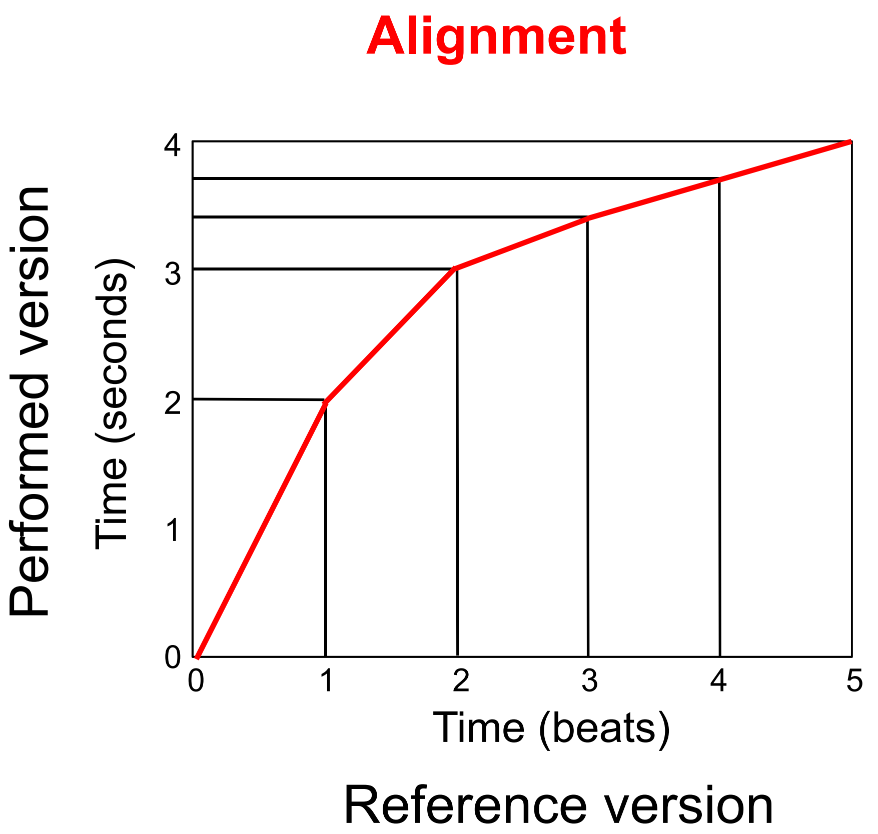
Music Synchronization: MIDI-Audio

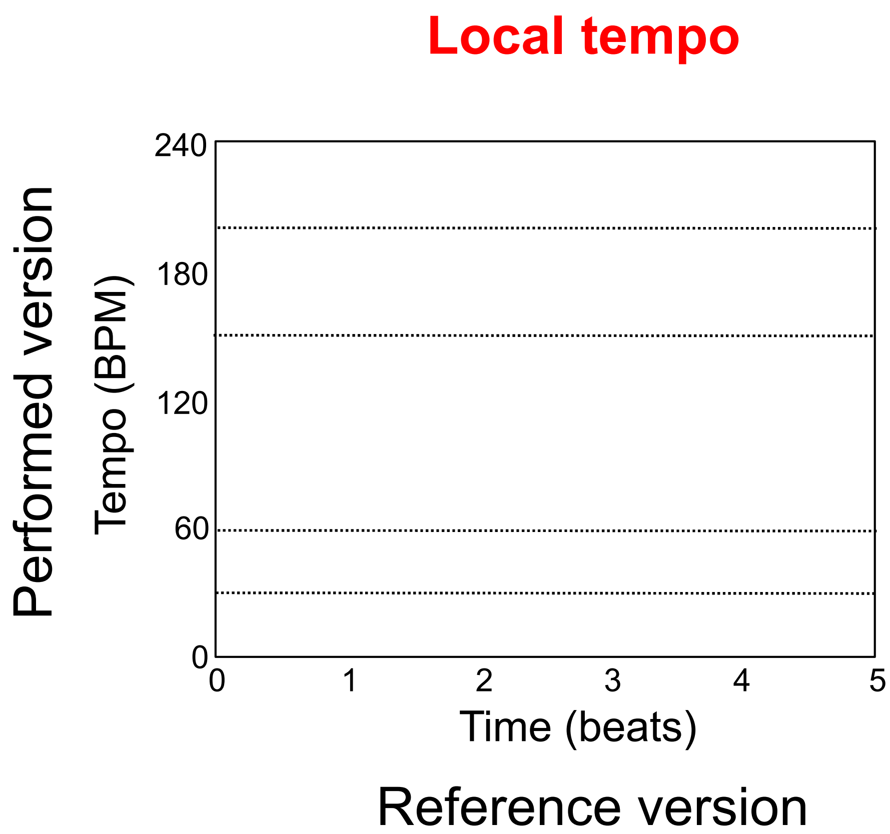
MIDI = reference (score)

