Fundamentals of Music Processing

Chapter 5: Chord Recognition

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www.music-processing.de

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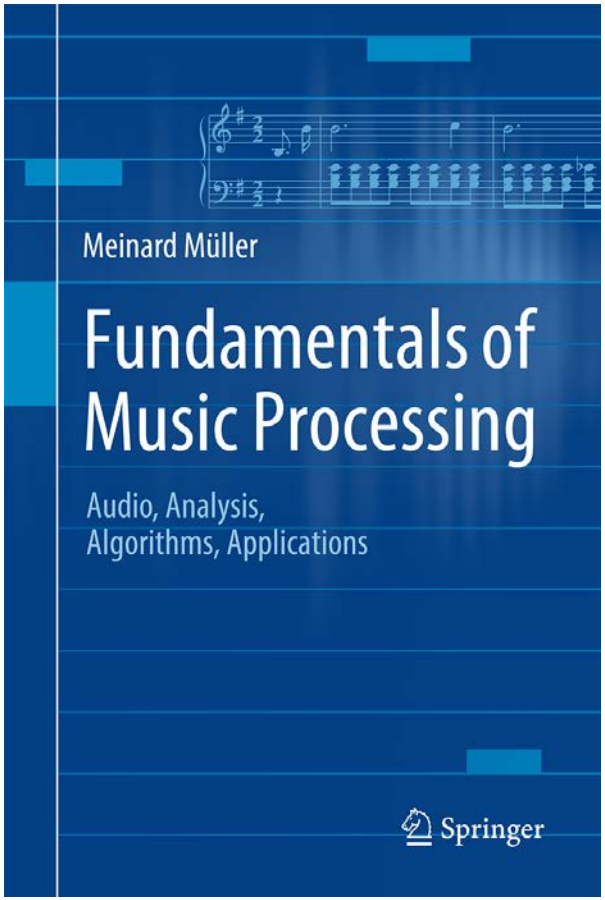
Book: Fundamentals of Music Processing



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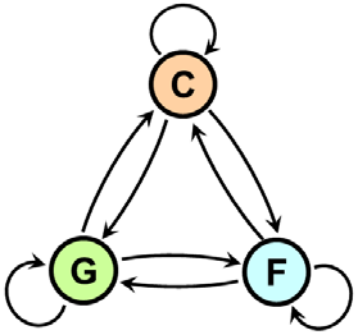
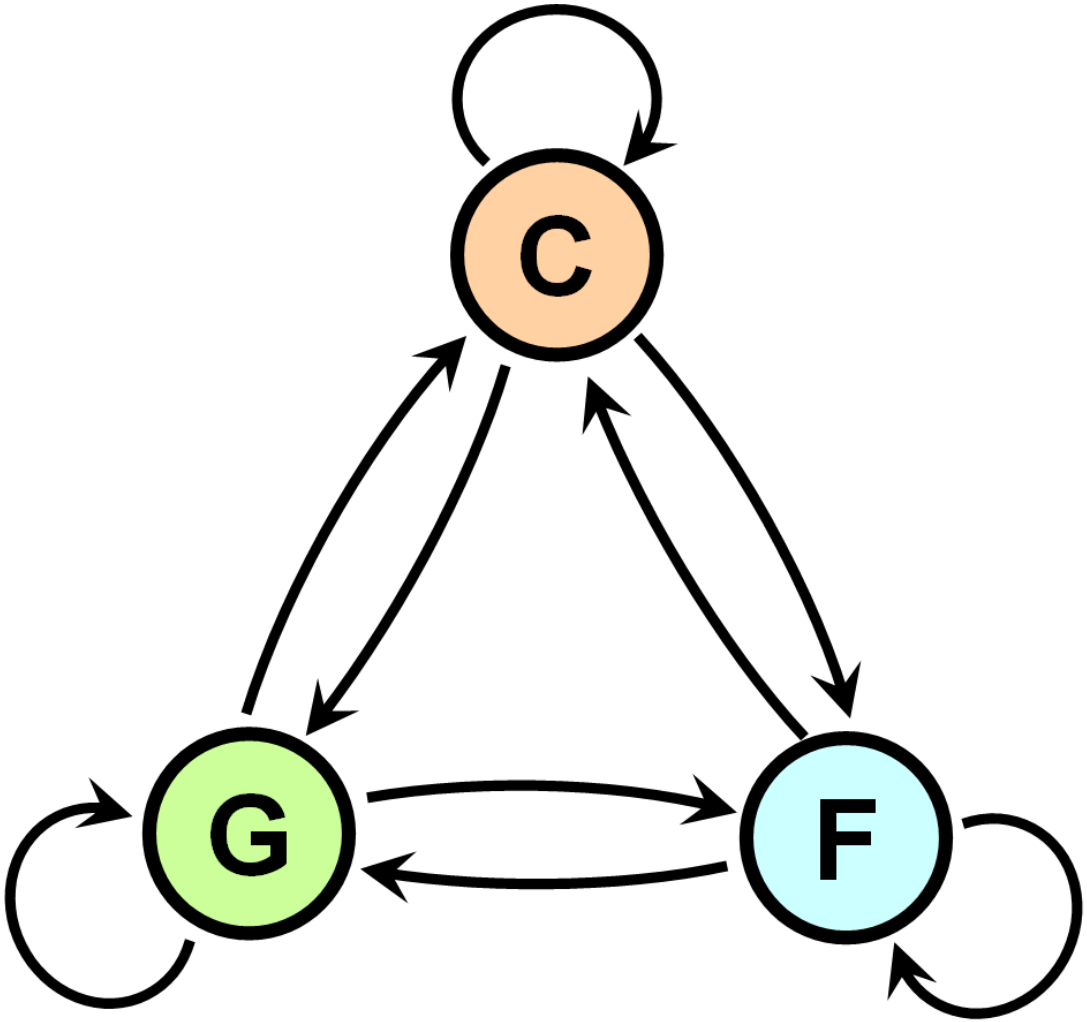
Chapter 5: Chord Recognition



In Chapter 5, we consider the problem of analyzing harmonic properties of a piece of music by determining a descriptive progression of chords from a given audio recording. We take this opportunity to first discuss some basic theory of harmony including concepts such as intervals, chords, and scales. Then, motivated by the automated chord recognition scenario, we introduce template-based matching procedures and hidden Markov models—a concept of central importance for the analysis of temporal patterns in time-dependent data streams including speech, gestures, and music.