Tempo information

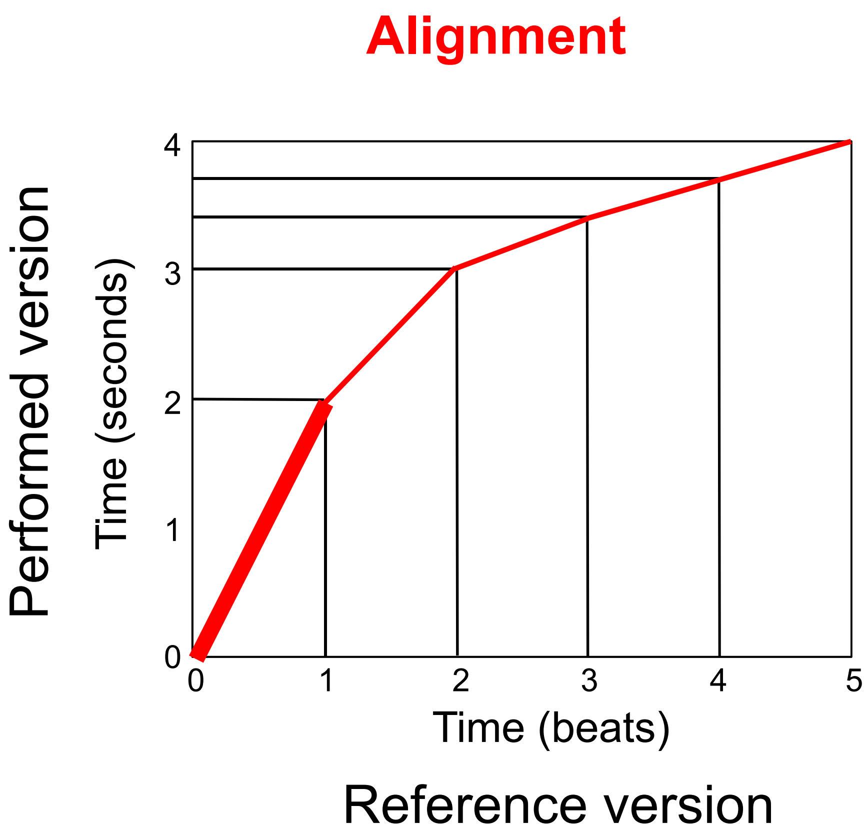
Audio recording

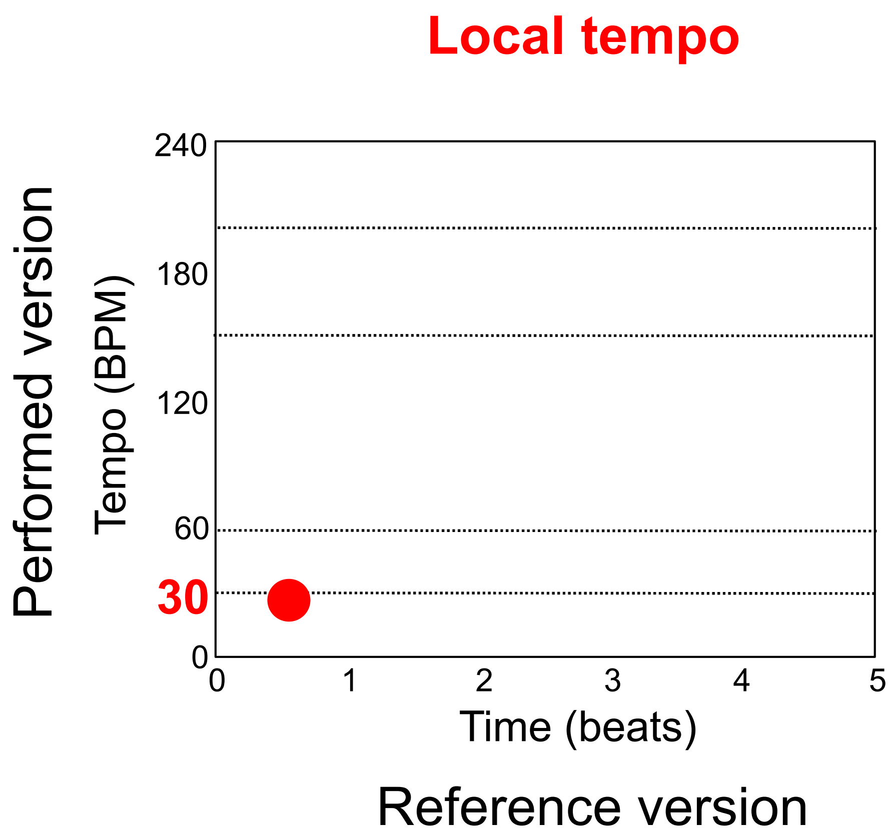
Performance Analysis: Tempo Curves





Performance Analysis: Tempo Curves

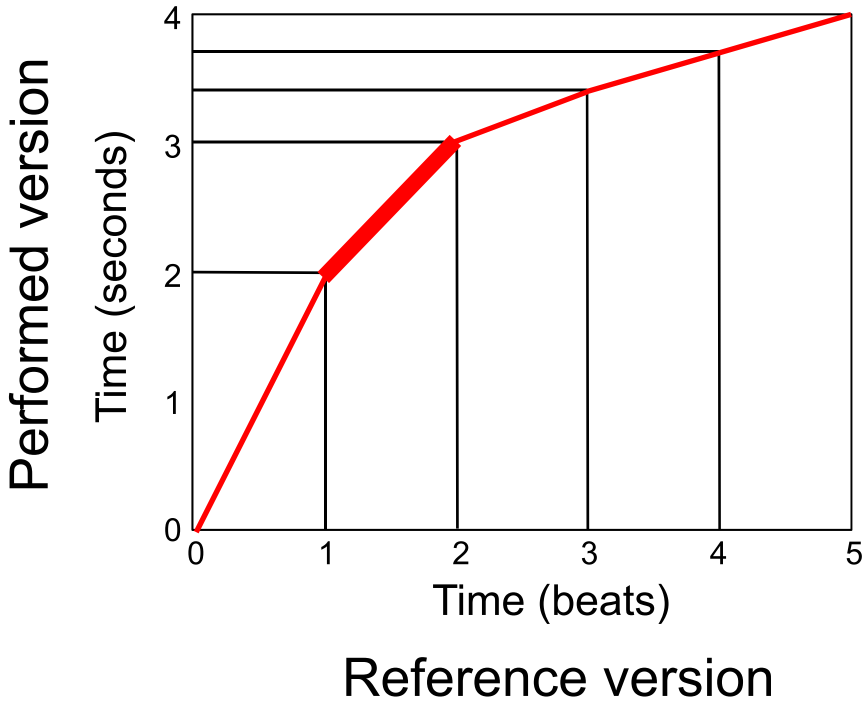




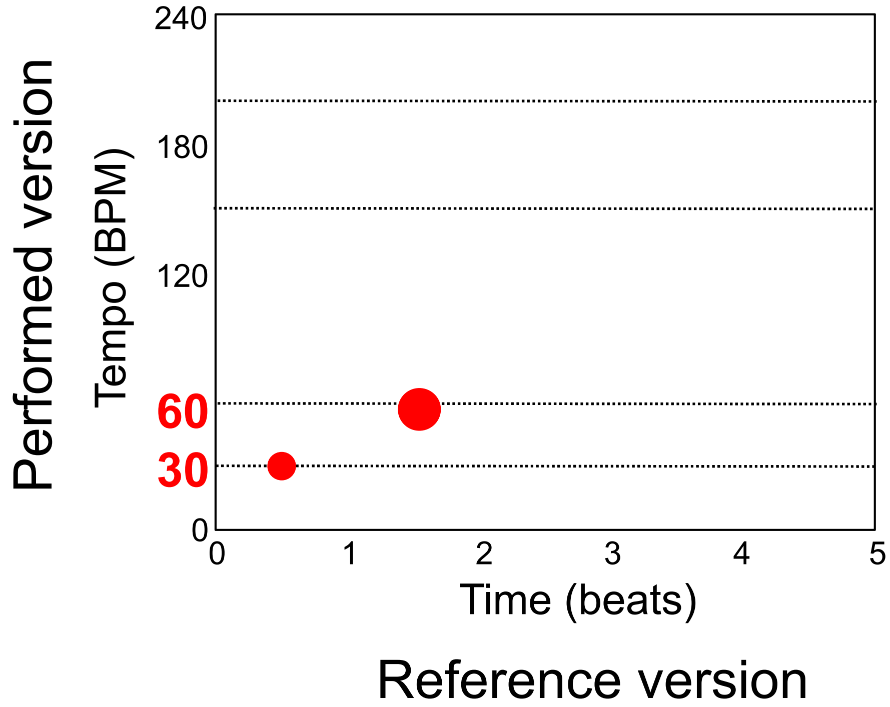
1 beat lasting 2 seconds ≙ 30 BPM

Performance Analysis: Tempo Curves

Alignment

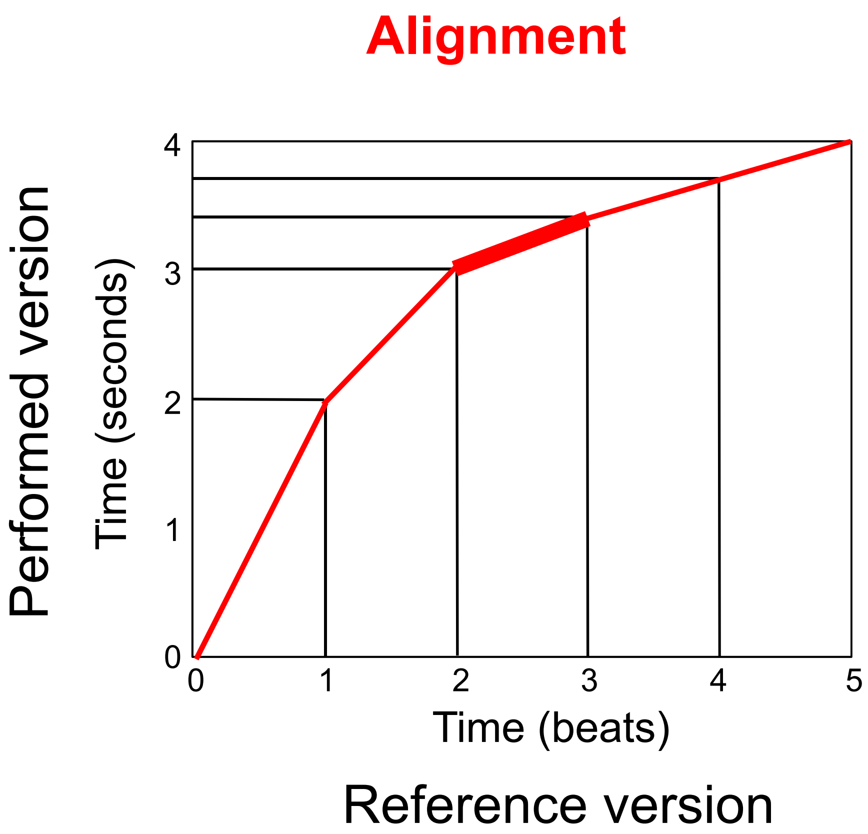


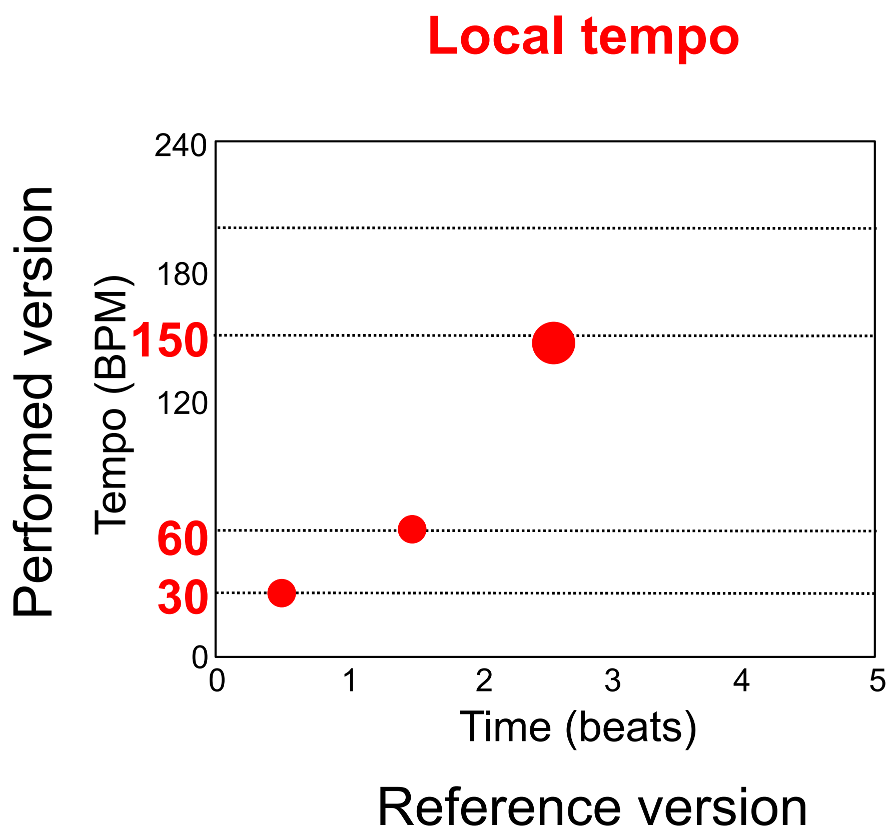
Local tempo



1 beat lasting 1 seconds ≙ 60 BPM

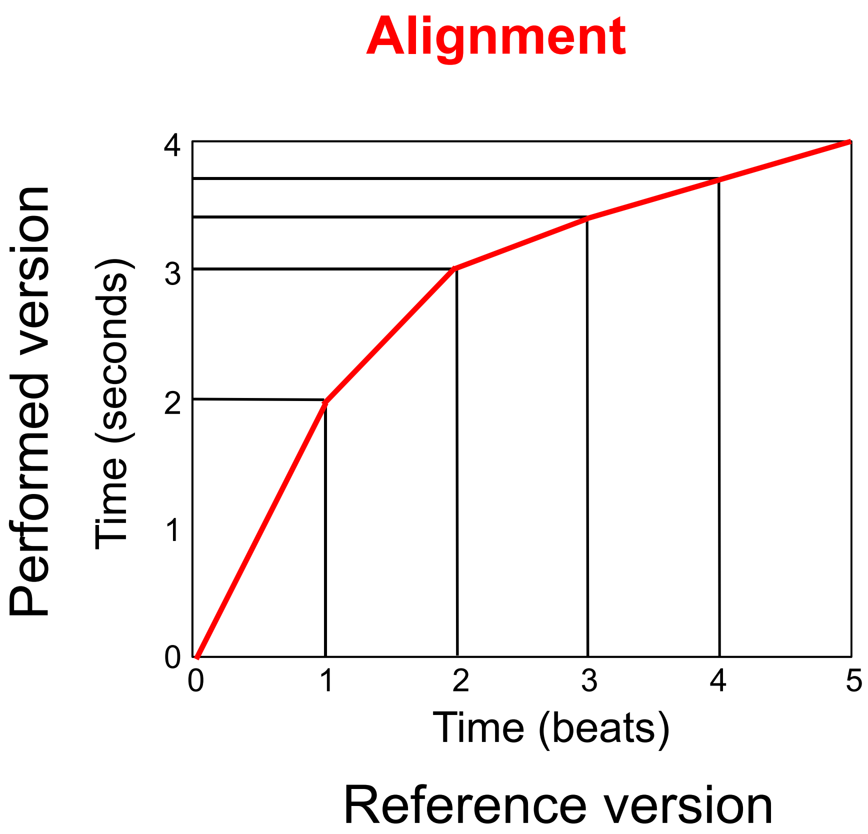
Performance Analysis: Tempo Curves

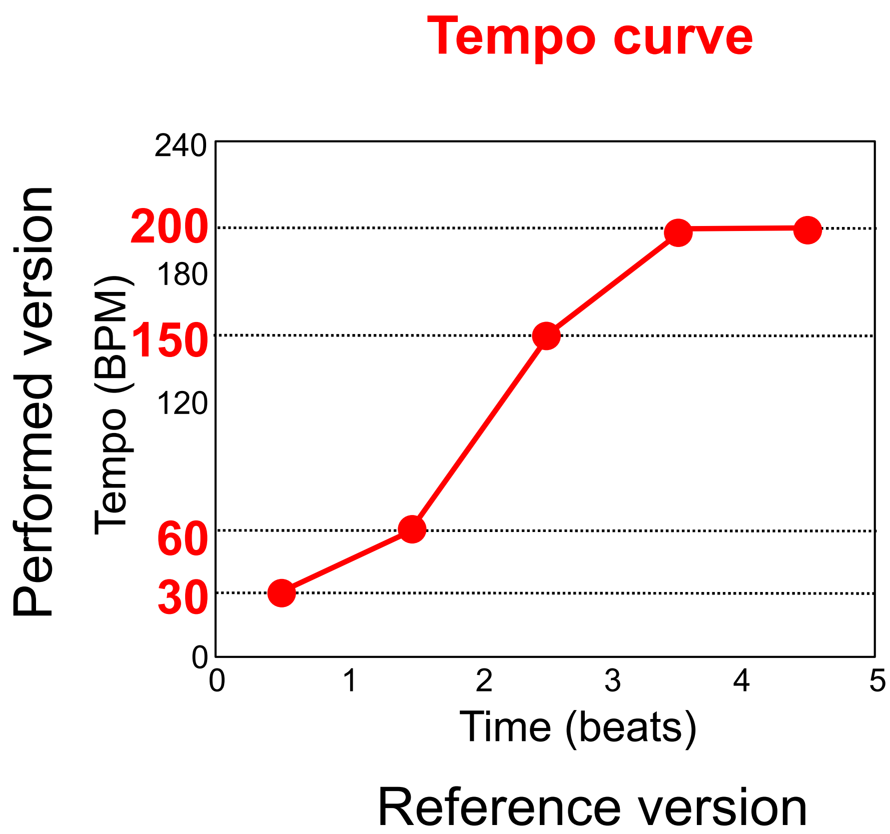




1 beat lasting 0.4 seconds ≙ 150 BPM

Performance Analysis: Tempo Curves



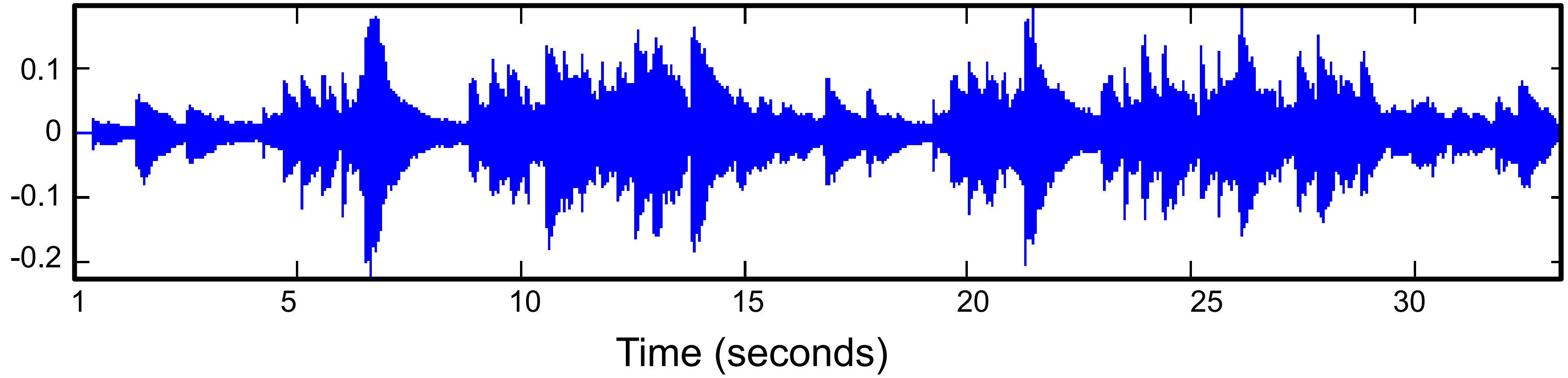


Tempo curve is optained by interpolation

Performance Analysis: Tempo Curves

Schumann: Träumerei

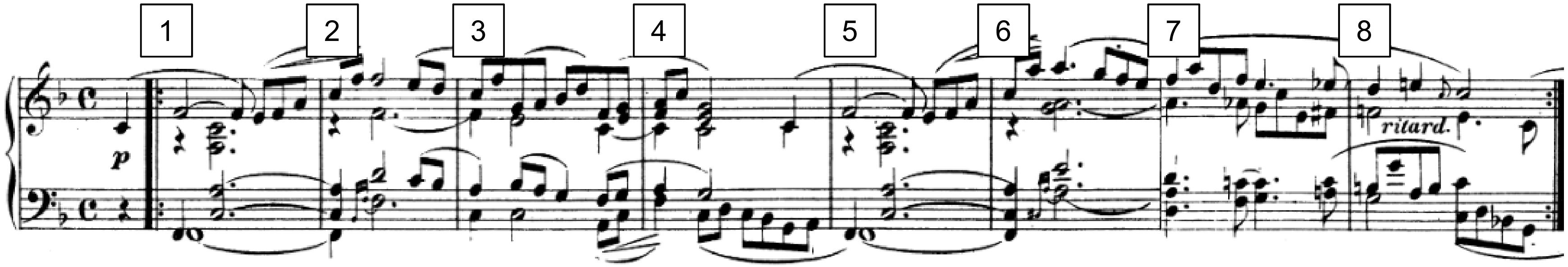
Performance:



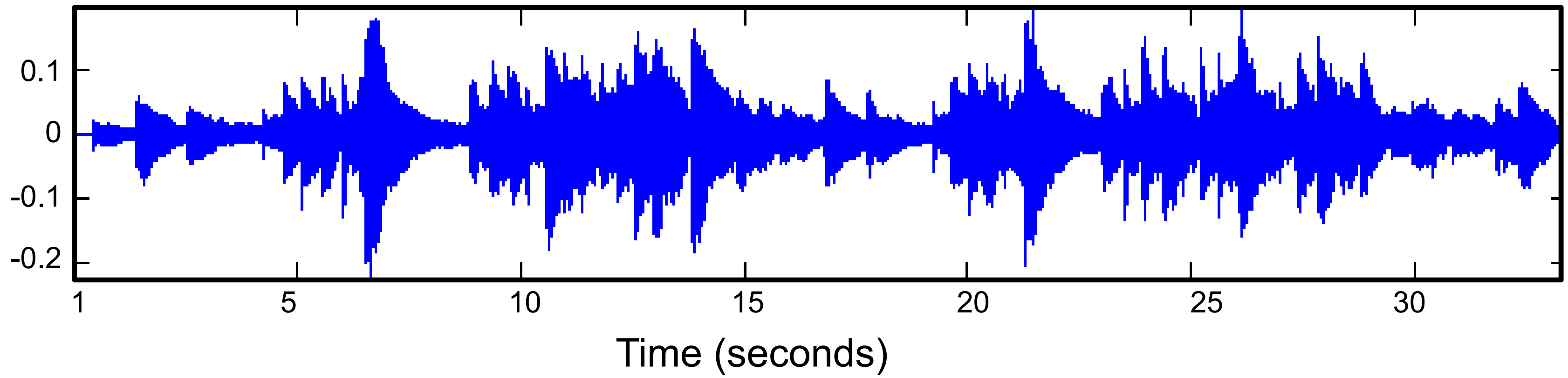
Performance Analysis: Tempo Curves

Schumann: Träumerei

Score (reference):



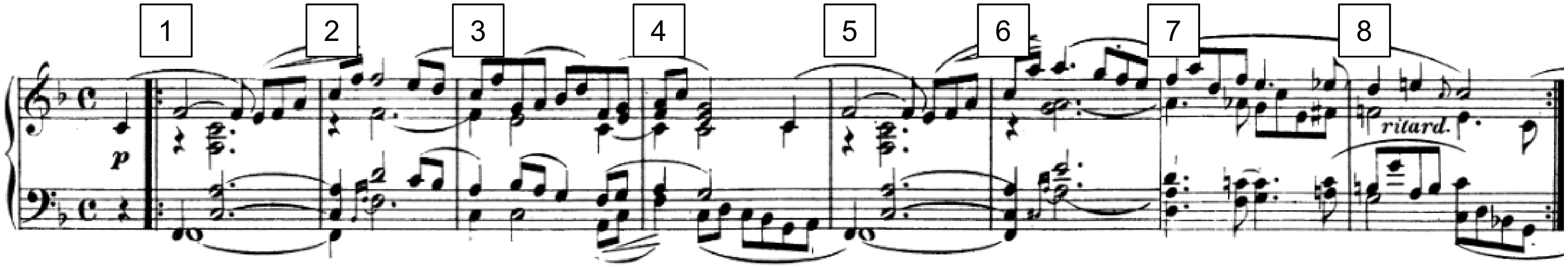
Performance:



Performance Analysis: Tempo Curves

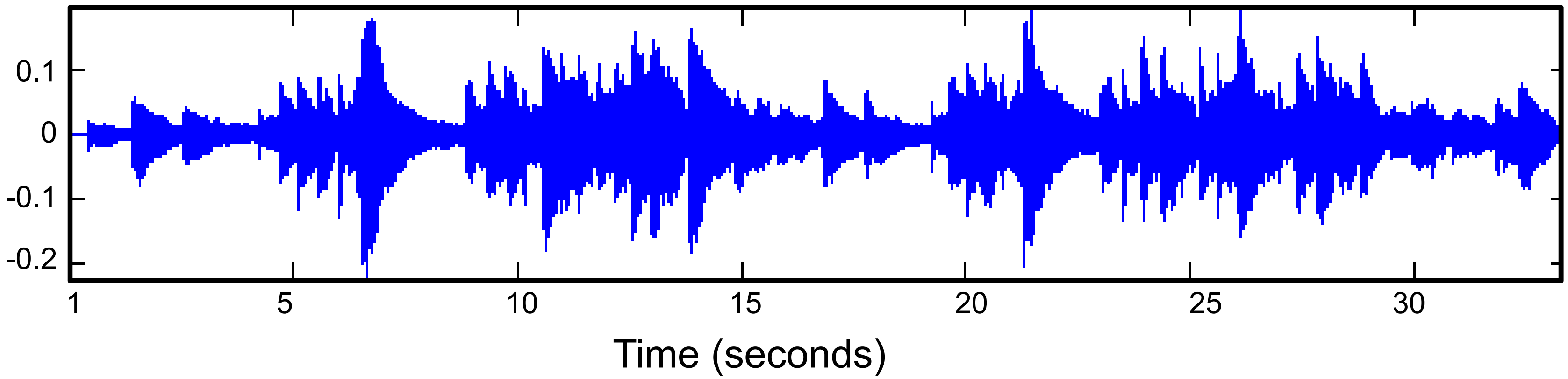
Schumann: Träumerei

Score (reference):



Strategy: Compute score-audio synchronization and derive tempo curve

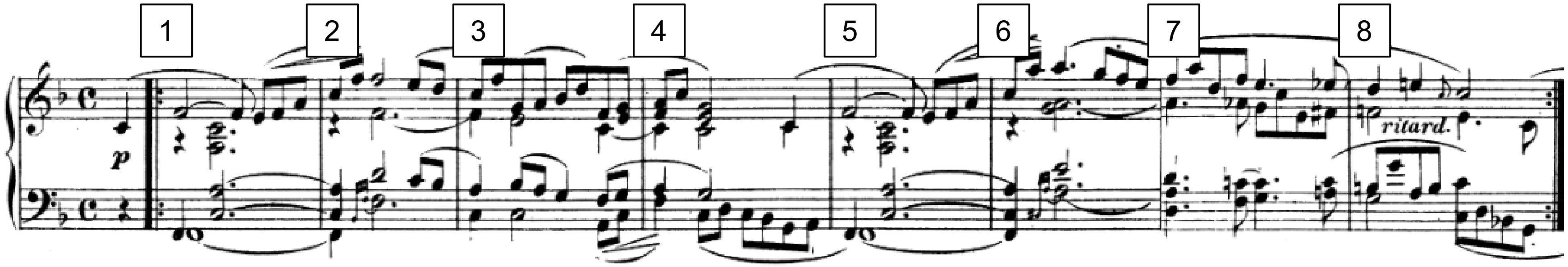
Performance:



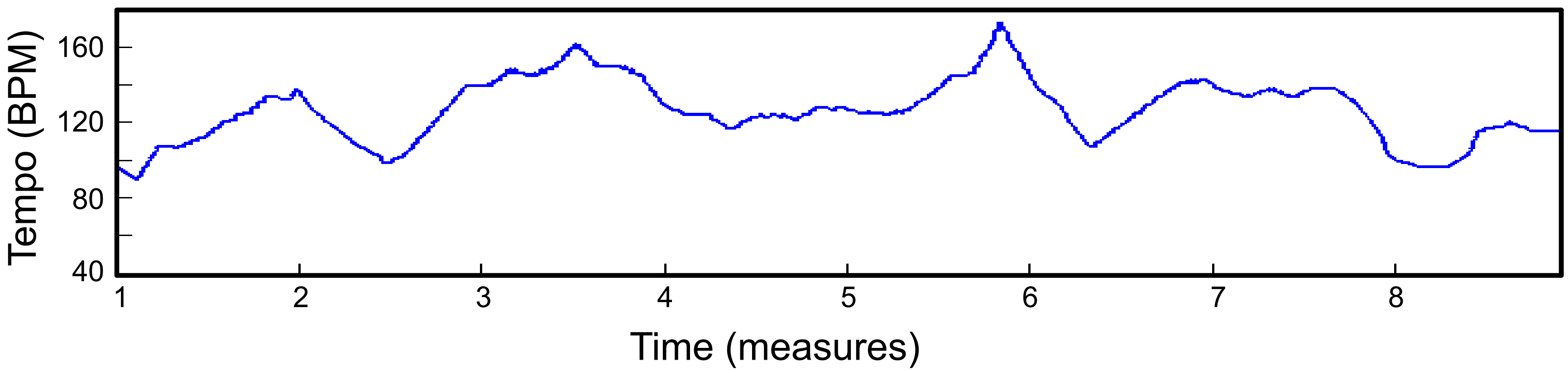
Performance Analysis: Tempo Curves

Schumann: Träumerei

Score (reference):



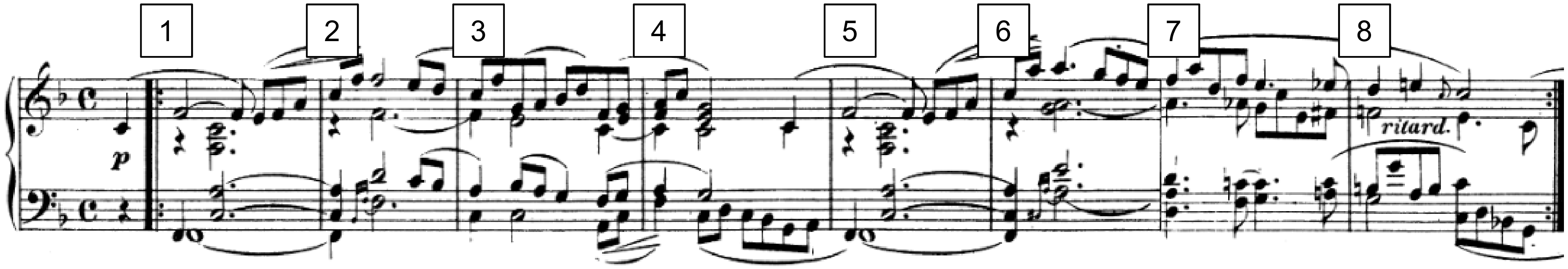
Tempo curve:



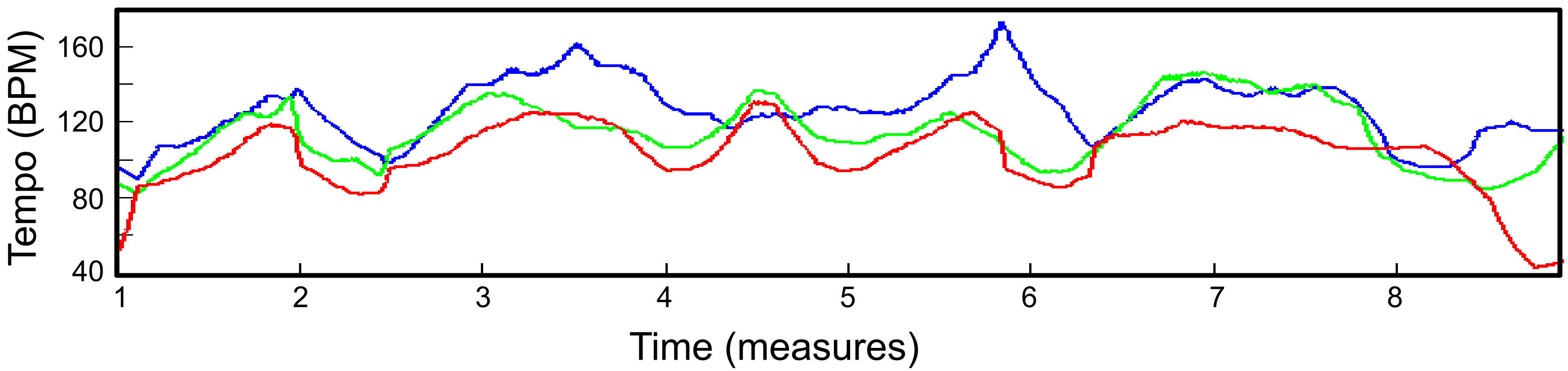
Performance Analysis: Tempo Curves

Schumann: Träumerei

Score (reference):



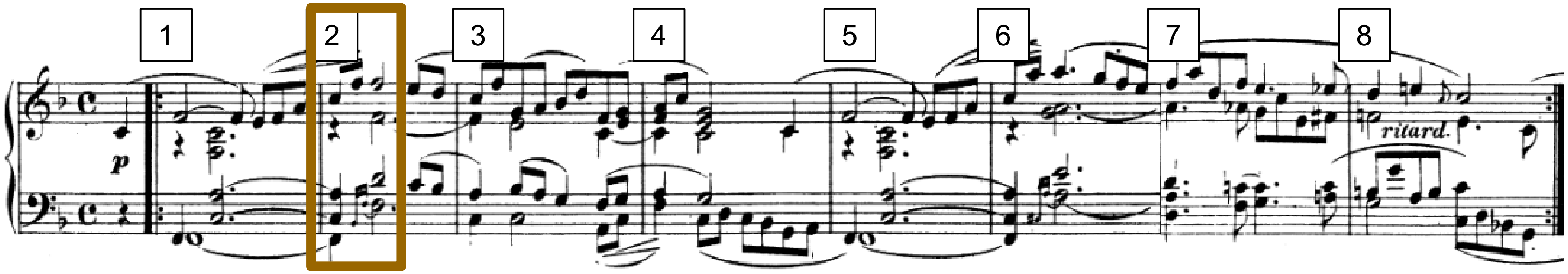
Tempo curves:



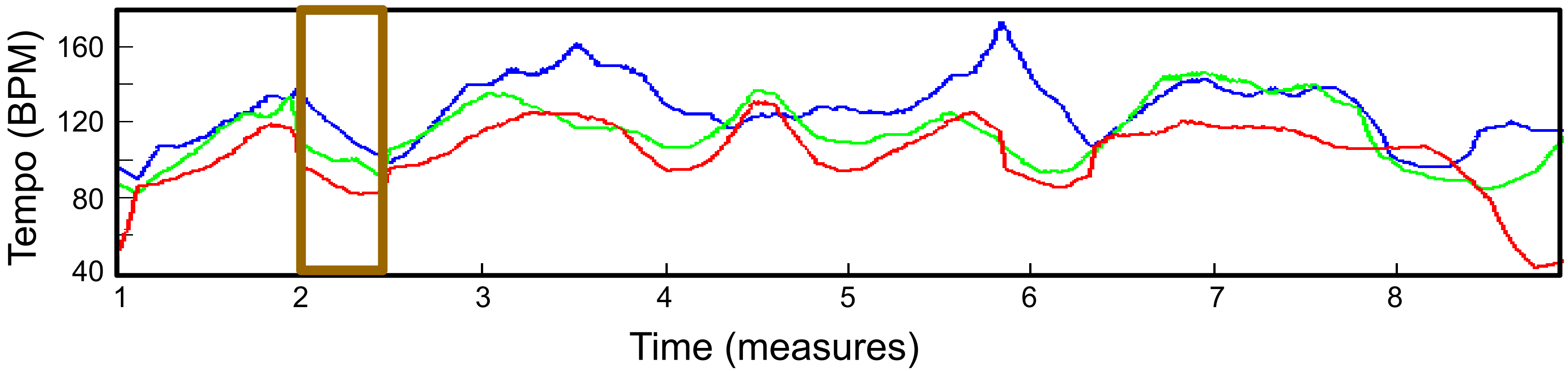
Performance Analysis: Tempo Curves

Schumann: Träumerei

Score (reference):



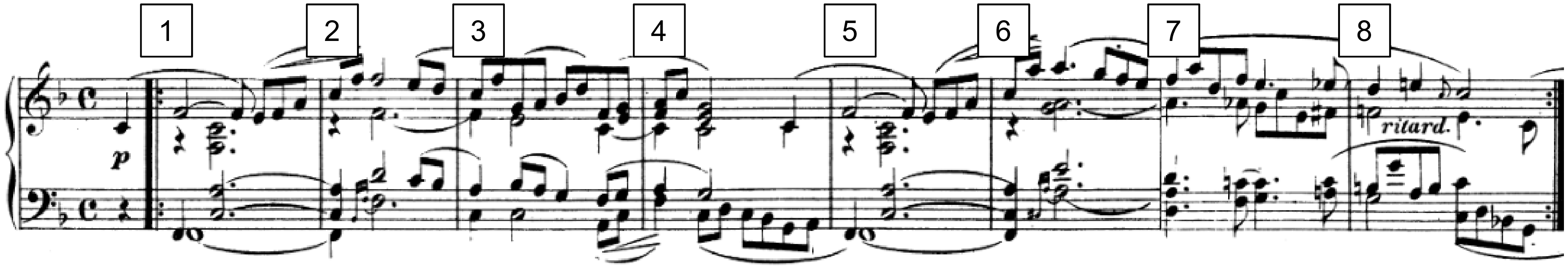
Tempo curves:



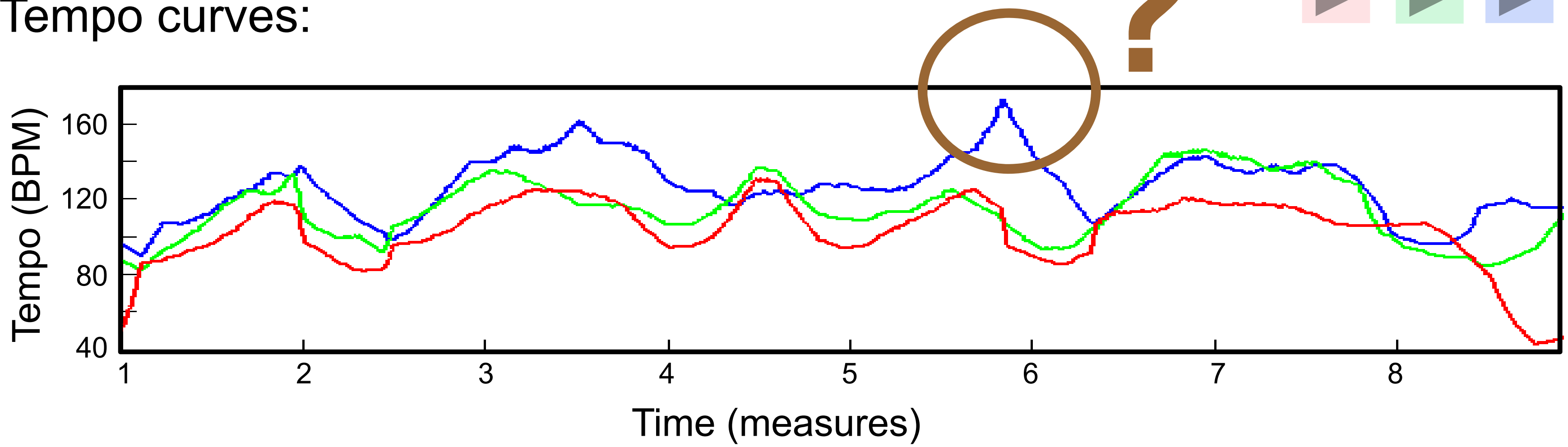
Performance Analysis: Tempo Curves

Schumann: Träumerei

Score (reference):



Tempo curves:

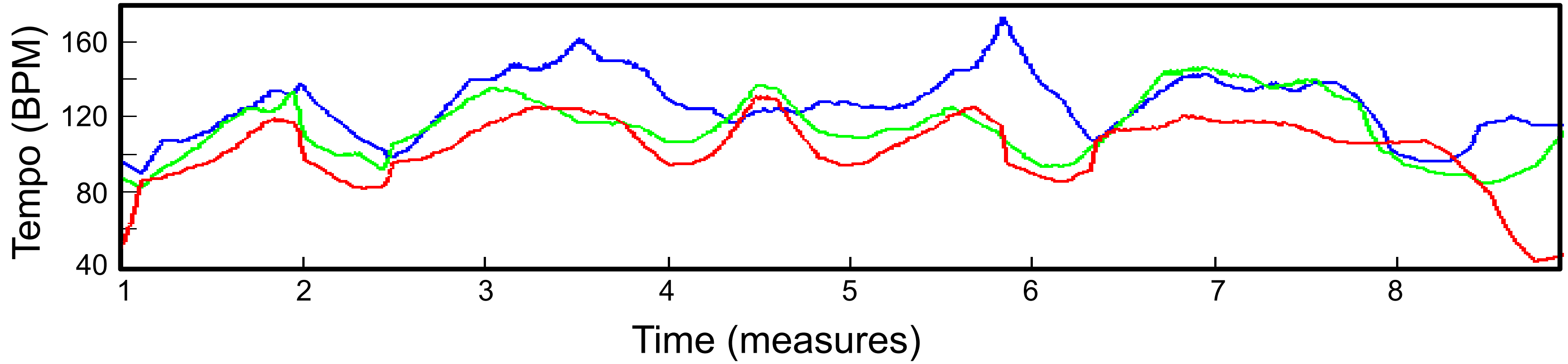


Performance Analysis: Tempo Curves

Schumann: Träumerei

What can be done if no reference is available?

Tempo curves:



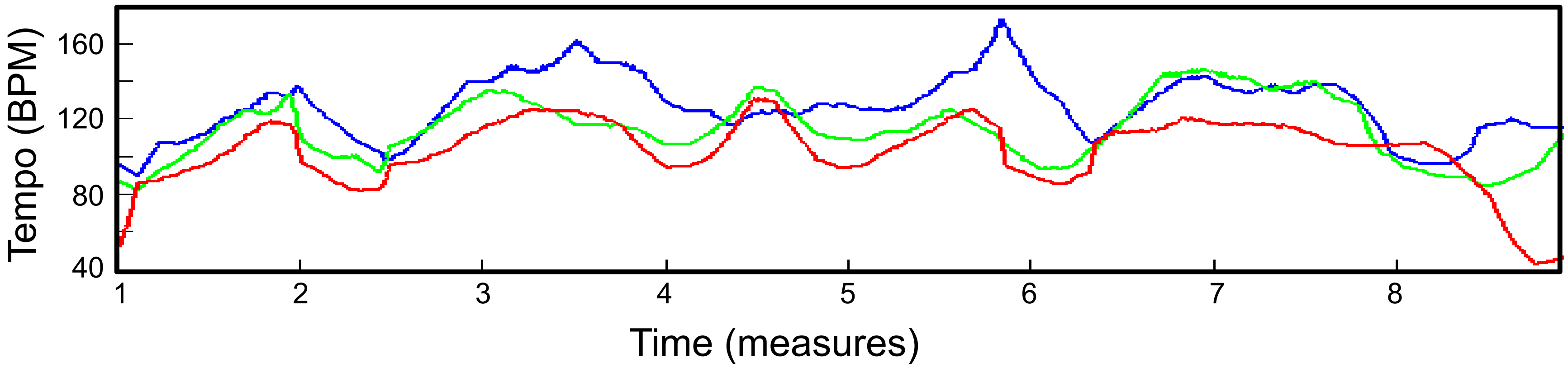
Performance Analysis: Tempo Curves

Schumann: Träumerei

What can be done if no reference is available?

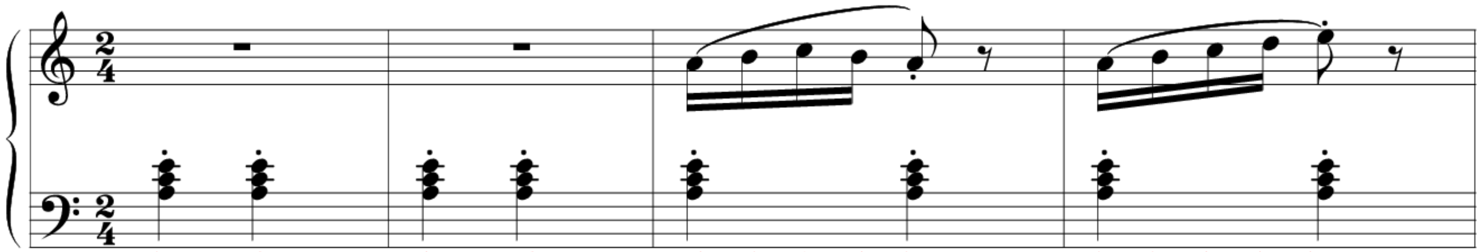
→ Tempo and Beat Tracking

Tempo curves:

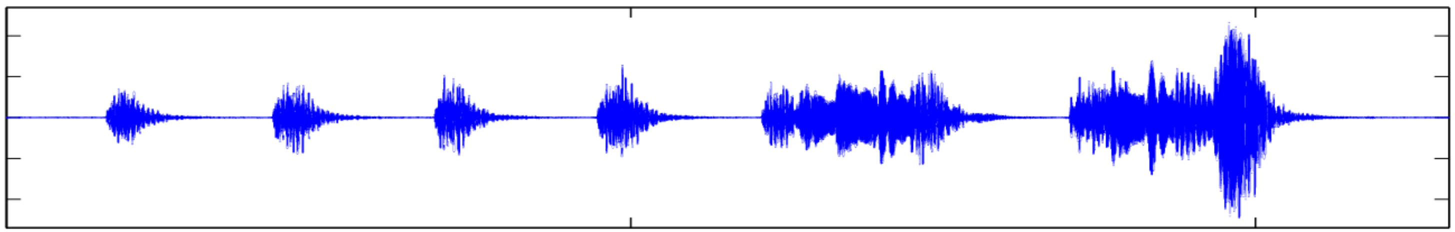


Music Synchronization: Image-Audio

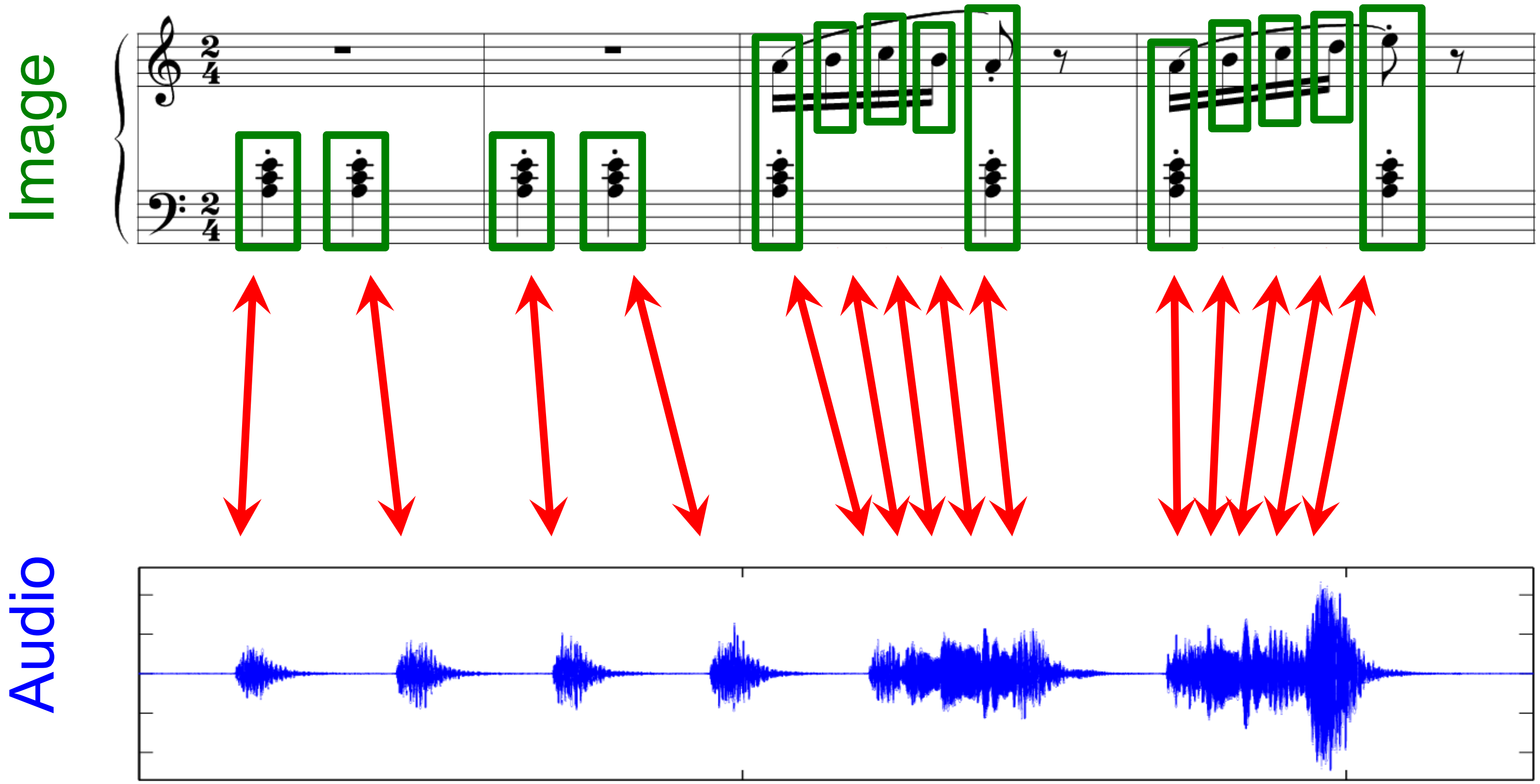
Image



Audio

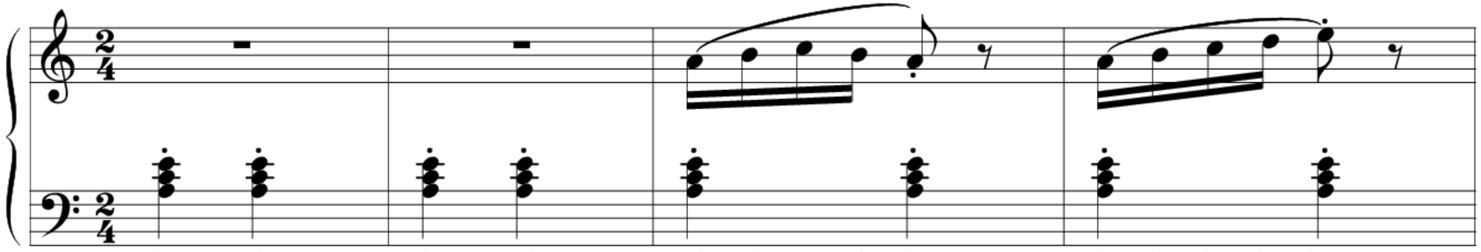


Music Synchronization: Image-Audio



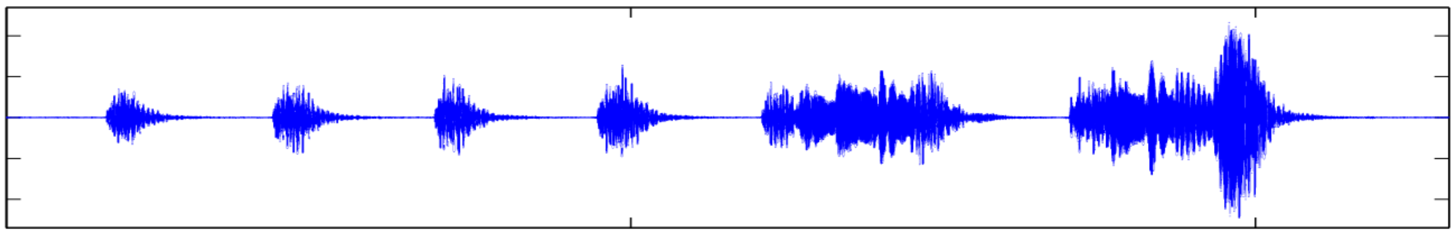
Music Synchronization: Image-Audio

Image



Convert data into common
mid-level feature representation

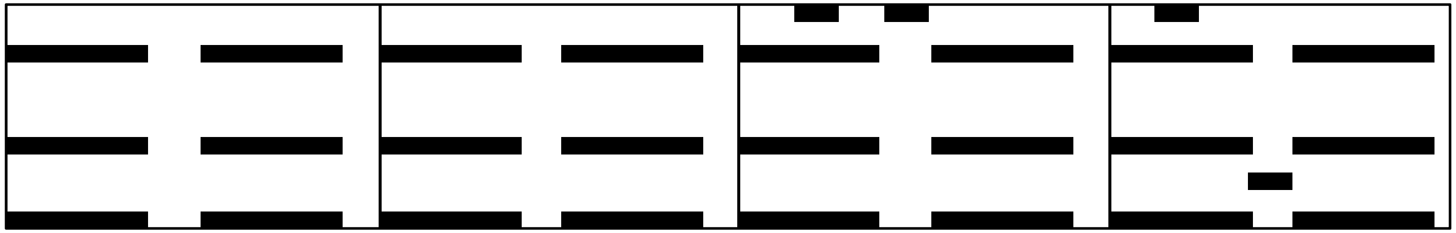
Audio



Music Synchronization: Image-Audio

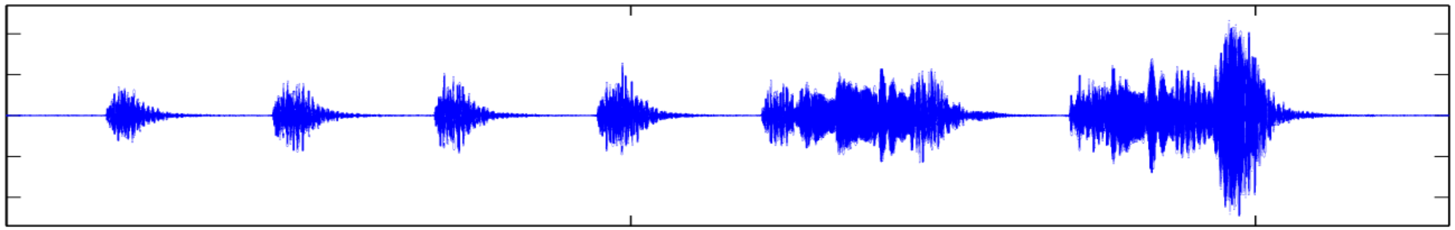
Image Processing: Optical Music Recognition