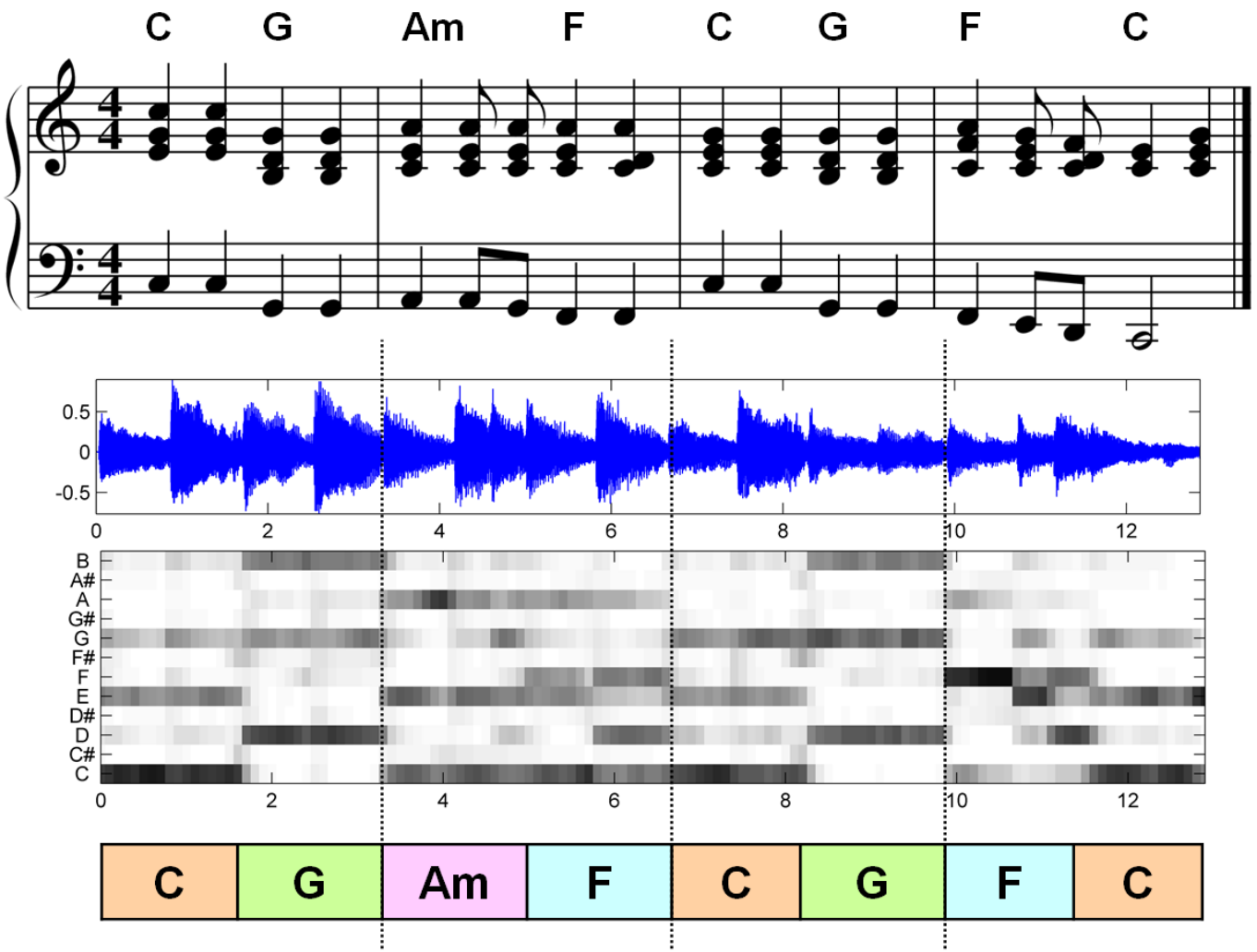
5 Chord Recognition

Teaser



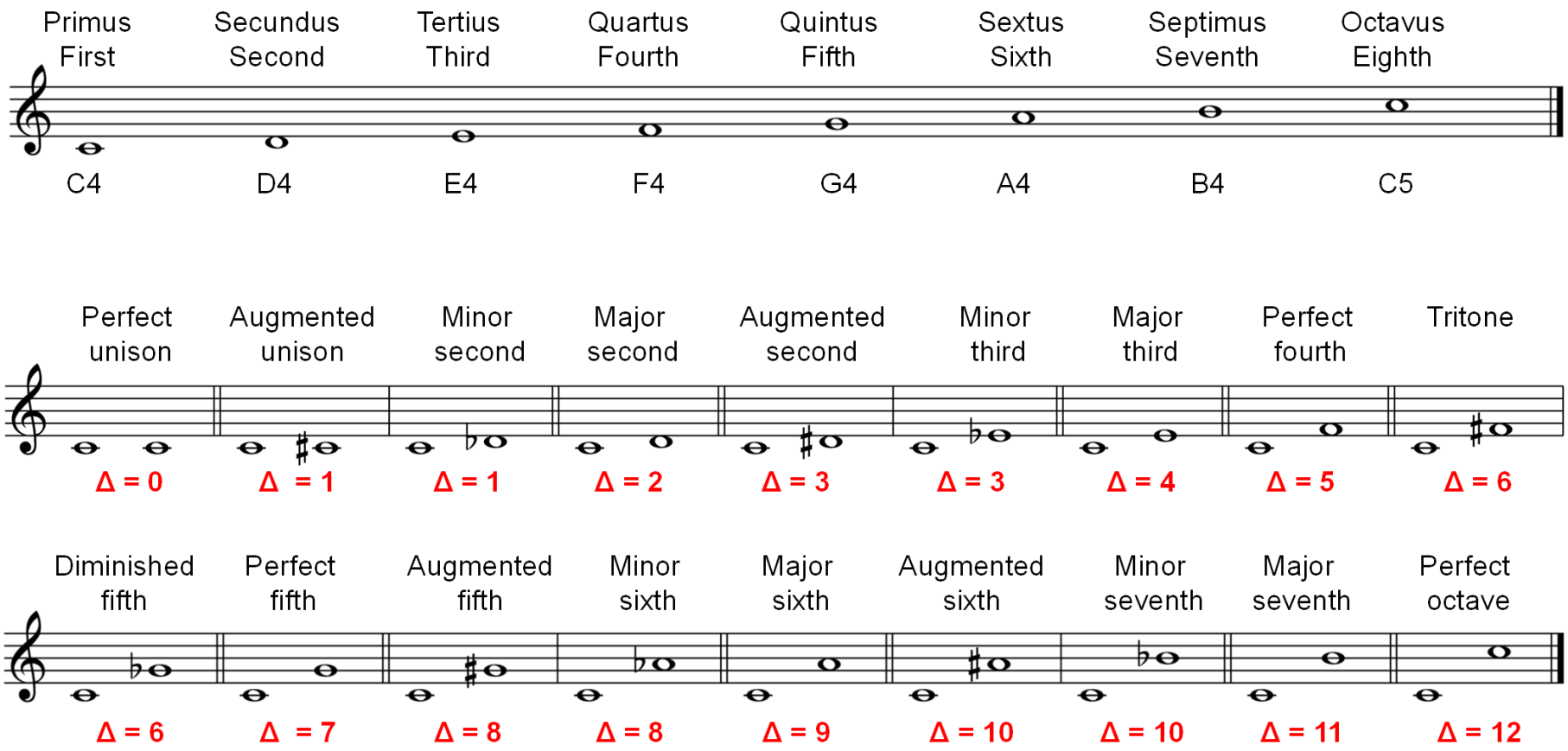
5 Chord Recognition

Fig. 5.1



5.1 Basic Theory of Harmony

Fig. 5.2



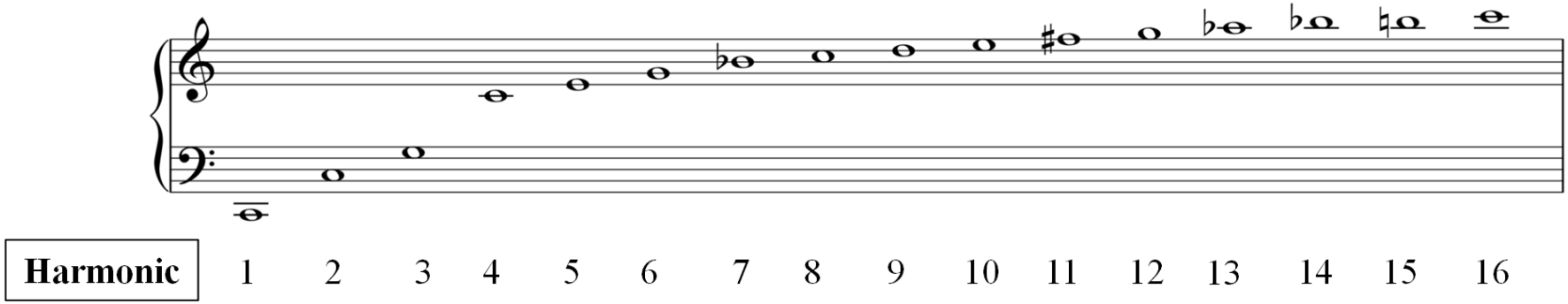
5.1 Basic Theory of Harmony

Fig. 5.3

Δ	Interval name	Interval	JI ratio	Pyt. ratio
0	(Perfect) unison	C4 – C4	1:1	1:1
1	Minor second	C4 – D♭4	15:16	$3^5:2^8$
2	Major second	C4 – D4	8:9	$2^3:3^2$
3	Minor third	C4 – E♭4	5:6	$3^3:2^5$
4	Major third	C4 – E4	4:5	$2^6:3^4$
5	(Perfect) fourth	C4 – F4	3:4	$3:2^2$
6	Tritone	C4 – F♯4	32:45	$2^9:3^6$ or $3^6:2^{10}$
7	(Perfect) fifth	C4 – G4	2:3	2:3
8	Minor sixth	C4 – A♭4	5:8	$3^4:2^7$
9	Major sixth	C4 – A4	3:5	$2^4:3^3$
10	Minor seventh	C4 – B♭4	5:9	$3^2:2^4$
11	Major seventh	C4 – B4	8:15	$2^7:3^5$
12	(Perfect) octave	C4 – C5	1:2	1:2