Image



Convert data into common
mid-level feature representation

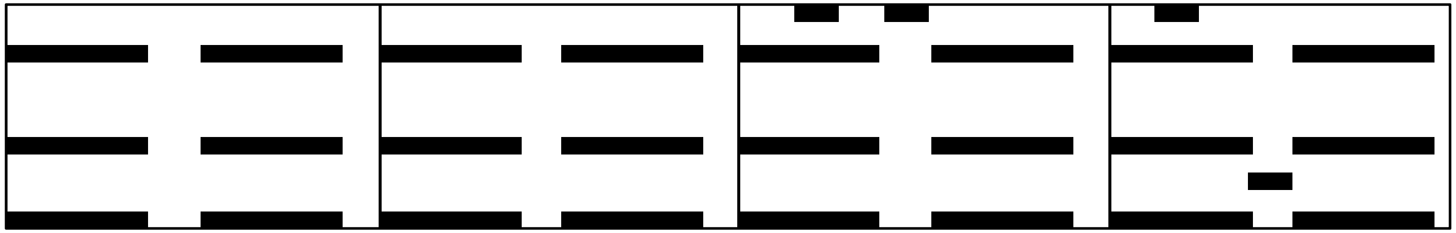
Audio



Music Synchronization: Image-Audio

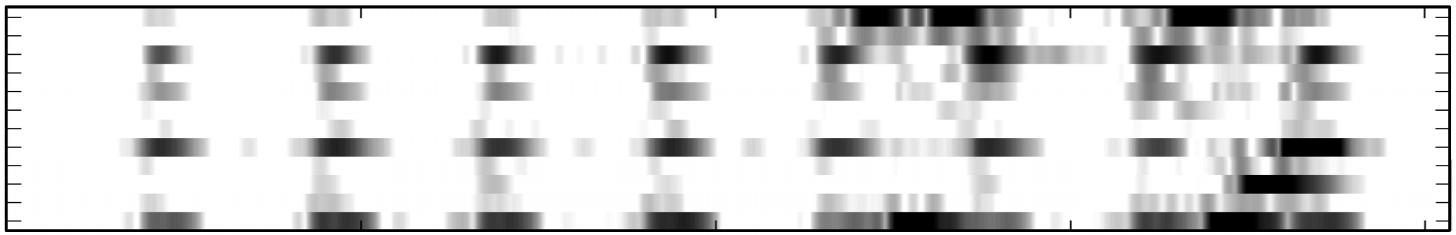
Image Processing: Optical Music Recognition

Image



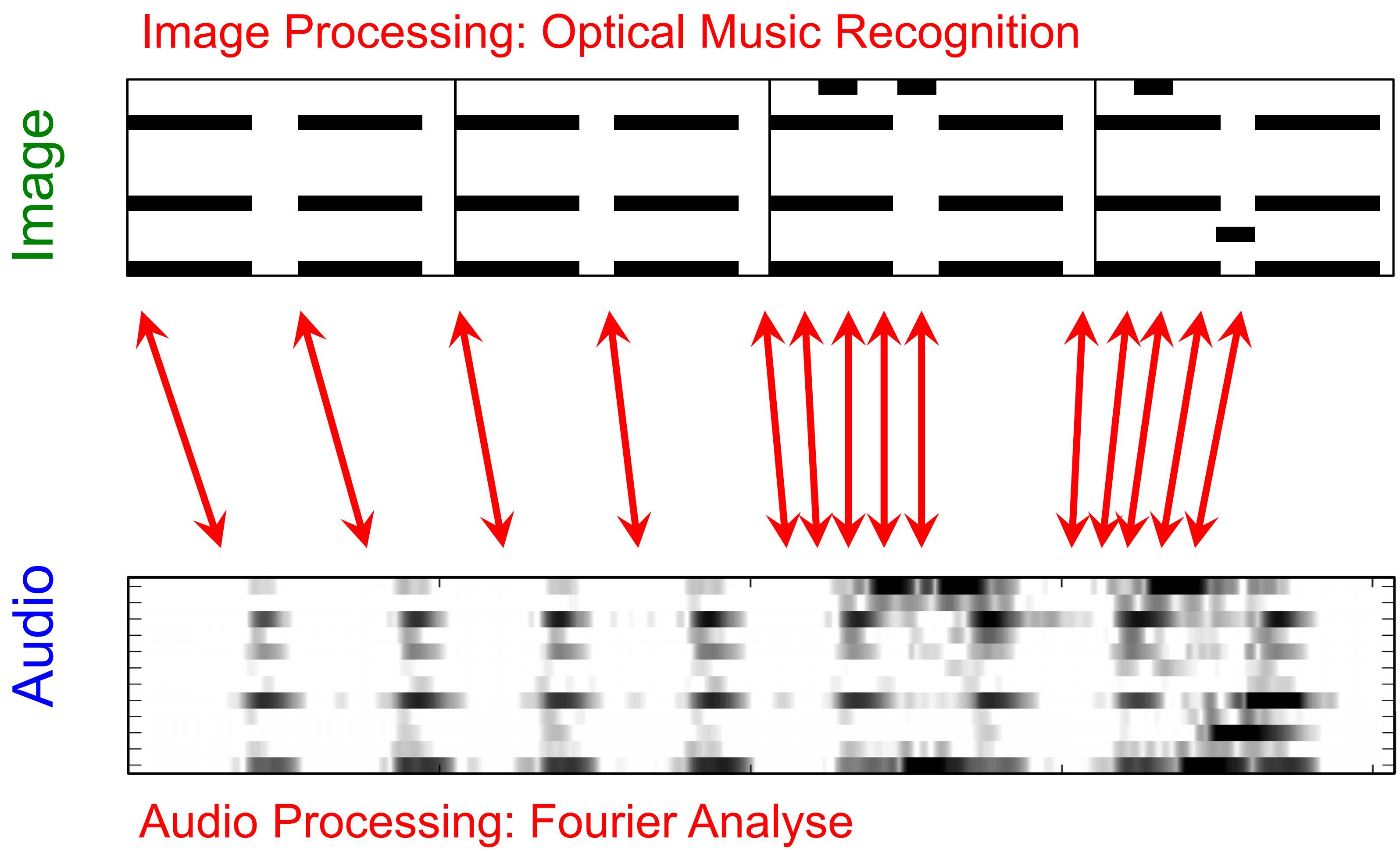
Convert data into common
mid-level feature representation

Audio



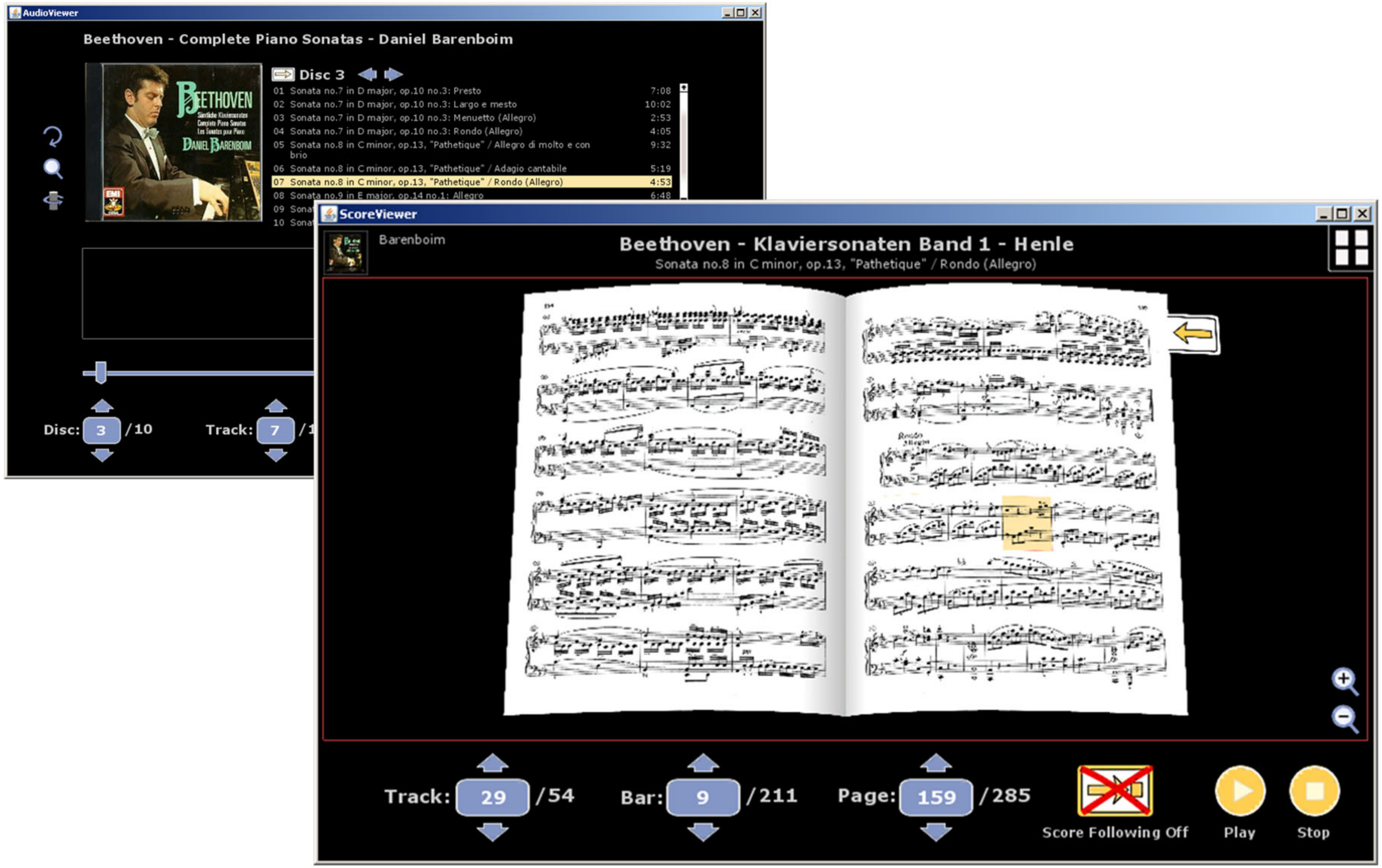
Audio Processing: Fourier Analyse

Music Synchronization: Image-Audio



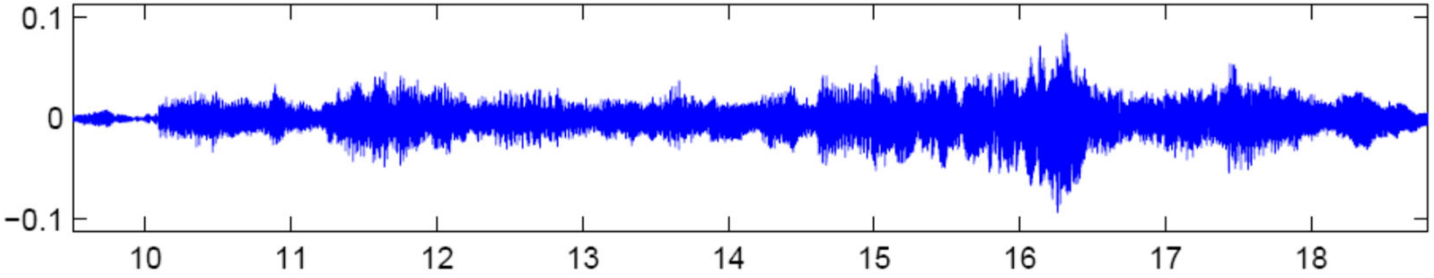
Music Synchronization: Image-Audio

Application: Score Viewer

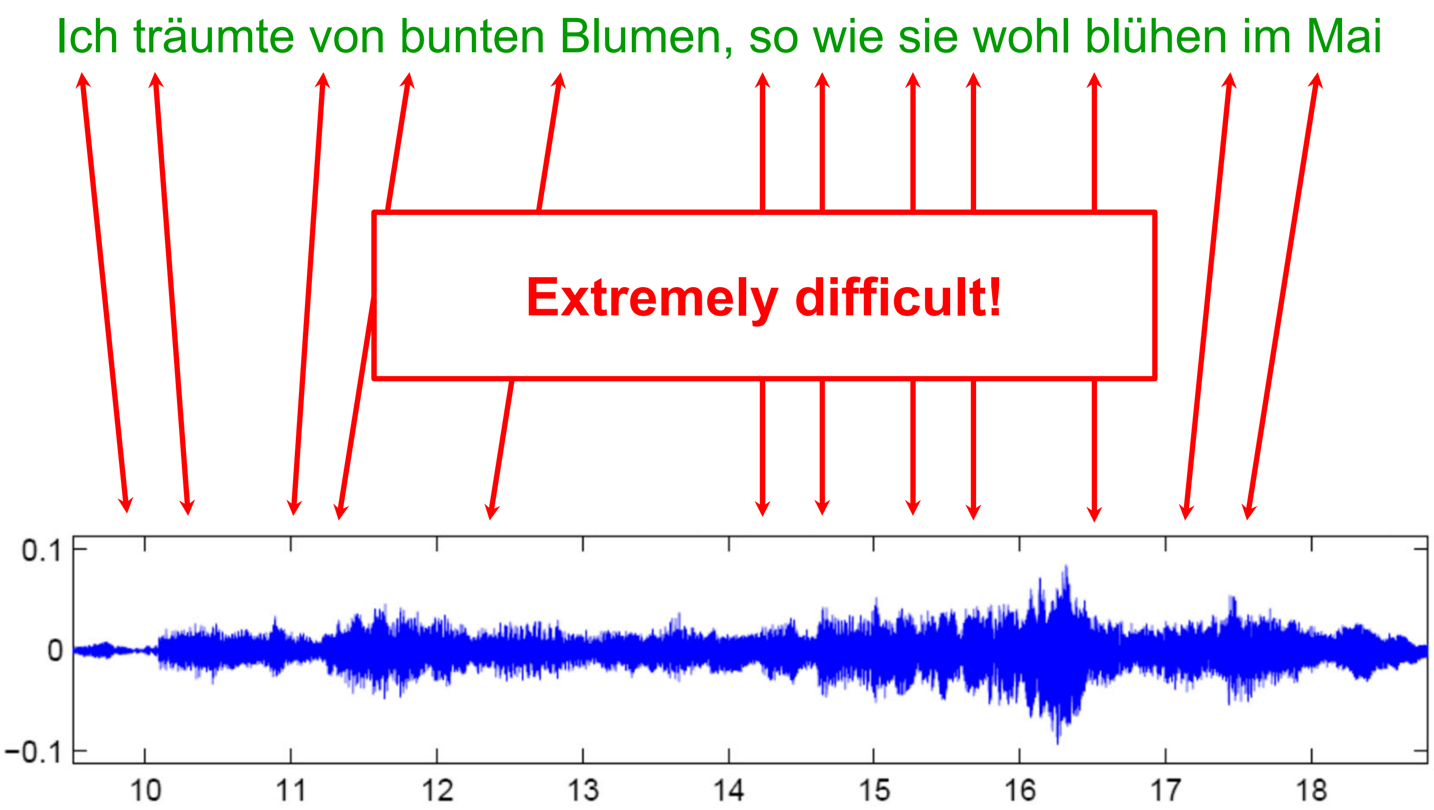


Music Synchronization: Lyrics-Audio

Ich träumte von bunten Blumen, so wie sie wohl blühen im Mai



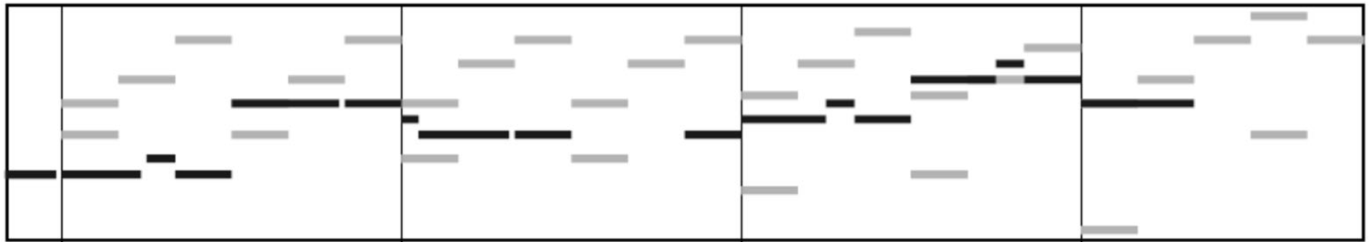
Music Synchronization: Lyrics-Audio

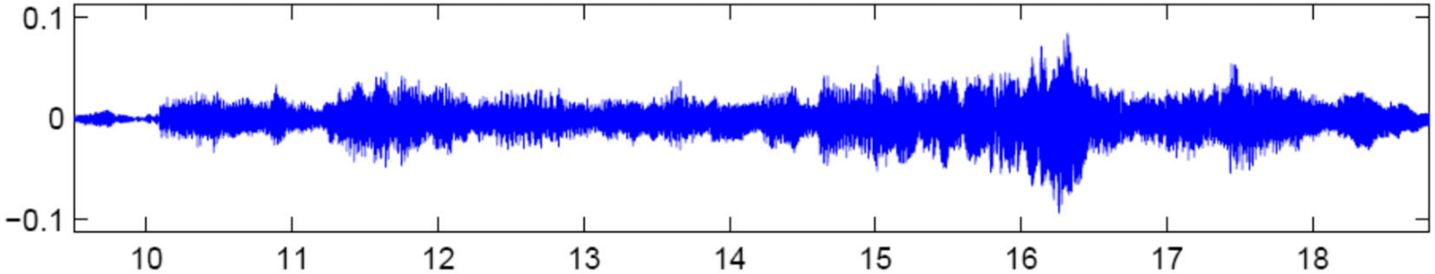


Music Synchronization: Lyrics-Audio

Lyrics-Audio $\rightarrow$ Lyrics-MIDI + MIDI-Audio

Ich träumte von bunten Blumen, so wie sie wohl blühen im Mai

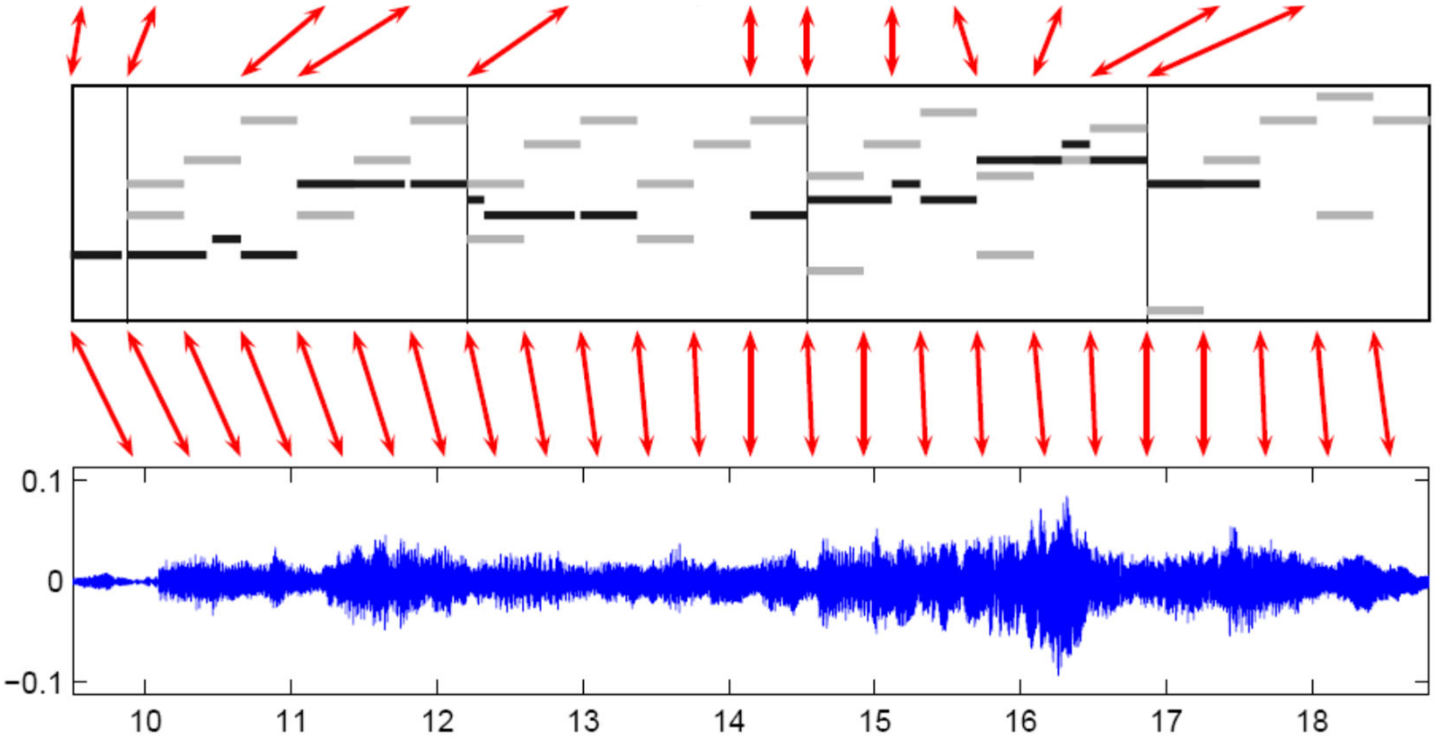




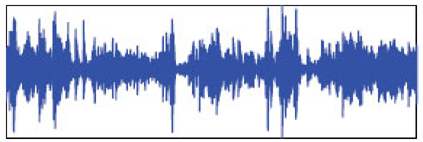
Music Synchronization: Lyrics-Audio

Lyrics-Audio → Lyrics-MIDI + MIDI-Audio

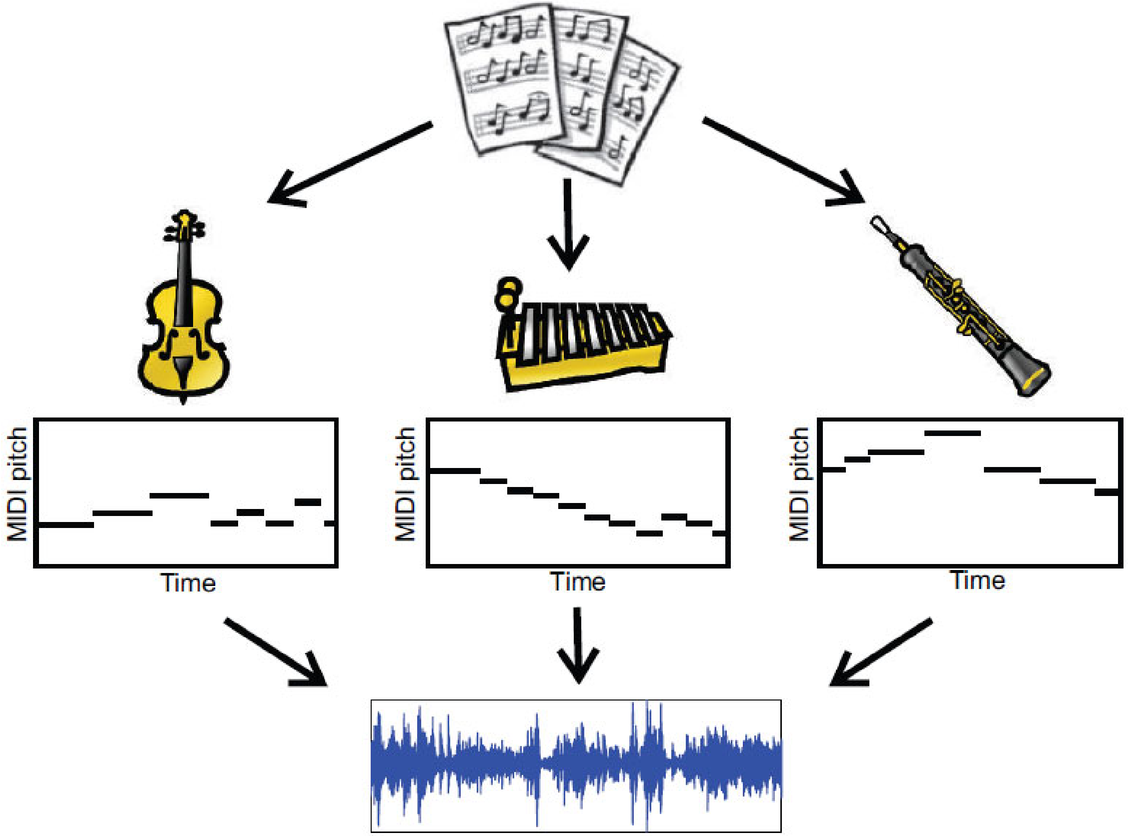
Ich träumte von bunten Blumen, so wie sie wohl blühen im Mai



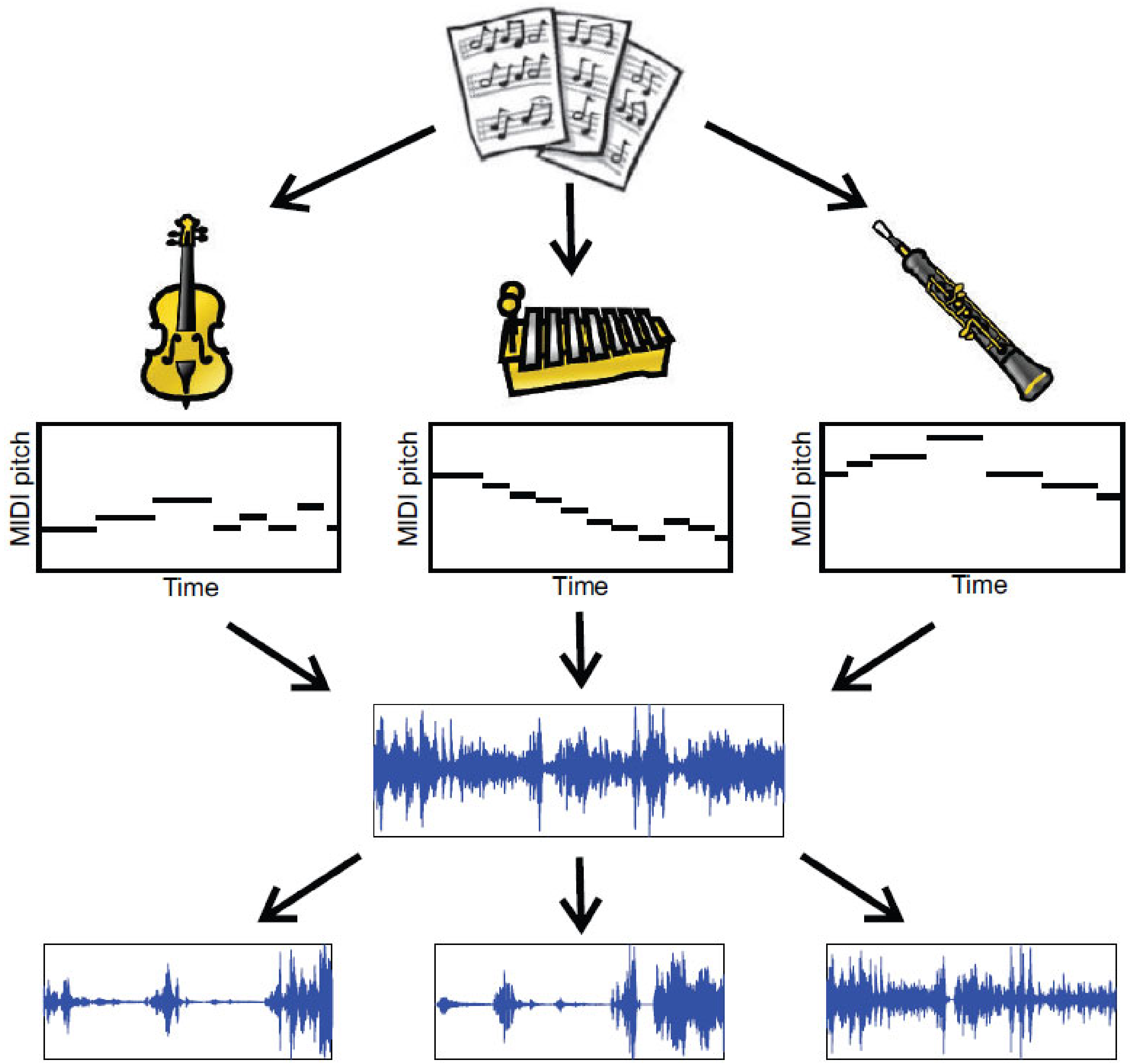
Score-Informed Source Separation



Score-Informed Source Separation

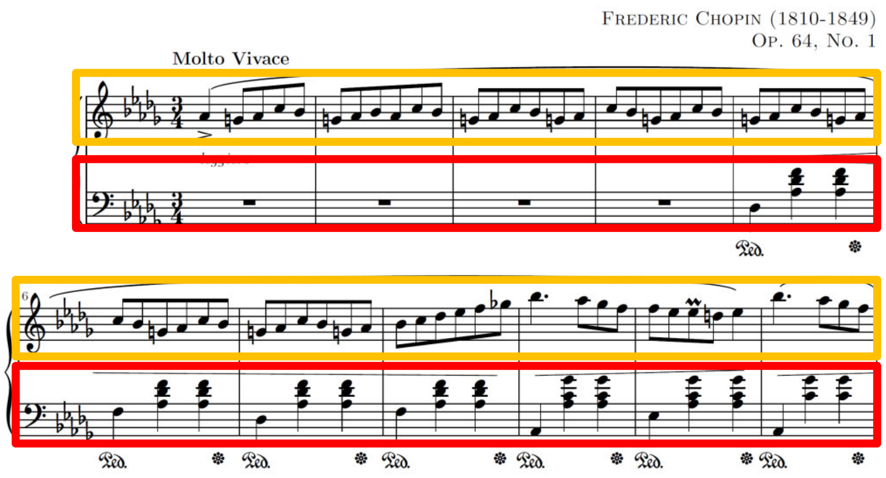


Score-Informed Source Separation



Score-Informed Source Separation

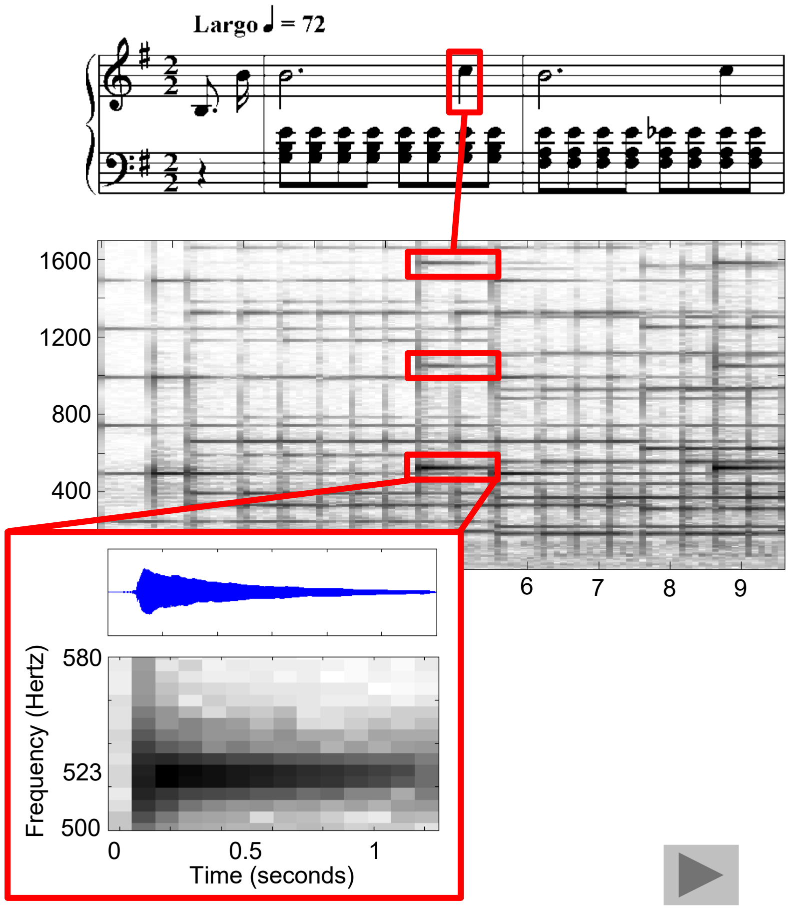
Experimental results for separating left and right hands for piano recordings:

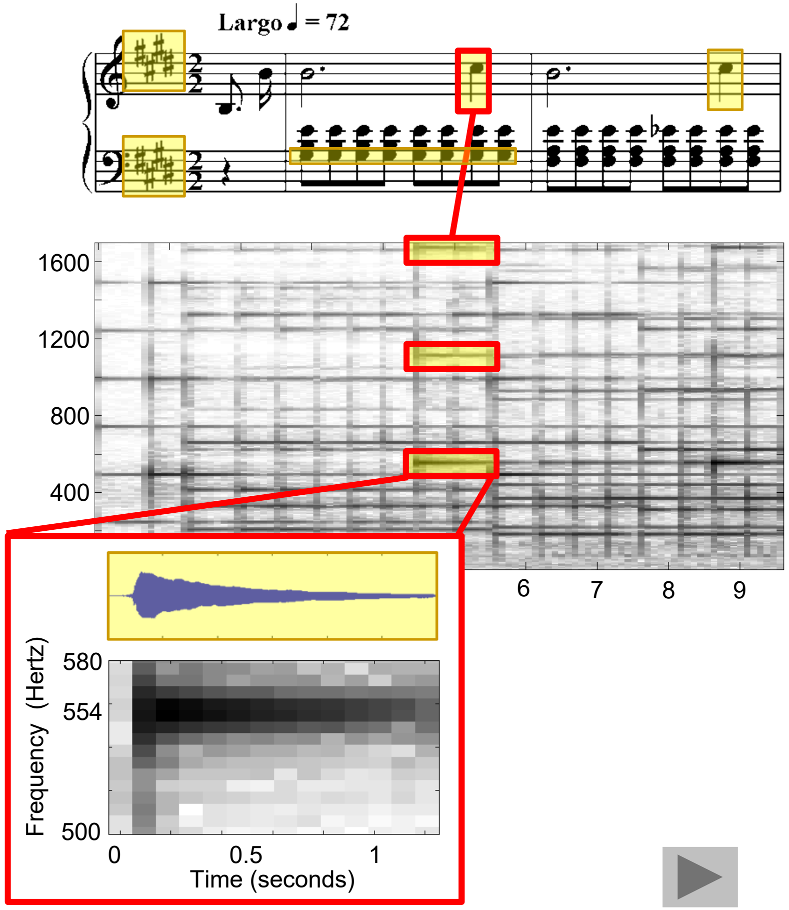


Composer	Piece	Database	Results			
			L	R	Eq	Org
Bach	BWV 875, Prelude	SMD				
Chopin	Op. 28, No. 15	SMD				
Chopin	Op. 64, No. 1	European Archive				

Score-Informed Source Separation

Audio editing





Dynamic Time Warping

Dynamic Time Warping

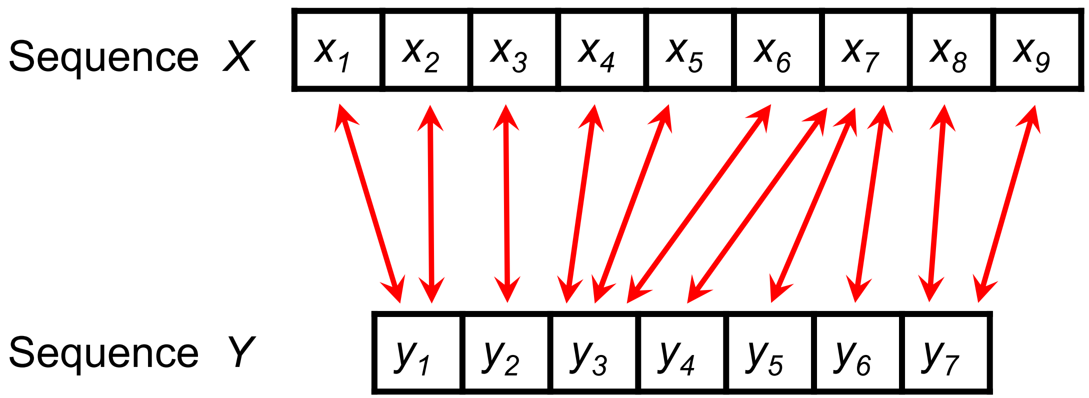
- Well-known technique to find an optimal alignment between two given (time-dependent) sequences under certain restrictions.
- Intuitively, sequences are warped in a non-linear fashion to match each other.
- Originally used to compare different speech patterns in automatic speech recognition

Dynamic Time Warping

Sequence X	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9

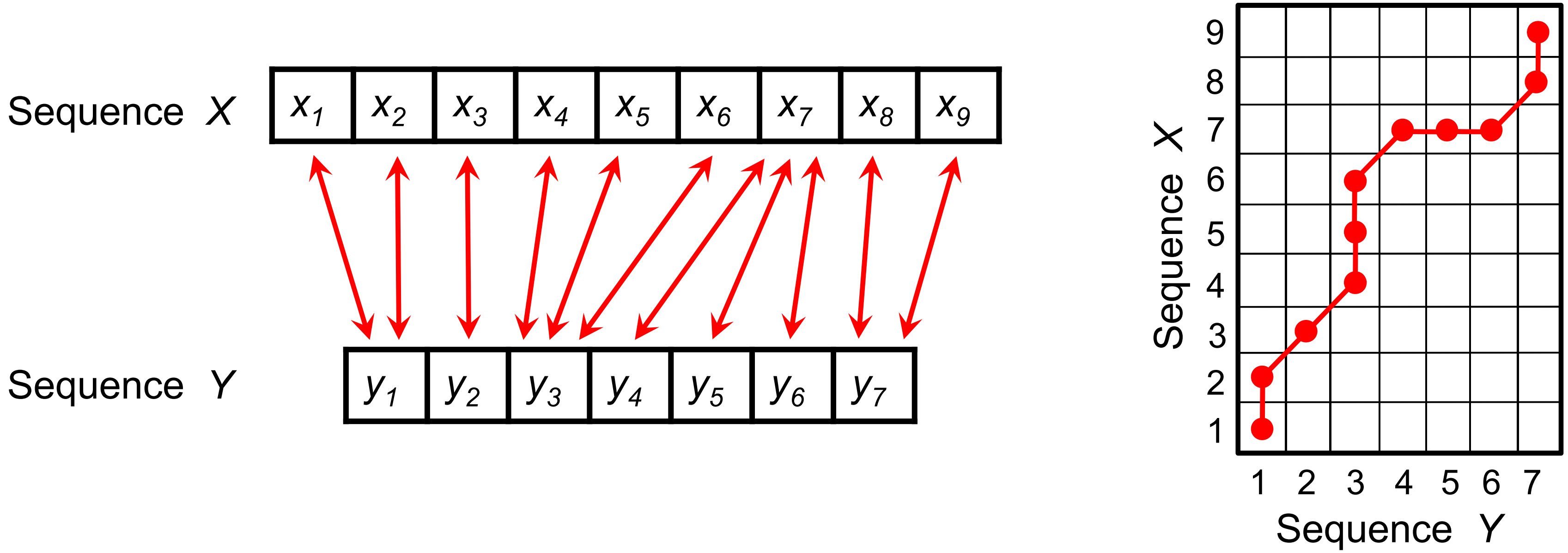
Sequence Y	y_1	y_2	y_3	y_4	y_5	y_6	y_7

Dynamic Time Warping



Time alignment of two time-dependent sequences, where the aligned points are indicated by the arrows.