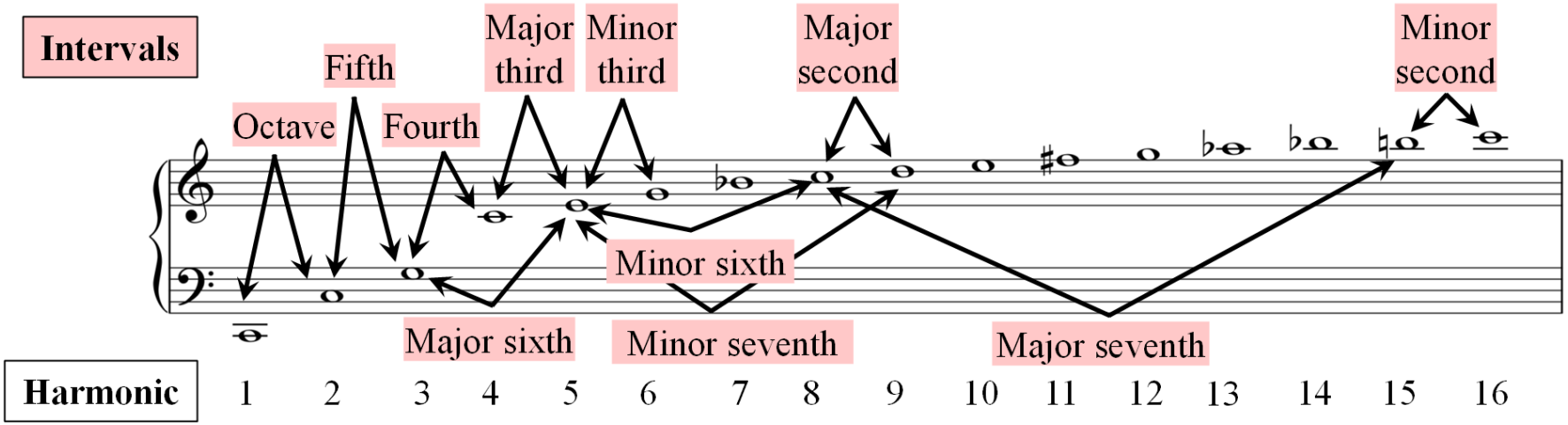
5.1 Basic Theory of Harmony

Fig. 5.4



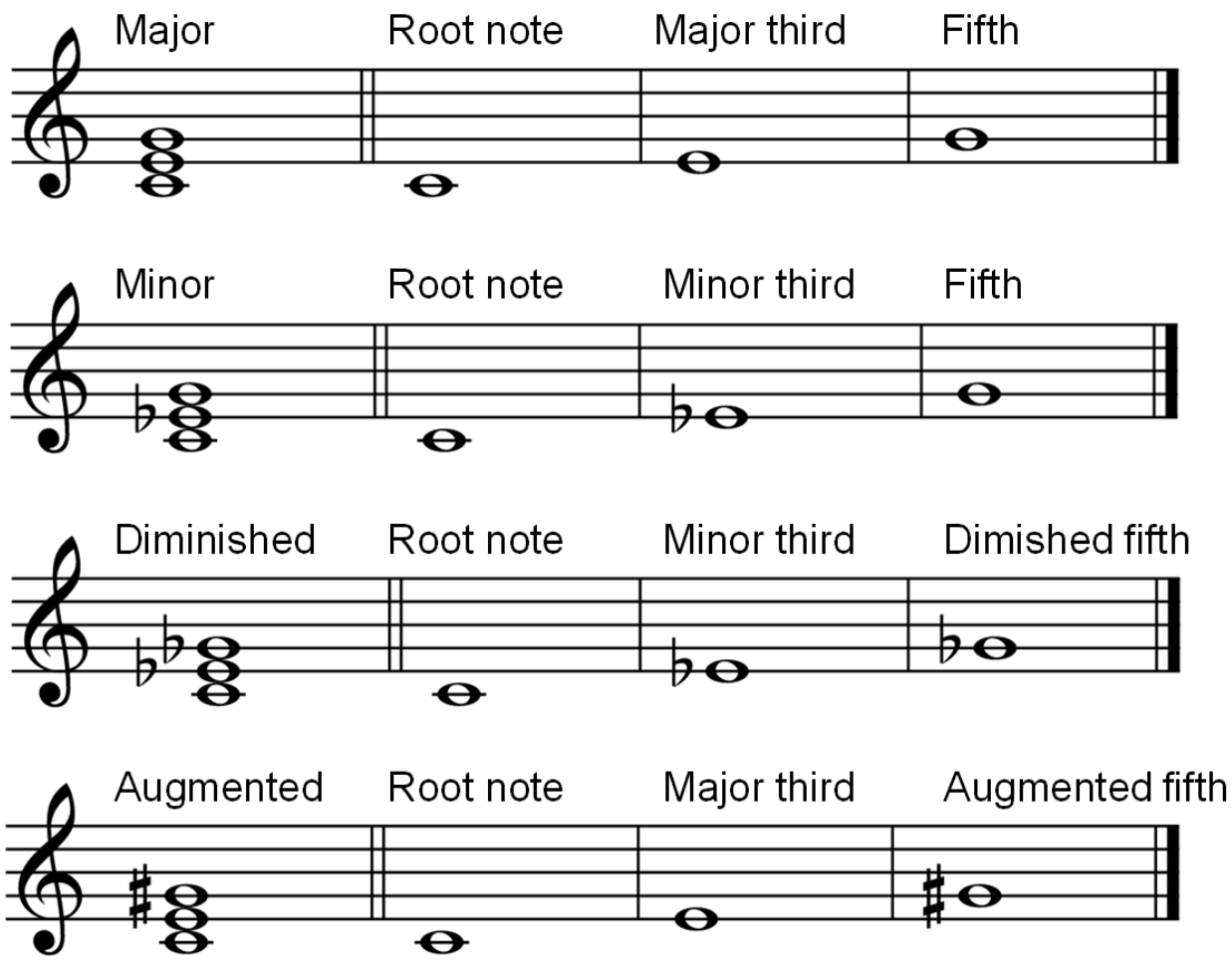
5.1 Basic Theory of Harmony

Fig. 5.4



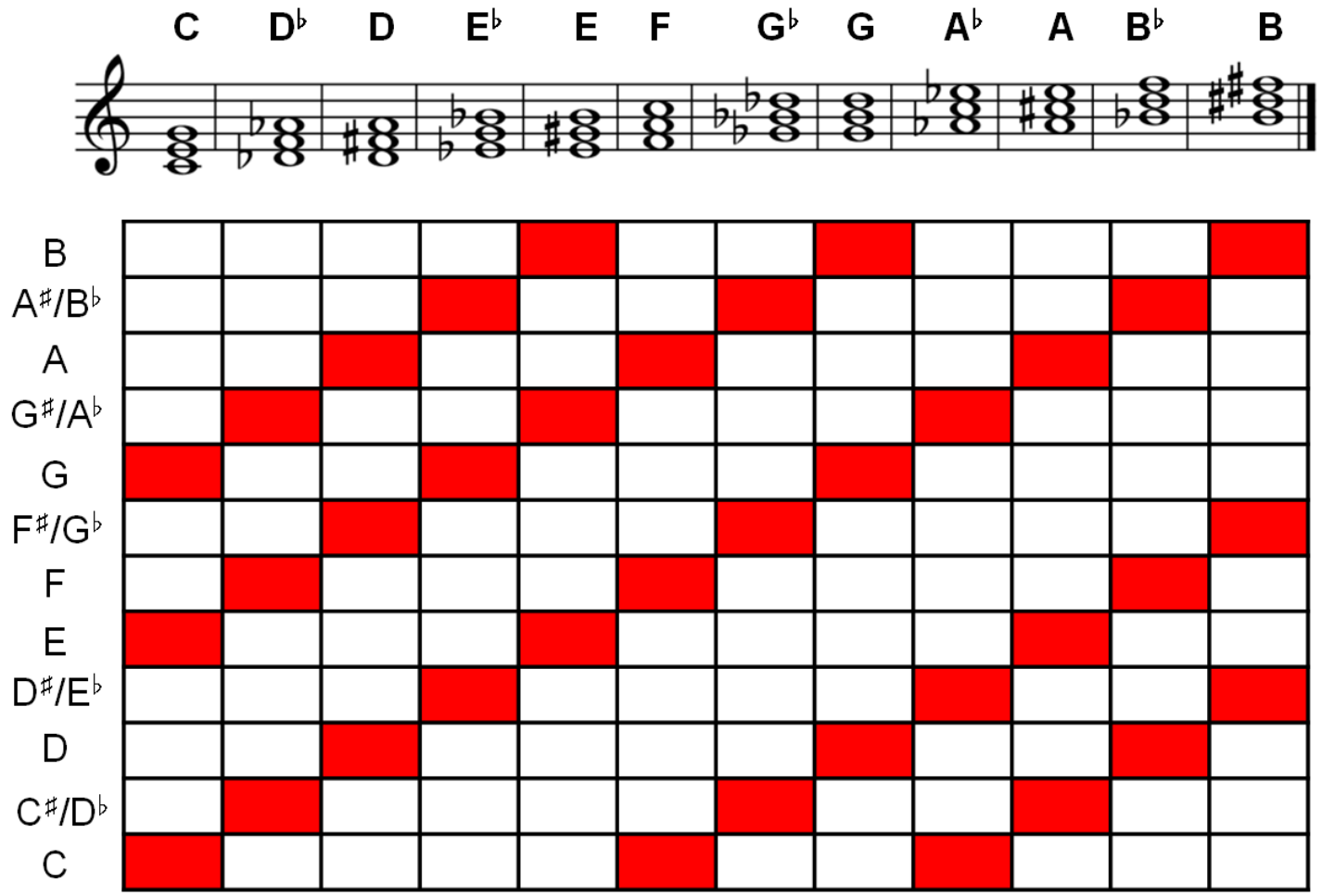
5.1 Basic Theory of Harmony

Fig. 5.5



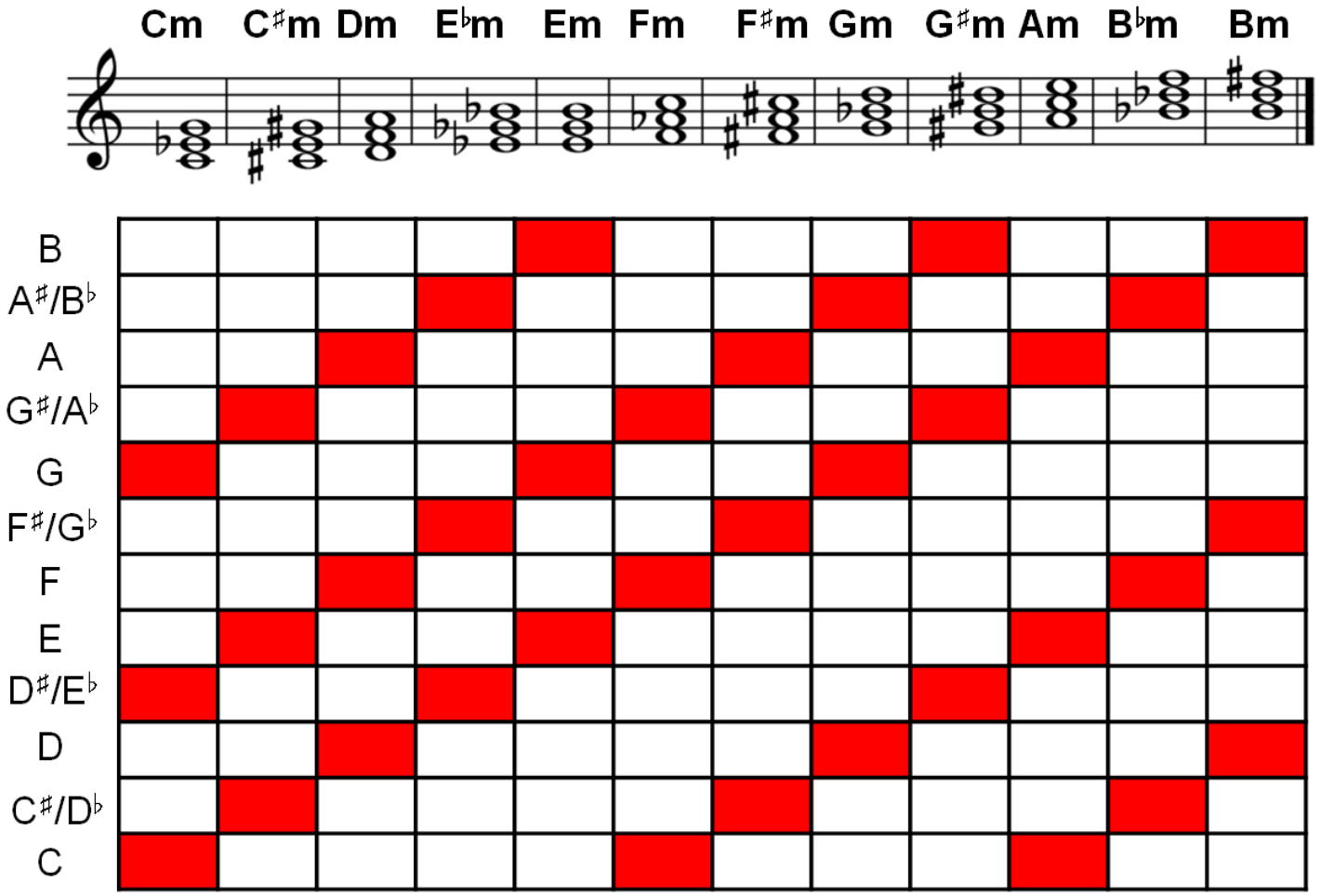