Dynamic Time Warping



Time alignment of two time-dependent sequences, where the aligned points are indicated by the arrows.

Dynamic Time Warping

The objective of DTW is to compare two (time-dependent) sequences

$$X := (x_1, x_2, \ldots, x_N)$$

of length $N \in \mathbb{N}$ and

$$Y := (y_1, y_2, \ldots, y_M)$$

of length $M \in \mathbb{N}$. Here,

$$x_n, y_m \in \mathcal{F},\ n \in [1:N],\ m \in [1:M],$$

are suitable features that are elements from a given feature space denoted by $\mathcal{F}$.

Dynamic Time Warping

To compare two different features $x, y \in \mathcal{F}$ one needs a local cost measure which is defined to be a function

$$c : \mathcal{F} \times \mathcal{F} \to \mathbb{R}_{\geq 0}$$

Typically, $c(x, y)$ is small (low cost) if x and y are similar to each other, and otherwise $c(x, y)$ is large (high cost).

Dynamic Time Warping

Evaluating the local cost measure for each pair of elements of the sequences X and Y, one obtains the cost matrix

$$C \in \mathbb{R}^{N \times M}$$

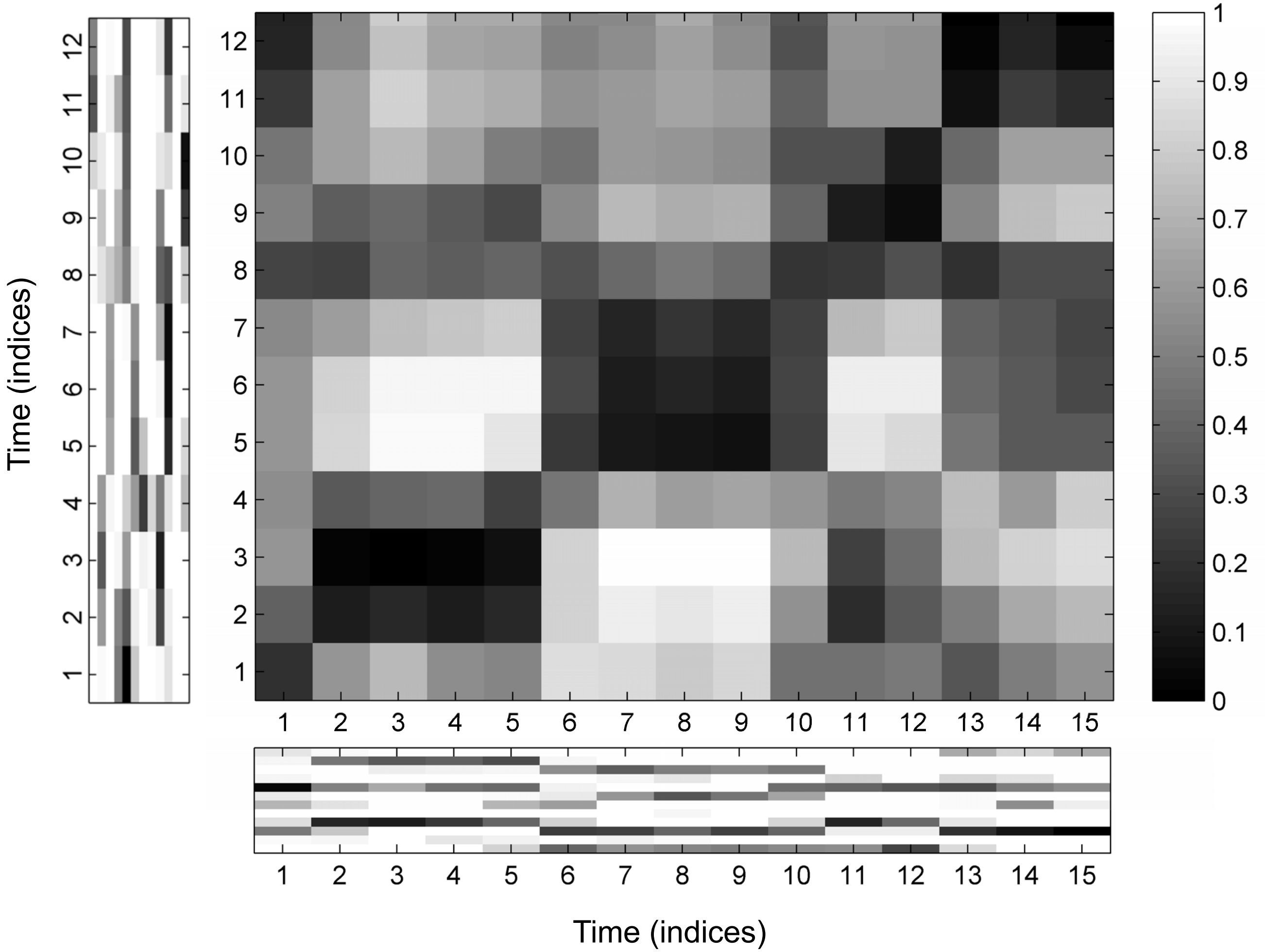
denfined by

$$C(n, m) := c(x_n, y_m).$$

Then the goal is to find an alignment between X and Y having minimal overall cost. Intuitively, such an optimal alignment runs along a “valley” of low cost within the cost matrix C .

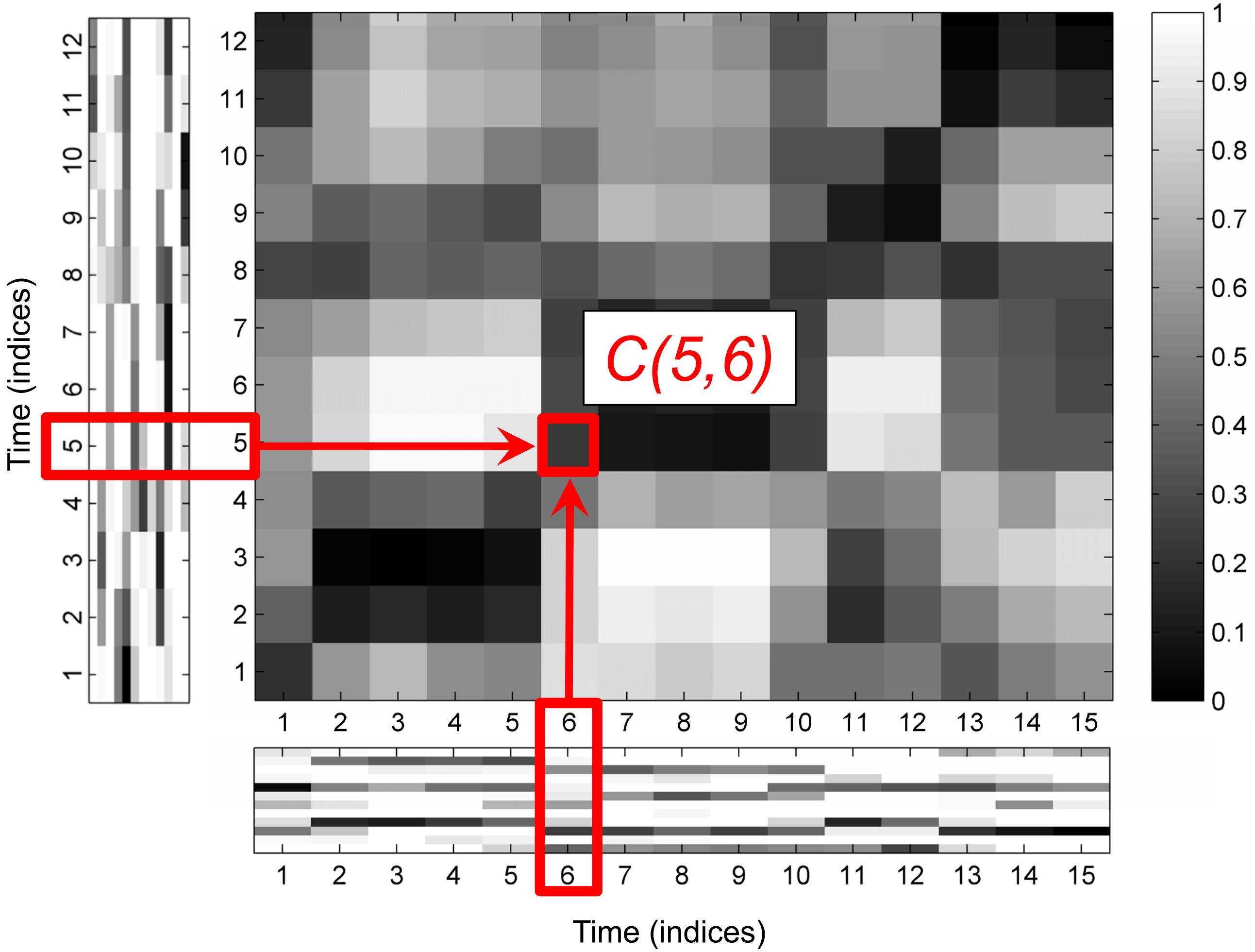
Dynamic Time Warping

Cost matrix *C*



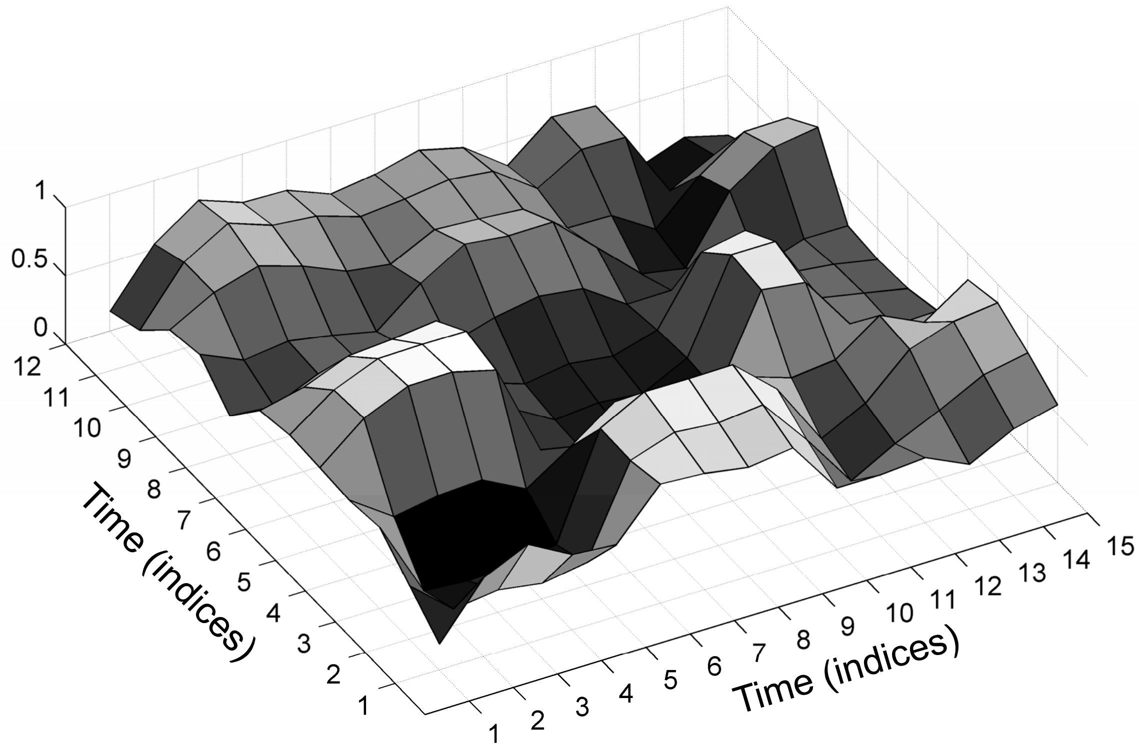
Dynamic Time Warping

Cost matrix *C*



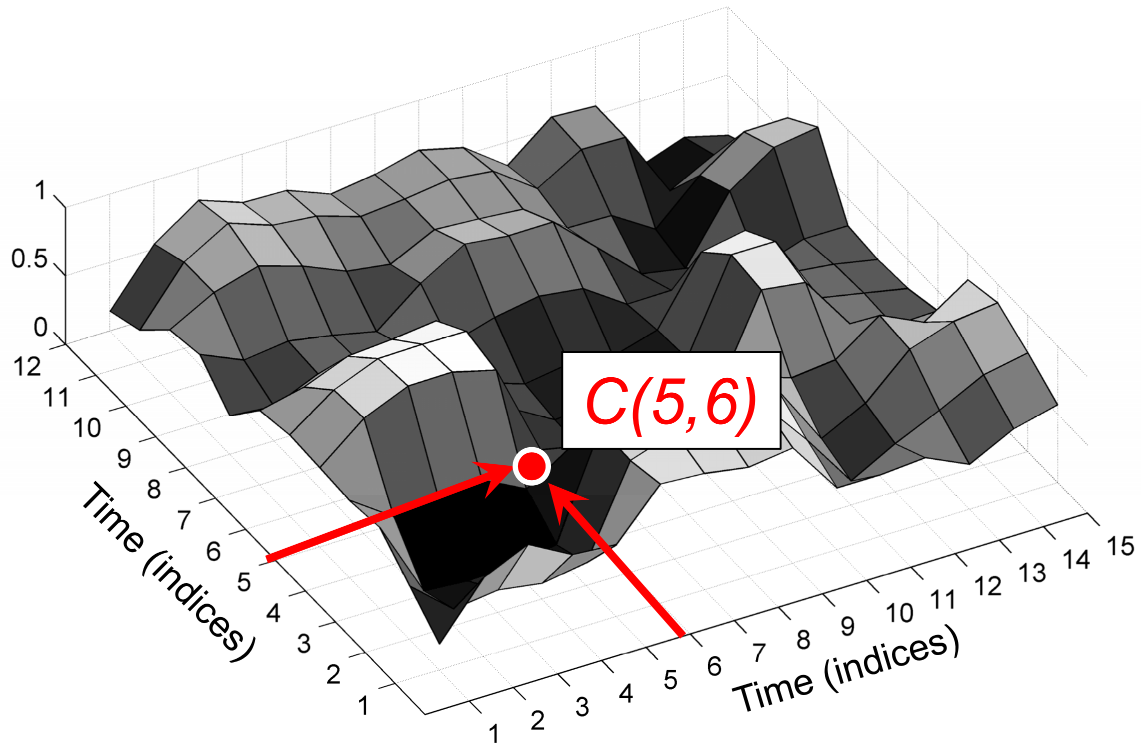
Dynamic Time Warping

Cost matrix *C*



Dynamic Time Warping

Cost matrix *C*



Dynamic Time Warping

The next definition formalizes the notion of an alignment.

A **warping path** is a sequence $p = (p_1, \ldots, p_L)$ with

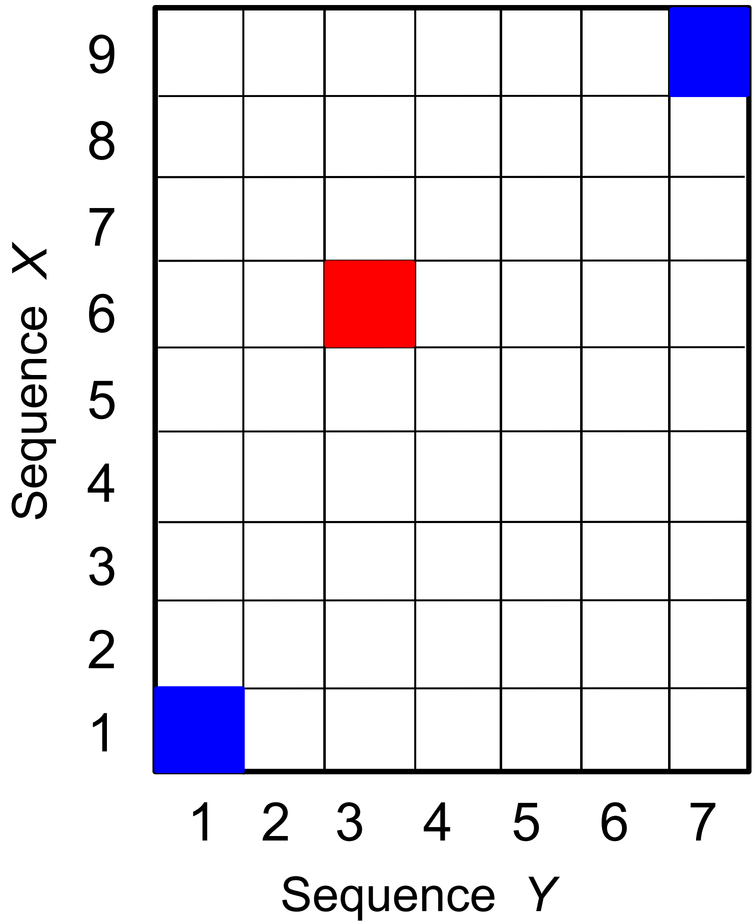
$$p_\ell = (n_\ell, m_\ell) \in [1:N] \times [1:M]$$

for $\ell \in [1:L]$ satisfying the following three conditions:

- Boundary condition: $p_1 = (1,1)$ and $p_L = (N,M)$
- Monotonicity condition: $n_1 \leq n_2 \leq \ldots \leq n_L$ and $m_1 \leq m_2 \leq \ldots \leq m_L$
- Step size condition: $p_{\ell+1} - p_\ell \in \{(1,0),(0,1),(1,1)\}$ for $\ell \in [1:L-1]$

Dynamic Time Warping

Warping path



Each matrix entry (cell) corresponds to a pair of indices.

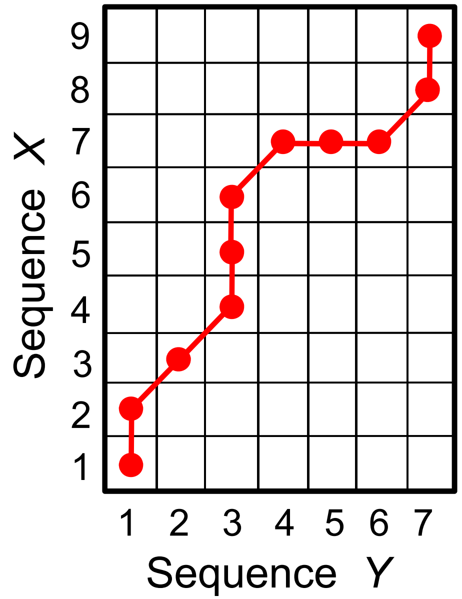
Cell = (6,3)

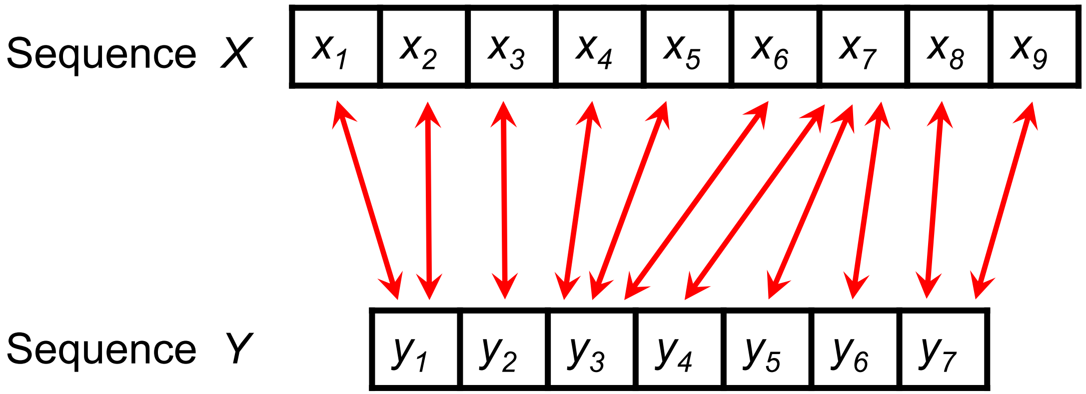
Boundary cells:
$p_1 = (1,1)$
$p_L = (N,M) = (9,7)$

Dynamic Time Warping

Warping path

Correct warping path

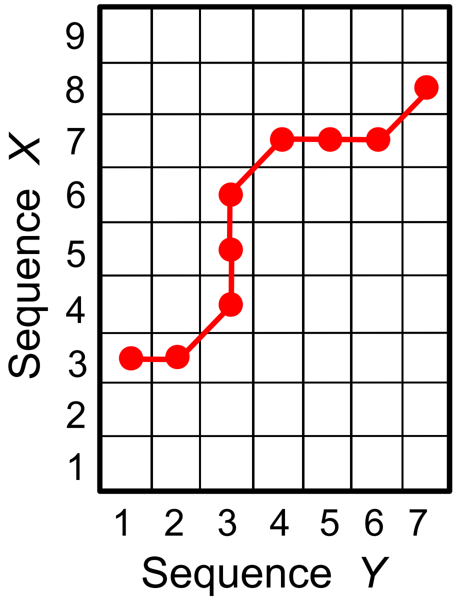


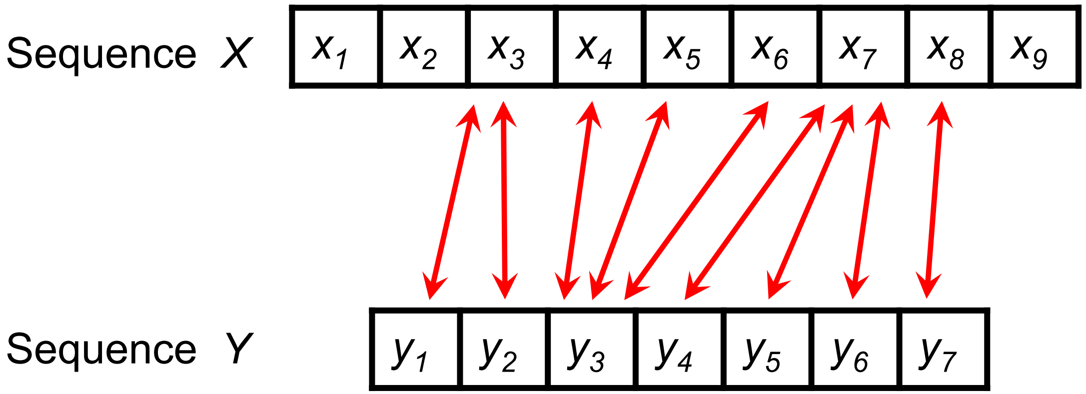


Dynamic Time Warping

Warping path

Violation of boundary condition

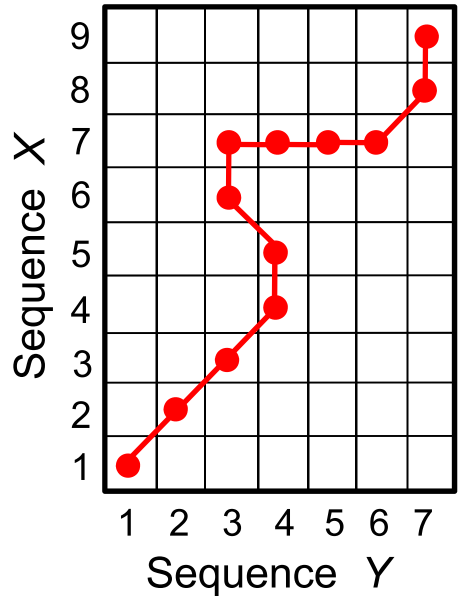


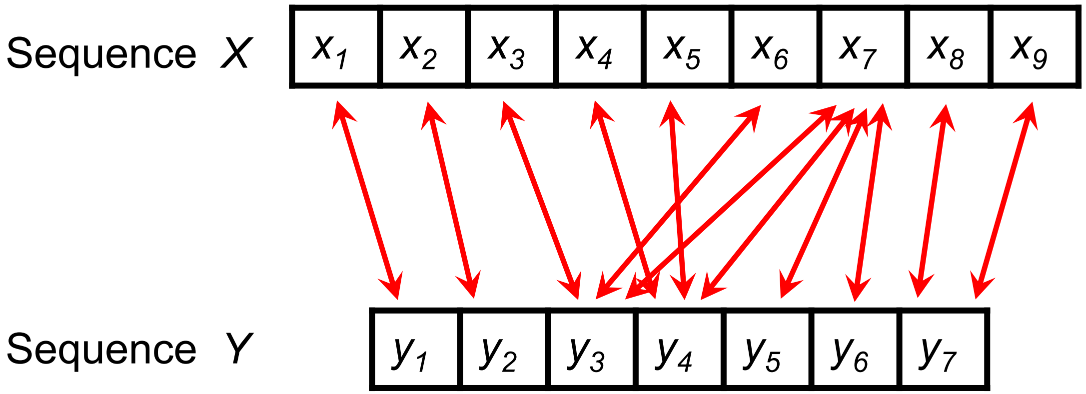


Dynamic Time Warping

Warping path

Violation of monotonicity condition

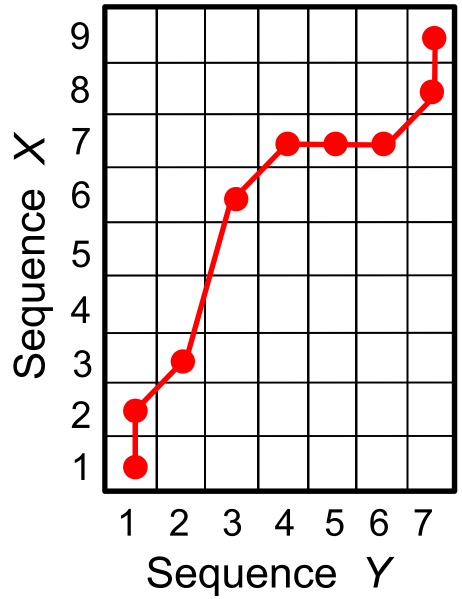


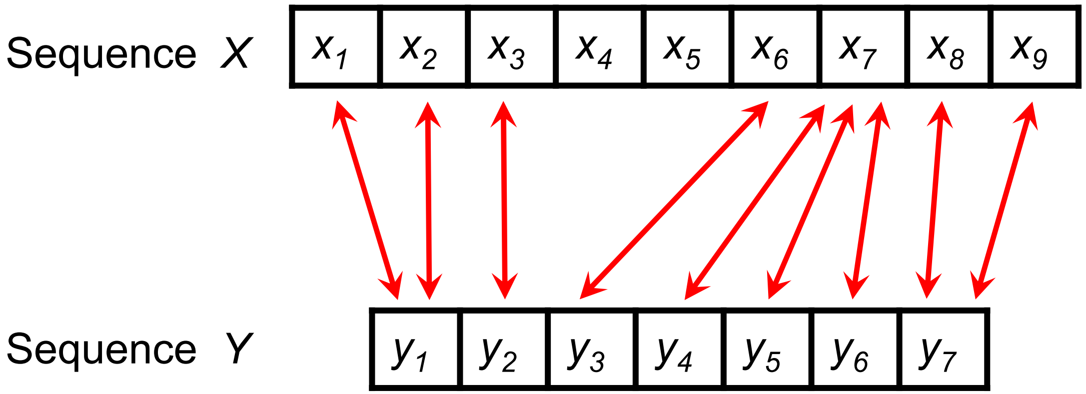


Dynamic Time Warping

Warping path

Violation of
step size condition





Dynamic Time Warping

The total cost $c_p(X,Y)$ of a warping path p between X and Y with respect to the local cost measure c is defined as

$$c_p(X,Y) := \sum_{\ell=1}^{L} c(x_{n_\ell}, y_{m_\ell})$$

Furthermore, an optimal warping path between X and Y is a warping path p^* having minimal total cost among all possible warping paths. The DTW distance $\mathrm{DTW}(X,Y)$ between X and Y is then defined as the total cost o p^*

$$\begin{aligned} \mathrm{DTW}(X,Y) &:= c_{p^*}(X,Y) \\ &= \min\{c_p(X,Y) \mid p \text{ is a warping path}\} \end{aligned}$$

Dynamic Time Warping

- The warping path p^* is not unique (in general).
- DTW does (in general) not definne a metric since it may not satisfy the triangle inequality.
- There exist exponentially many warping paths.
- How can p^* be computed efficiently?

Dynamic Time Warping

Notation:
$$\begin{aligned} X(1:n) &:= (x_1, \ldots, x_n), \quad 1 \leq n \leq N \\ Y(1:m) &:= (y_1, \ldots, y_m), \quad 1 \leq m \leq M \\ D(n,m) &:= \mathrm{DTW}(X(1:n), Y(1:m)) \end{aligned}$$

The matrix D is called the accumulated cost matrix.

The entry $D(n,m)$ specifies the cost of an optimal warping path that aligns $X(1:n)$ with $Y(1:m)$.

Dynamic Time Warping

Lemma:

$$
\begin{array}{llcl}
(i) & D(N,M) & = & \mathrm{DTW}(X,Y) \\
(ii) & D(1,1) & = & C(1,1) \\
(iii) & D(n,1) & = & \sum_{k=1}^{n} C(k,1) \\
 & D(1,m) & = & \sum_{k=1}^{m} C(1,k) \\
(iv) & D(n,m) & = & \min \begin{pmatrix} D(n-1,m-1) \\ D(n-1,m) \\ D(n,m-1) \end{pmatrix} + C(n,m)
\end{array}
$$

for $n > 1,\ m > 1$

Proof: *(i)* – *(iii)* are clear by definition

Dynamic Time Warping

Proof of *(iv)*: Induction via n, m :

Let $n > 1,\ m > 1$ and $q = (q_1, \ldots, p_{L-1}, p_L)$ be an optimal warping path for $X(1:n))$ and $Y(1:m))$.
Then $q_L = (n, m)$ (boundary condition).

Let $q_{L-1} = (a, b)$. The step size condition implies

$$(a, b) \in \{(n-1, m-1), (n-1, m), (n, m-1)\}$$

The warping path $(q_1, \ldots, q_{L-1})$ must be optimal for $X(1:a)$, $Y(1:b)$. Thus,

$$D(n, m) = c_{(q_1, \ldots, q_{L-1})}(X(1:a), Y(1:b)) + C(n, m)$$

■

Dynamic Time Warping

Accumulated cost matrix

Given the two feature sequences X and Y, the matrix D is computed recursively.

- Initialize D using *(ii)* and *(iii) of the lemma.*
- Compute $D(n, m)$ for $n > 1$, $m > 1$ using *(iv)*.
- $\mathrm{DTW}(X, Y) = D(N, M)$ *using (i).*

Note:

- Complexity *O(NM)*.
- Dynamic programming: “overlapping-subproblem property”

Dynamic Time Warping

Optimal warping path

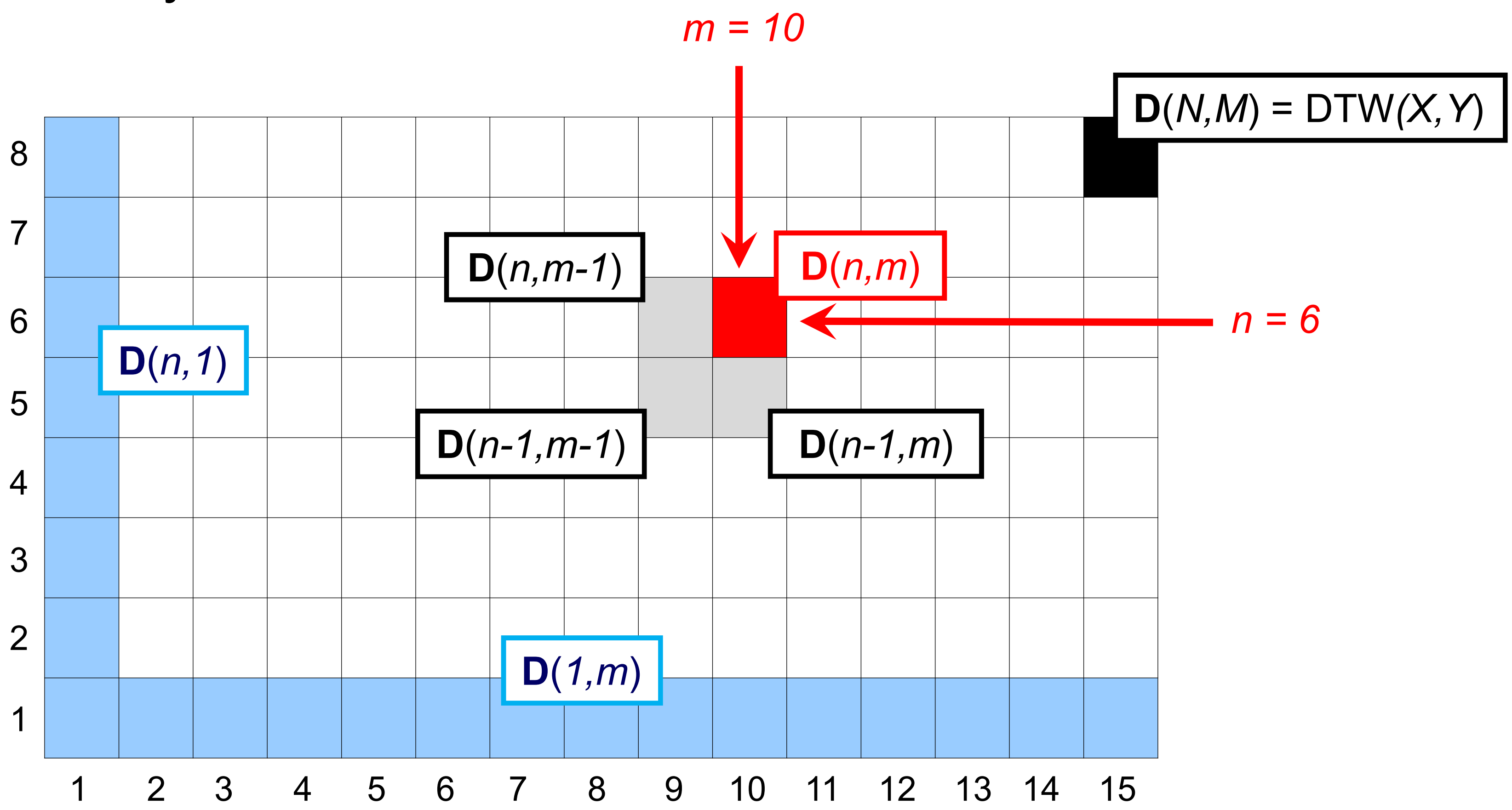
Given to the algorithm is the accumulated cost matrix D.
The optimal path $p^* = (p_1, \ldots, p_L)$ is computed in reverse order of the indices starting with $p_L = (N, M)$.
Suppose $p_\ell = (n, m)$ has been computed. In case $(n, m) = (1, 1)$, one must have $\ell = 1$ and we are done.
Otherwise,

$$p_{\ell-1} := \begin{cases} (1, m-1), & \text{if } n = 1 \\ (n-1, 1), & \text{if } m = 1 \\ \operatorname{argmin}\{D(n-1, m-1), & \\ \qquad D(n-1, m), D(n, m-1)\}, & \text{otherwise,} \end{cases}$$

where we take the lexicographically smallest pair in case “argmin” is not unique.

Dynamic Time Warping

Summary



Dynamic Time Warping

Summary

Algorithm: DTW

Input: Cost matrix $\mathbf{C}$ of size $N \times M$
Output: Accumulated cost matrix $\mathbf{D}$
Optimal warping path P^*

Procedure: Initialize $(N \times M)$ matrix $\mathbf{D}$ by $\mathbf{D}(n,1) = \sum_{k=1}^{n} \mathbf{C}(k,1)$ for $n \in [1:N]$ and $\mathbf{D}(1,m) = \sum_{k=1}^{m} \mathbf{C}(1,k)$ for $m \in [1:M]$. Then compute in a nested loop for $n = 2,\ldots,N$ and $m = 2,\ldots,M$:

$$\mathbf{D}(n,m) = \mathbf{C}(n,m) + \min\{\mathbf{D}(n-1,m-1), \mathbf{D}(n-1,m), \mathbf{D}(n,m-1)\}.$$

Set $\ell = 1$ and $q_\ell = (N,M)$. Then repeat the following steps until $q_\ell = (1,1)$:

Increase ℓ by one and let $(n,m) = q_{\ell-1}$.
If $n = 1$, then $\quad q_\ell = (1,m-1)$,
else if $m = 1$, then $\quad q_\ell = (n-1,m)$,
else $\quad q_\ell = \operatorname{argmin}\{\mathbf{D}(n-1,m-1), \mathbf{D}(n-1,m), \mathbf{D}(n,m-1)\}$.
(If 'argmin' is not unique, take lexicographically smallest cell.)

Set $L = \ell$ and return $P^* = (q_L, q_{L-1}, \ldots, q_1)$ as well as $\mathbf{D}$.

Dynamic Time Warping

Example

$$X = (1,3,3,8,1)$$

$$Y = (2,0,0,8,7,2)$$

$$c(x,y) = |x - y|,\ x,y \in \mathbb{R}$$

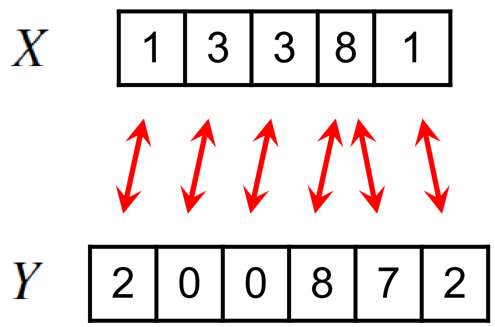
C

1	1	1	1	7	6	1
8	6	8	8	0	1	6
3	1	3	3	5	4	1
3	1	3	3	5	4	1
1	1	1	1	7	6	1
	2	0	0	8	7	2

D

1	10	10	11	14	13	9
8	9	11	13	7	8	14
3	3	5	7	10	12	13
3	2	4	5	8	12	13
1	1	2	3	10	16	17
	2	0	0	8	7	2