5.1 Basic Theory of Harmony

Fig. 5.6



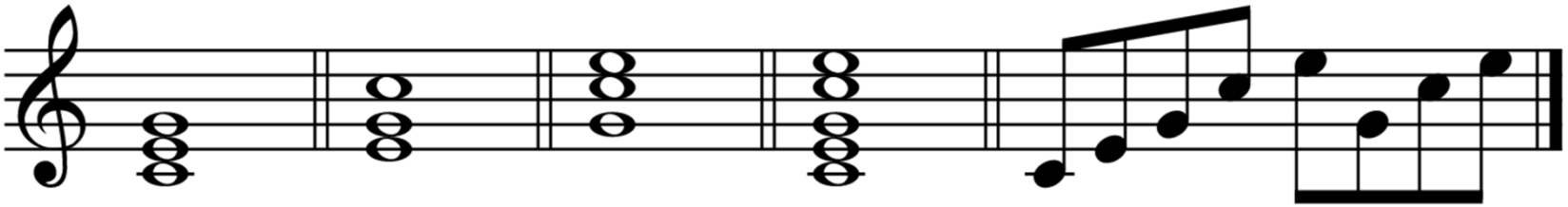
5.1 Basic Theory of Harmony

Fig. 5.6



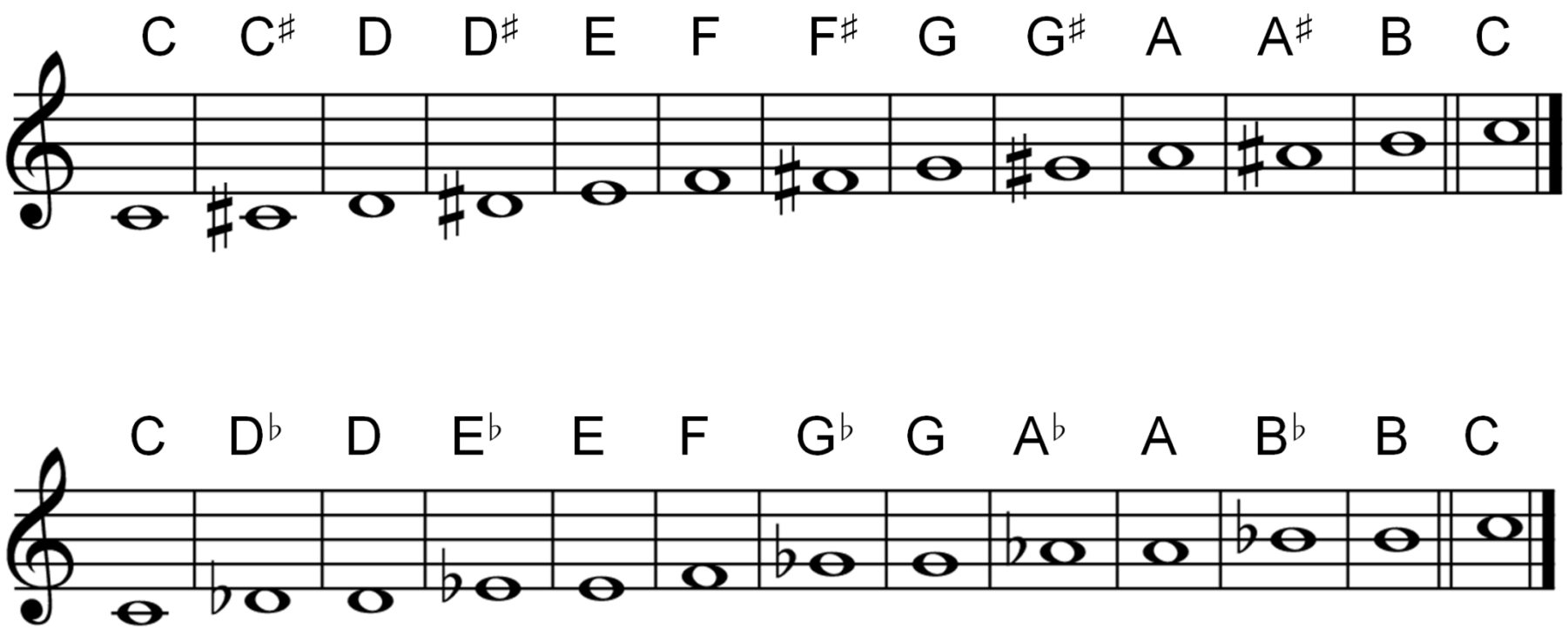
5.1 Basic Theory of Harmony

Fig. 5.7



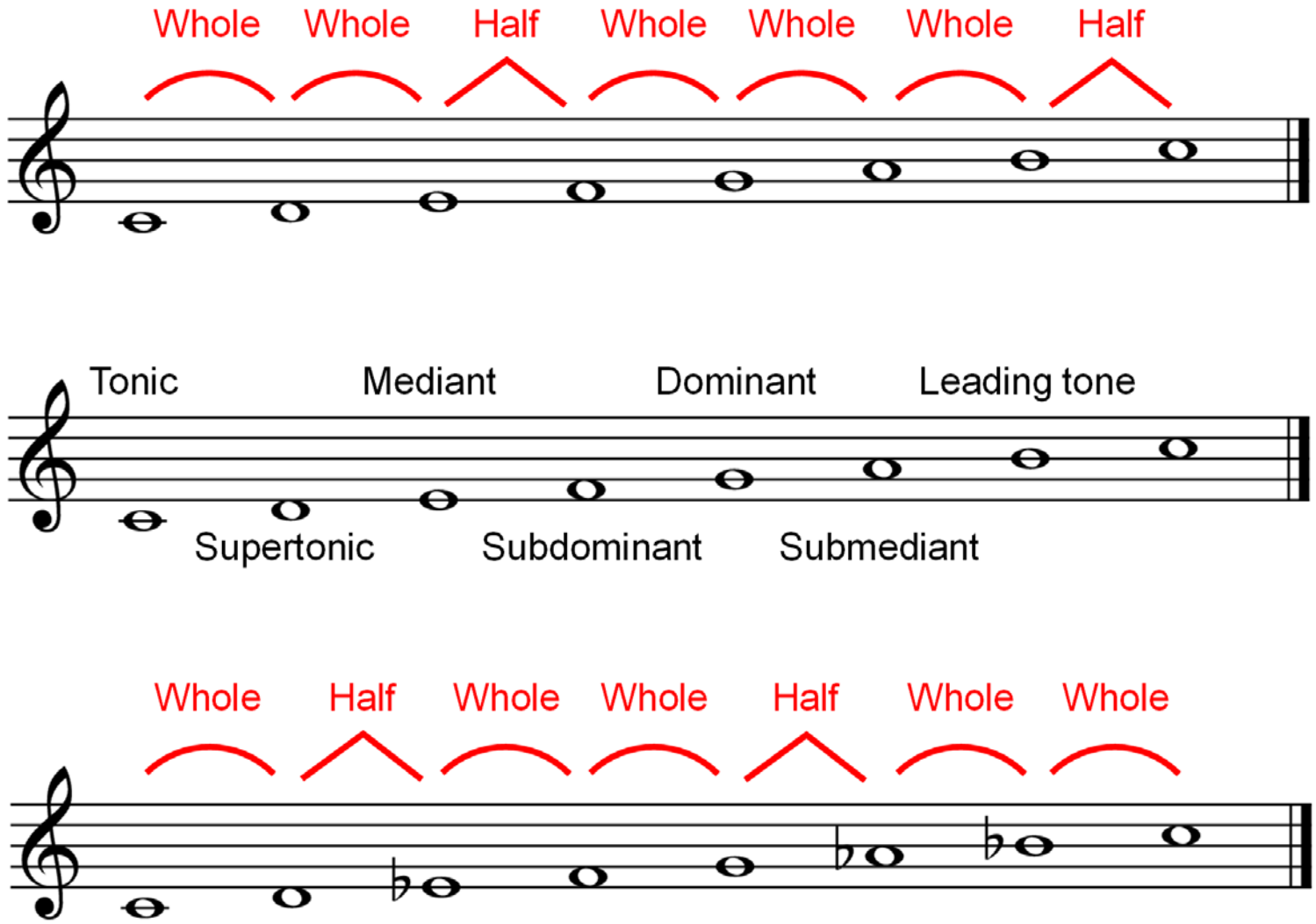
5.1 Basic Theory of Harmony

Fig. 5.8



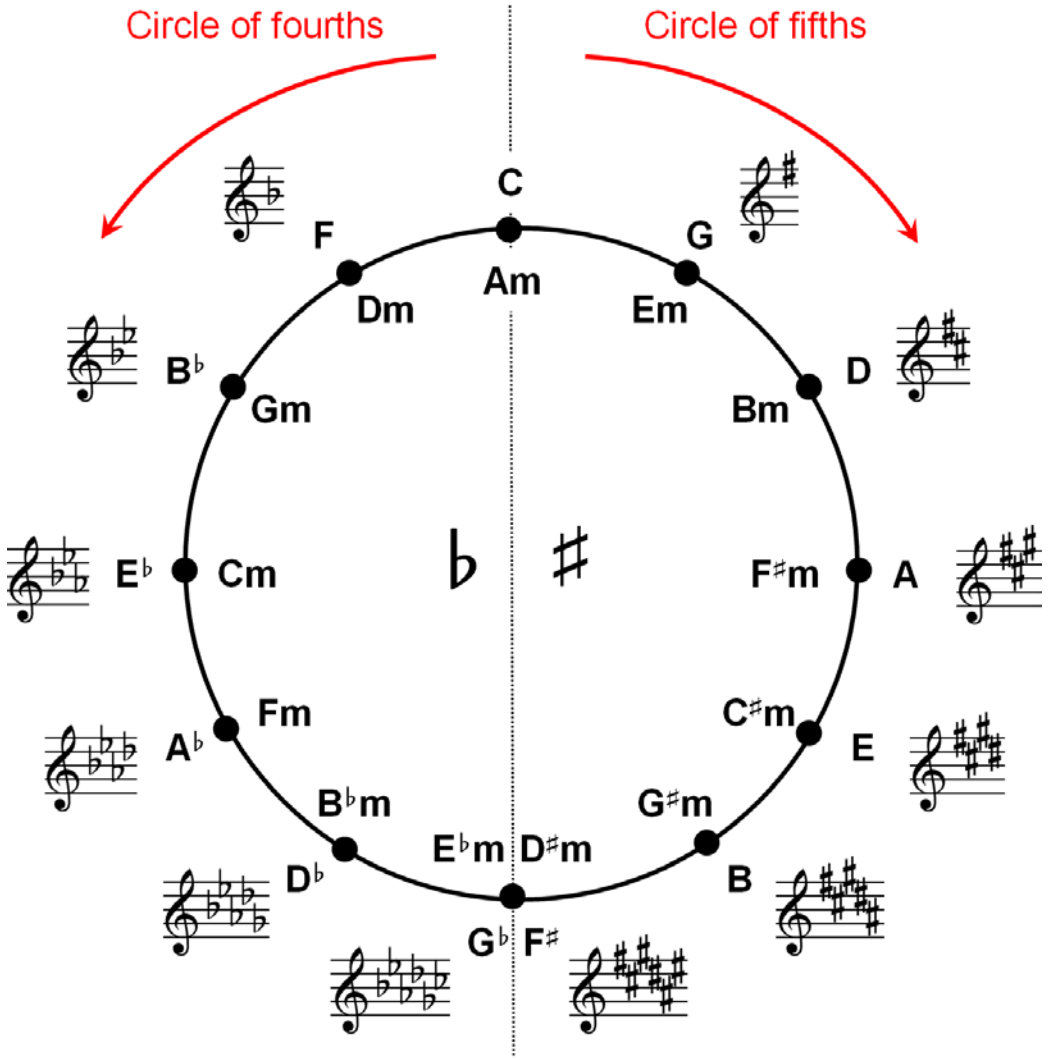
5.1 Basic Theory of Harmony

Fig. 5.9



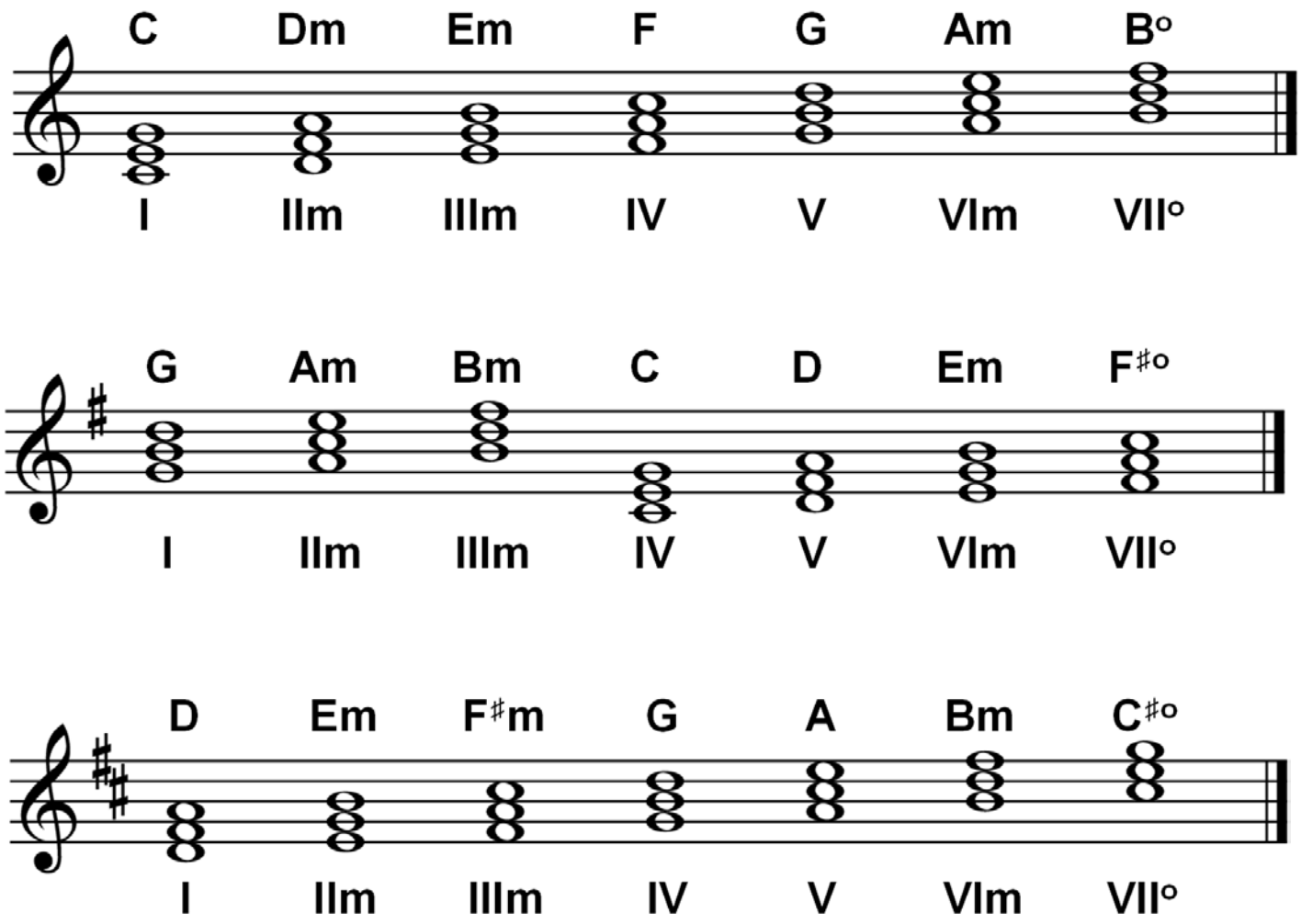
5.1 Basic Theory of Harmony

Fig. 5.10



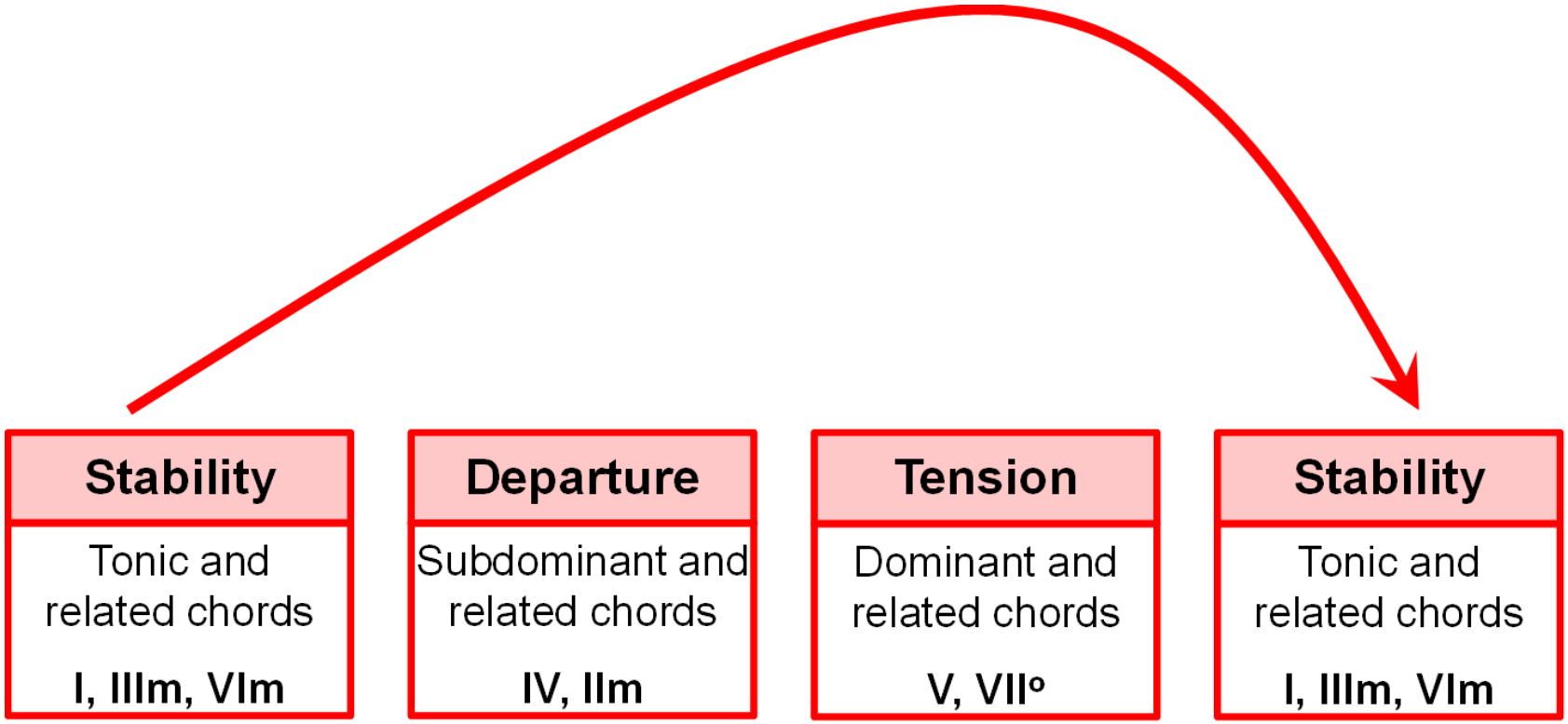
5.1 Basic Theory of Harmony