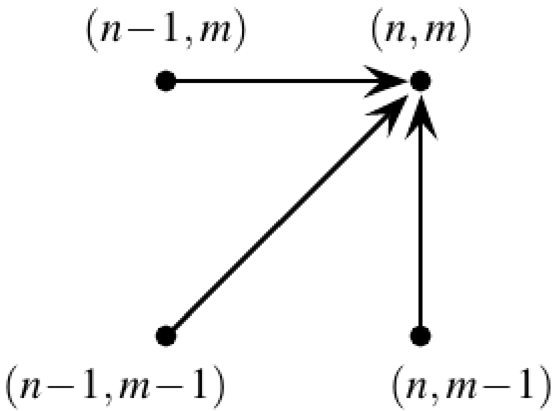
Alignment

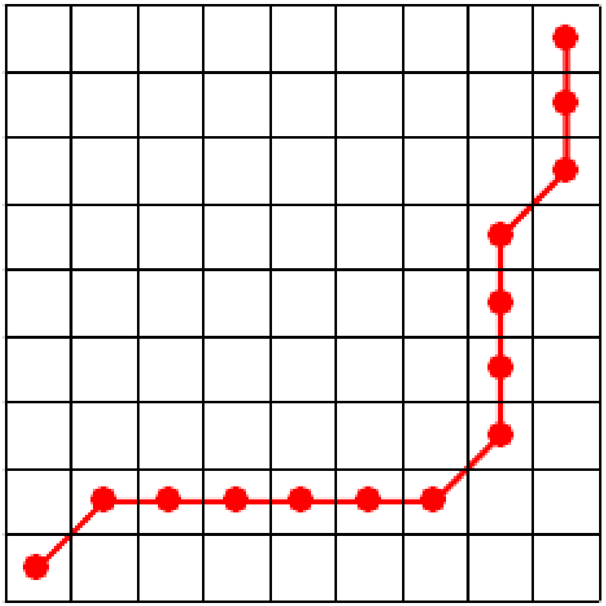


Optimal warping path: $P^* = ((1,1),(2,2),(3,3),(4,4),(4,5),(5,6))$

Dynamic Time Warping

Step size conditions

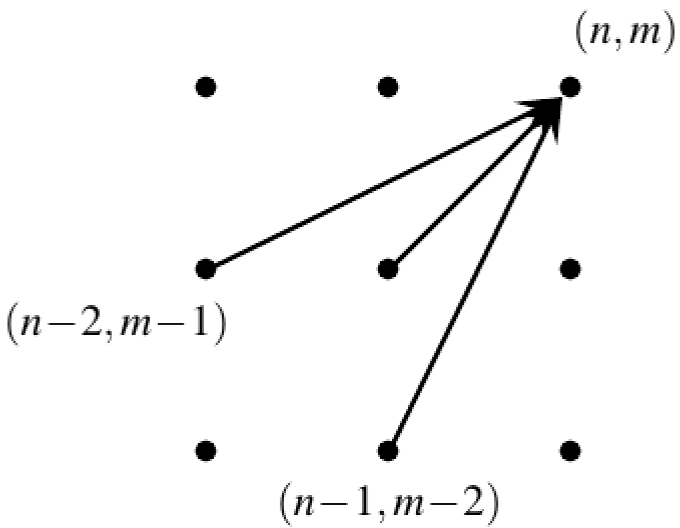


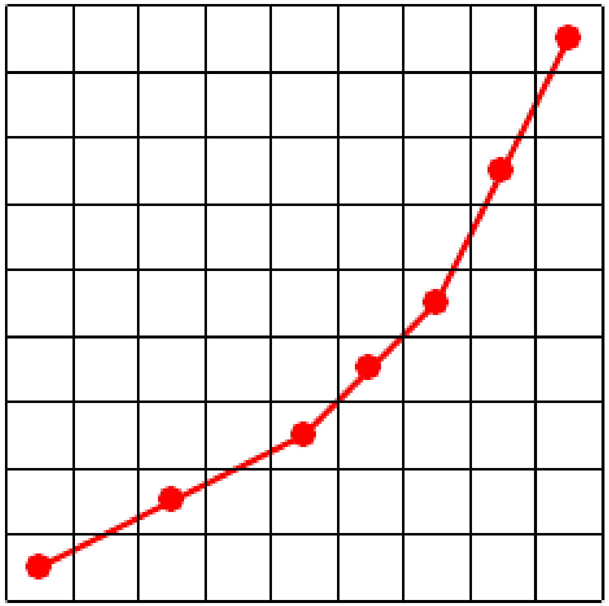


$$\Sigma = \{(1,0),(0,1),(1,1)\}$$

Dynamic Time Warping

Step size conditions

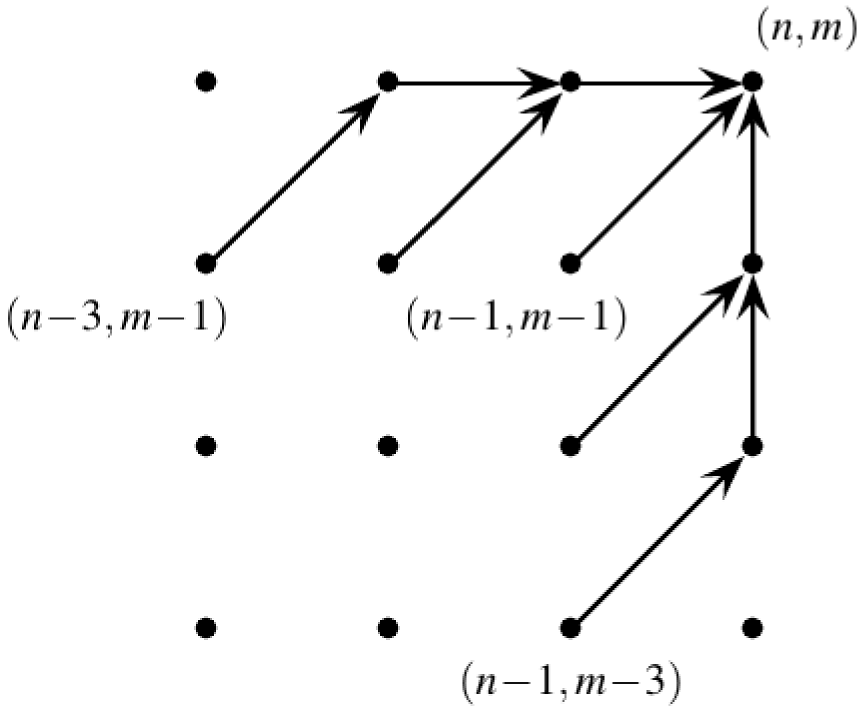


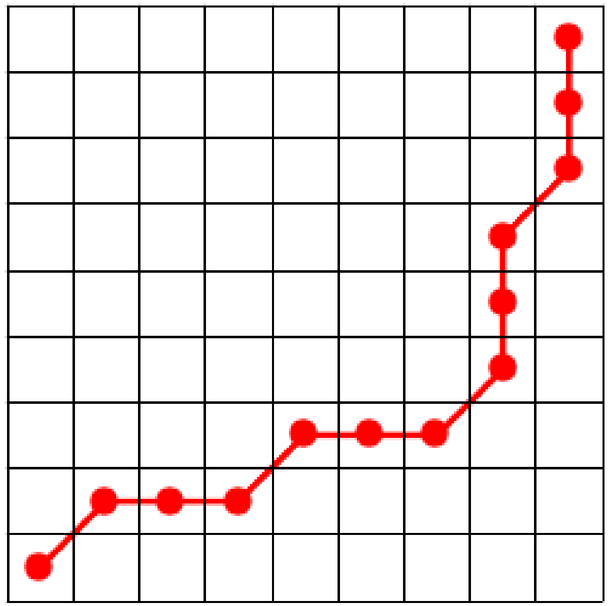


$$\Sigma = \{(2,1),(1,2),(1,1)\}$$

Dynamic Time Warping

Step size conditions





Dynamic Time Warping

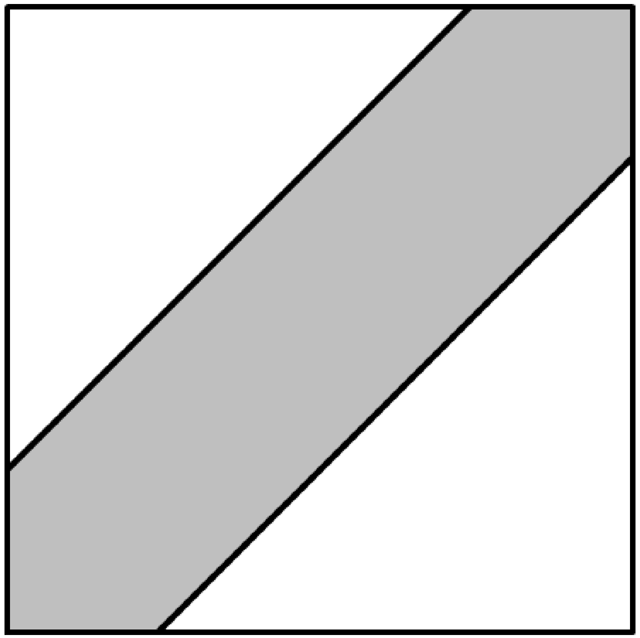
- Computation via dynamic programming
- Memory requirements and running time: $O(NM)$
- Problem: Infeasible for large N and M
- Example: Feature resolution 10 Hz, pieces 15 min

$$\Rightarrow \quad N, M \sim 10{,}000$$
$$\Rightarrow \quad N \cdot M \sim 100{,}000{,}000$$

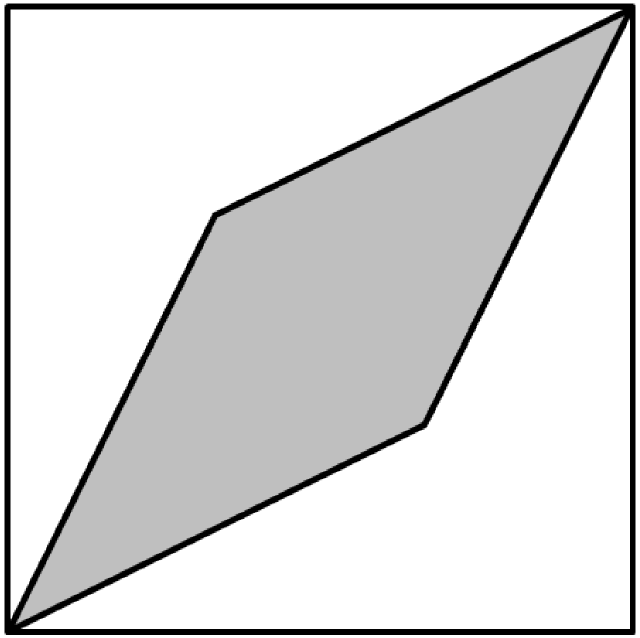
Dynamic Time Warping

Global constraints

Sakoe-Chiba band

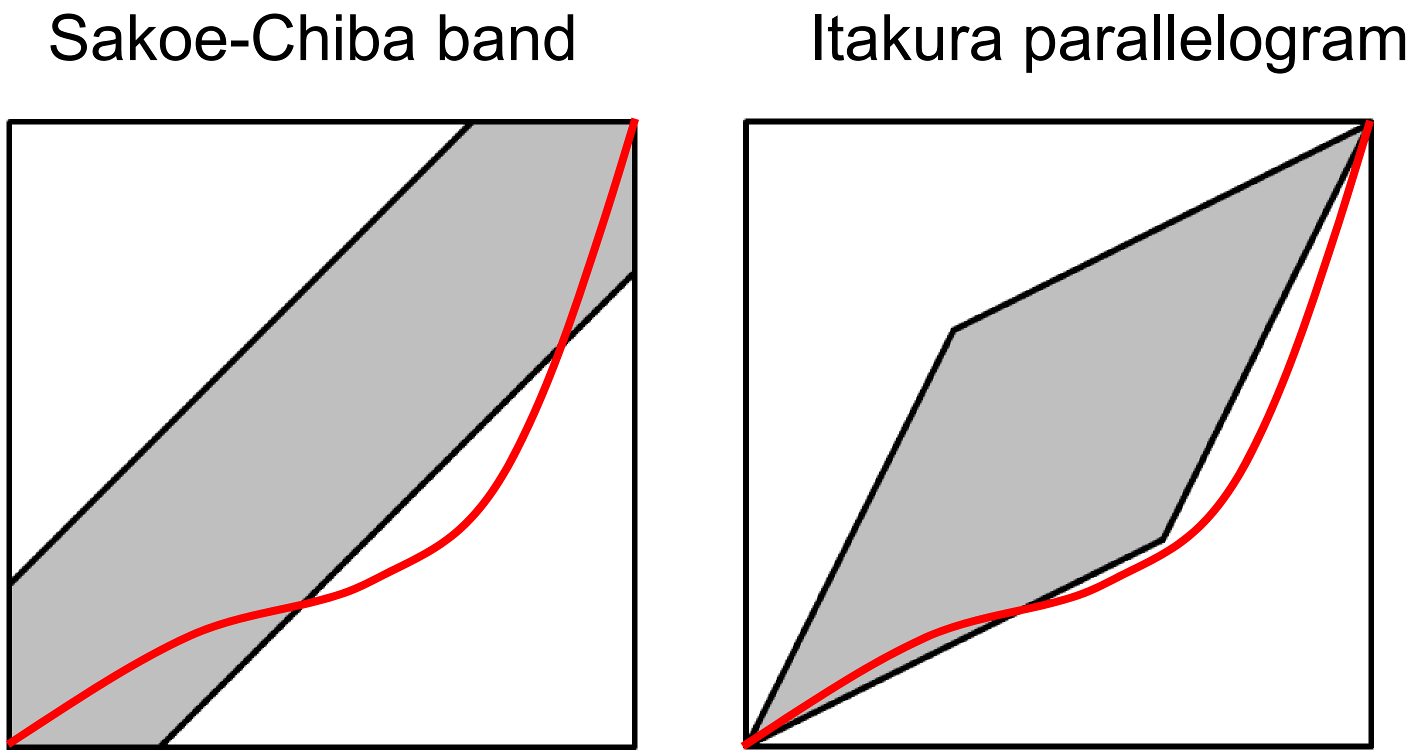


Itakura parallelogram



Dynamic Time Warping

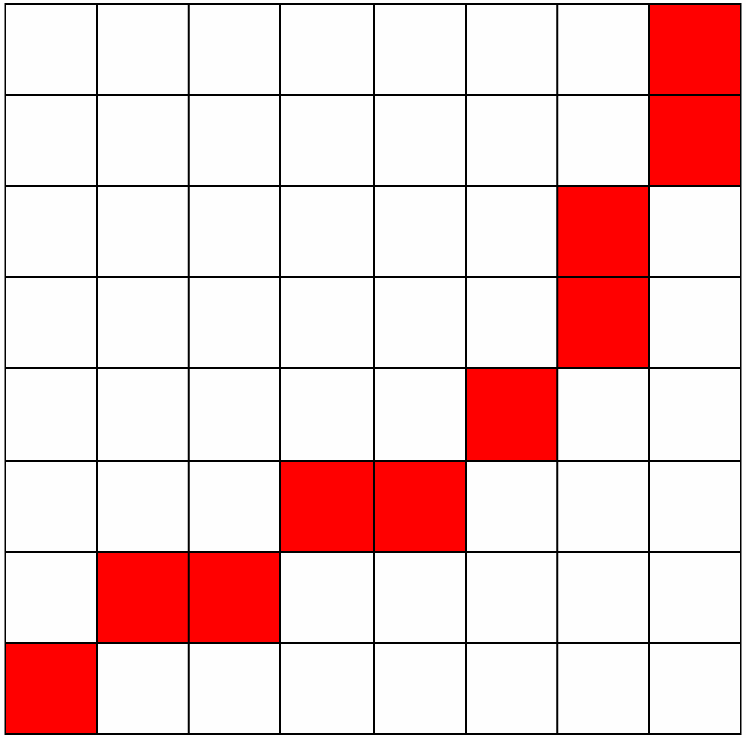
Global constraints



Problem: Optimal warping path not in constraint region

Dynamic Time Warping

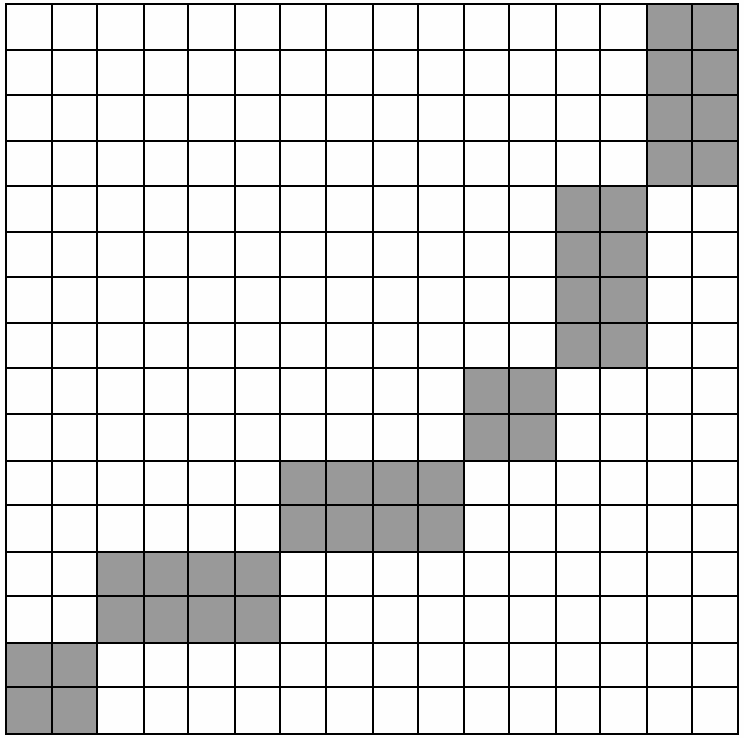
Multiscale approach



Compute optimal warping path on coarse level

Dynamic Time Warping

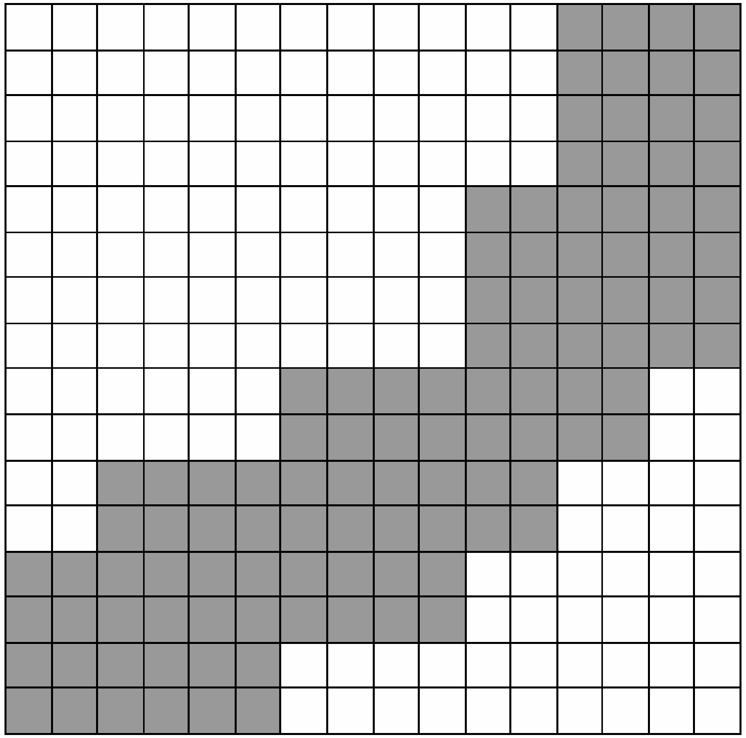
Multiscale approach



Project on fine level

Dynamic Time Warping

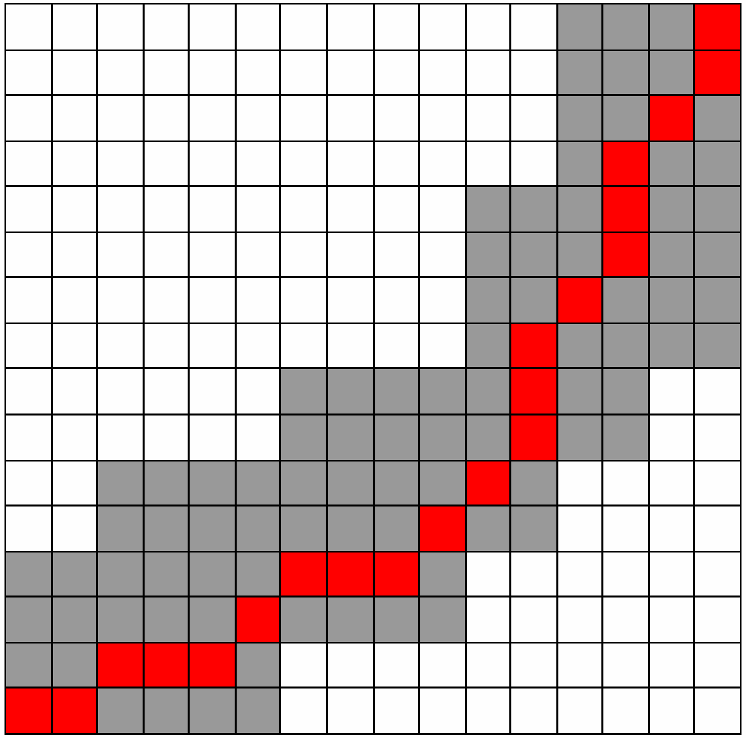
Multiscale approach



Specify constraint region

Dynamic Time Warping

Multiscale approach



Compute constrained optimal warping path

Dynamic Time Warping

Multiscale approach

- Suitable features?
- Suitable resolution levels?
- Size of constraint regions?

Good trade-off between efficiency and robustness?

Suitable parameters depend very much on application!