Fig. 5.11



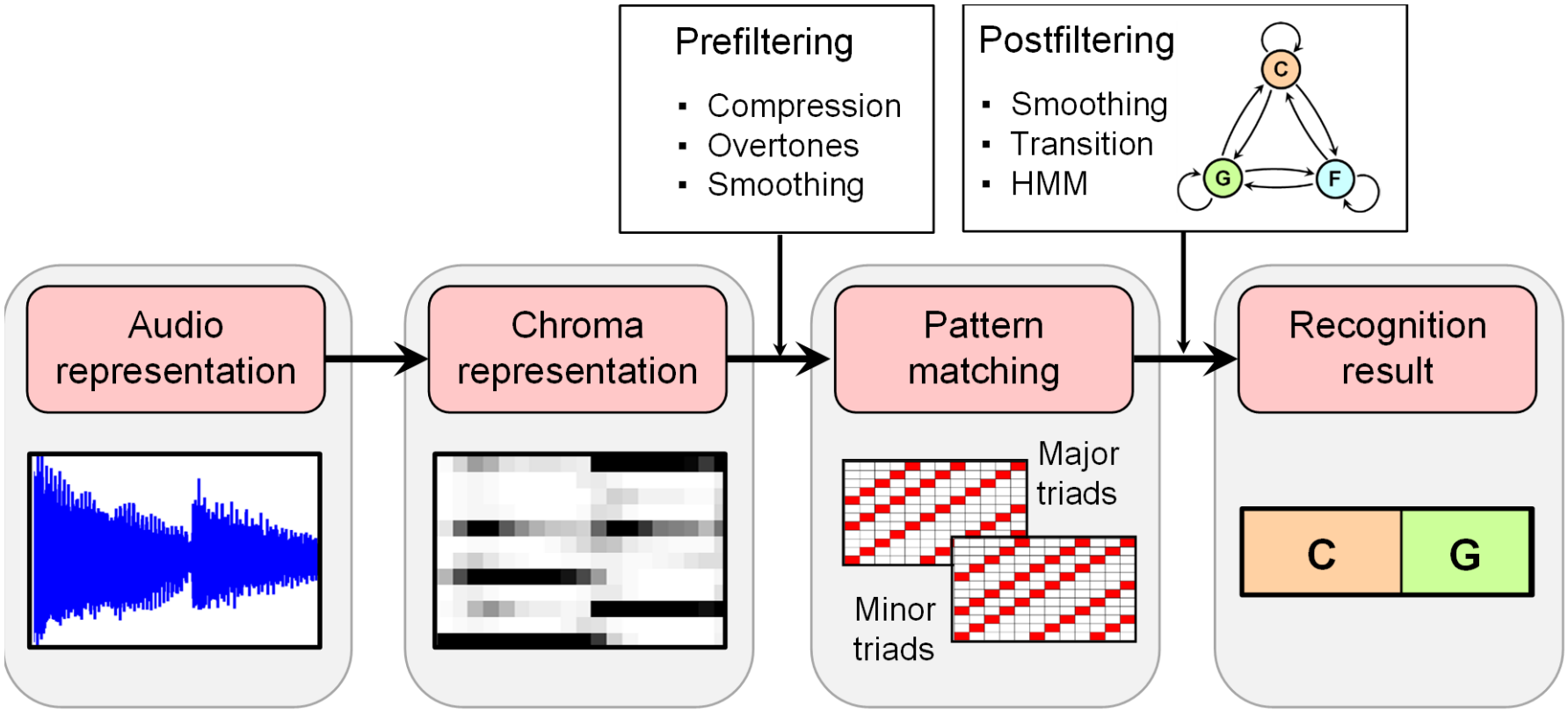
5.1 Basic Theory of Harmony

Fig. 5.12



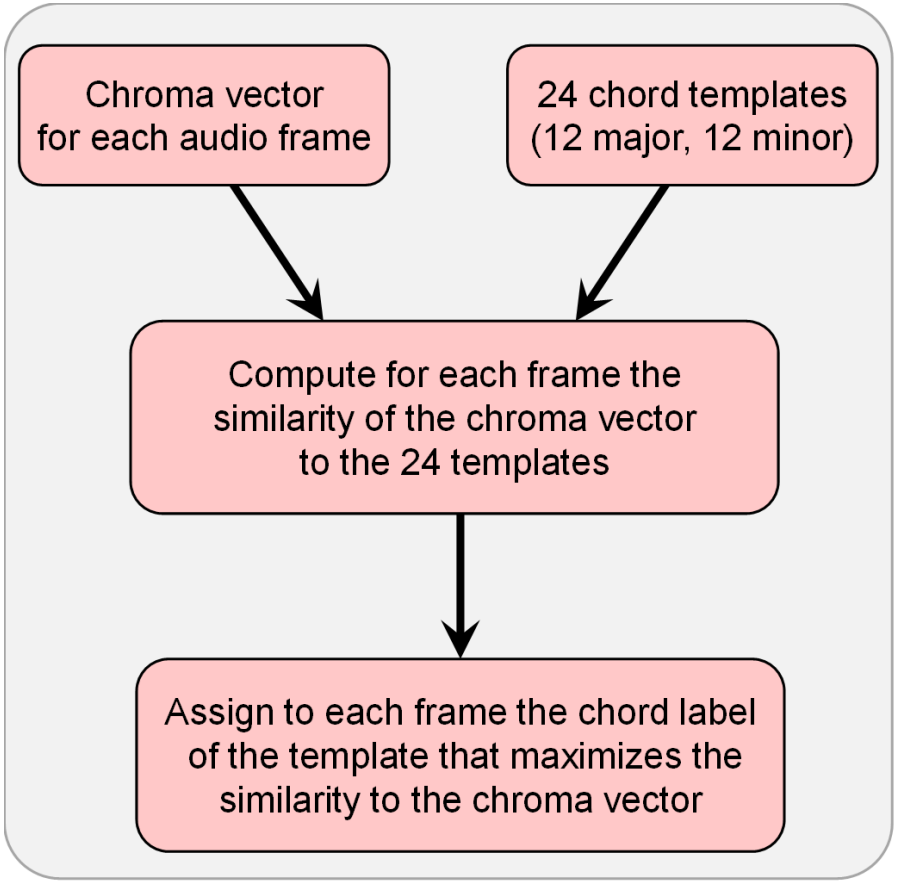
5.2 Template-Based Chord Recognition

Fig. 5.13



5.2 Template-Based Chord Recognition

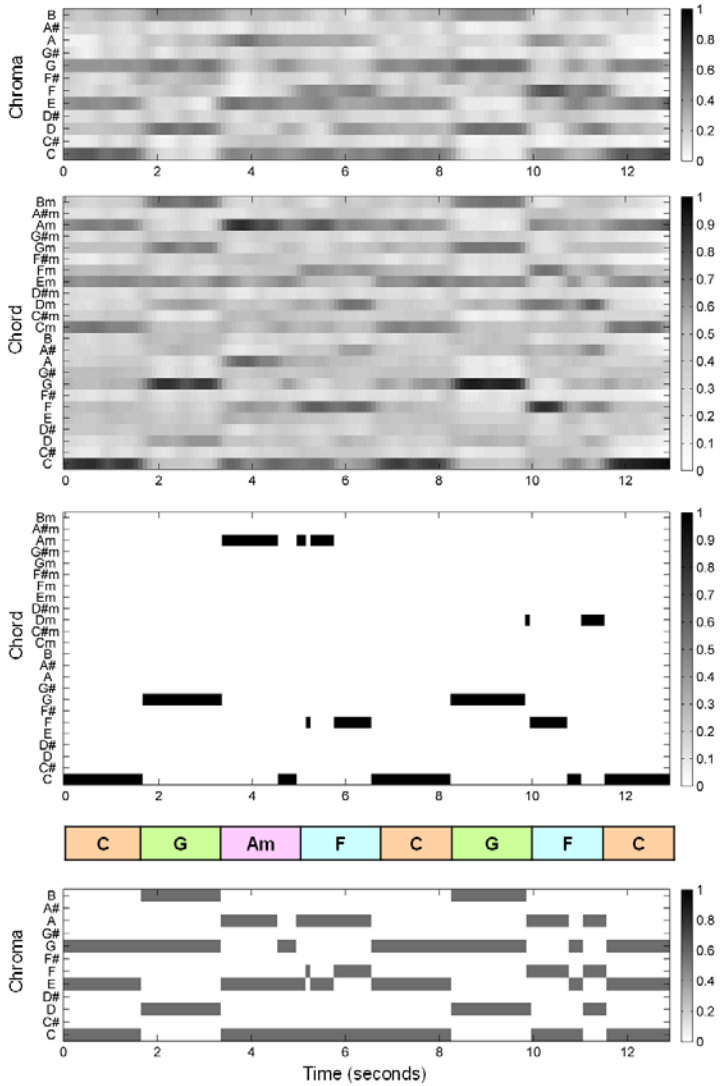
Fig. 5.14



	C	C♯	D	…	Cm	C♯m	Dm	…
B	0	0	0	…	0	0	0	…
A♯	0	0	0	…	0	0	0	…
A	0	0	1	…	0	0	1	…
G♯	0	1	0	…	0	1	0	…
G	1	0	0	…	1	0	0	…
F♯	0	0	1	…	0	0	0	…
F	0	1	0	…	0	0	1	…
E	1	0	0	…	0	1	0	…
D♯	0	0	0	…	1	0	0	…
D	0	0	1	…	0	0	1	…
C♯	0	1	0	…	0	1	0	…
C	1	0	0	…	1	0	0	…

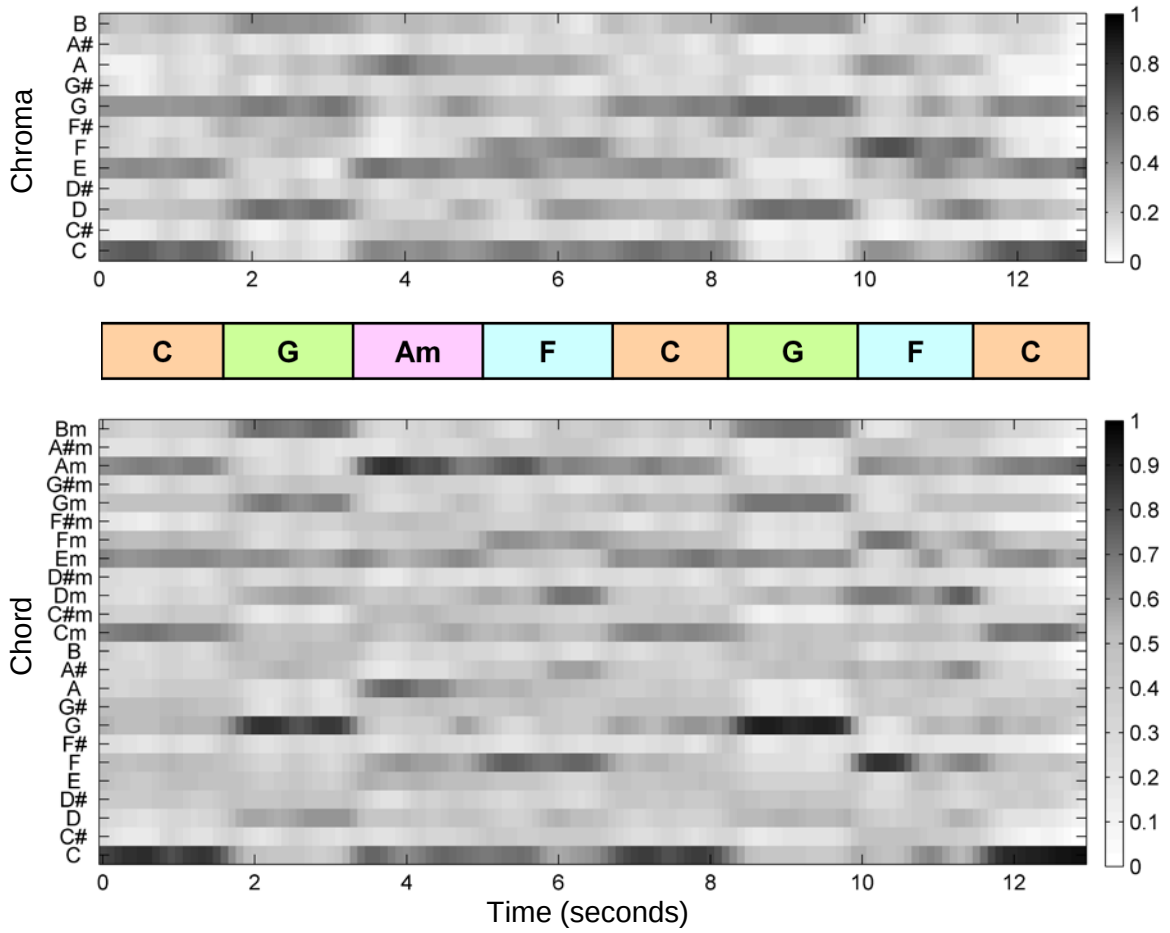
5.2 Template-Based Chord Recognition

Fig. 5.15



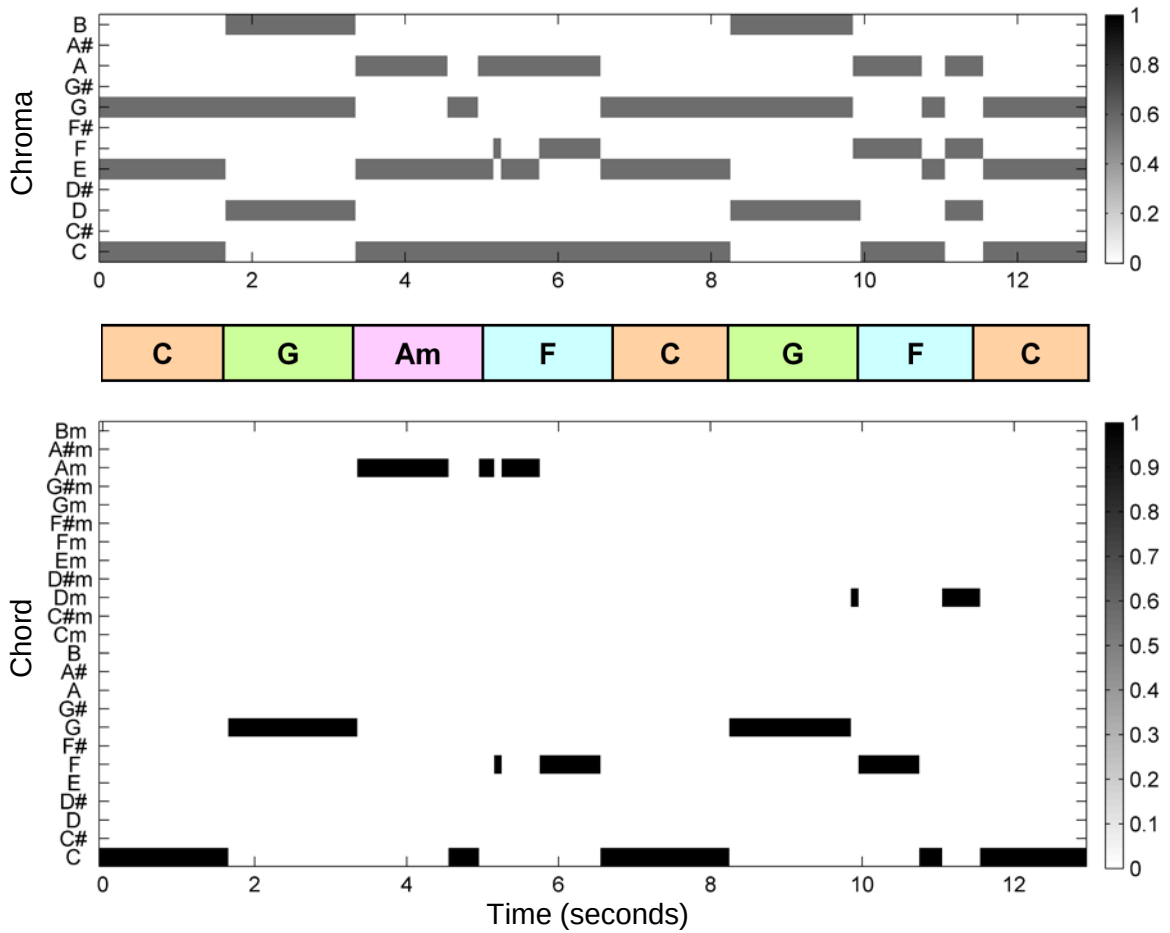
5.2 Template-Based Chord Recognition

Fig. 5.15



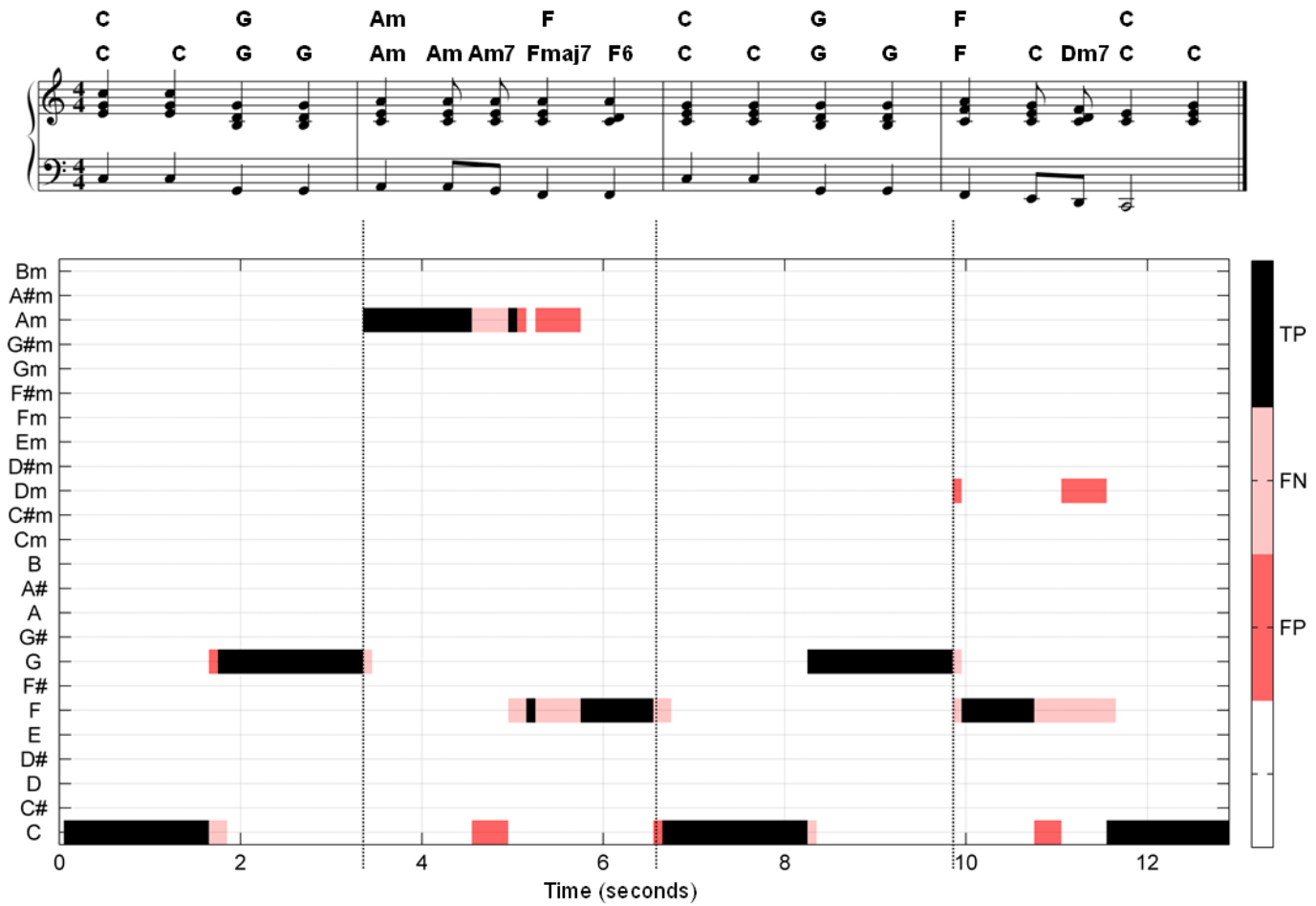
5.2 Template-Based Chord Recognition

Fig. 5.15



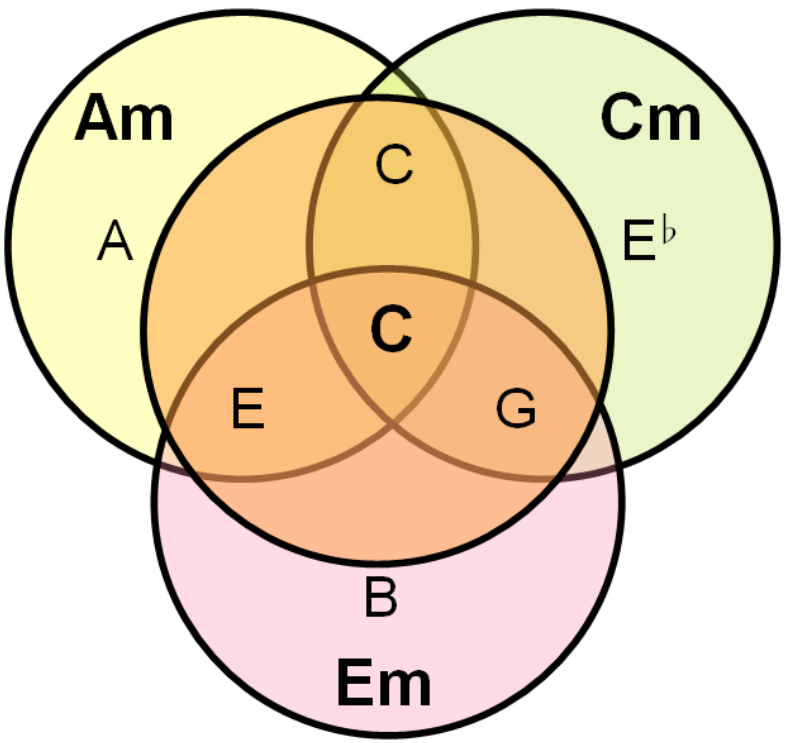
5.2 Template-Based Chord Recognition

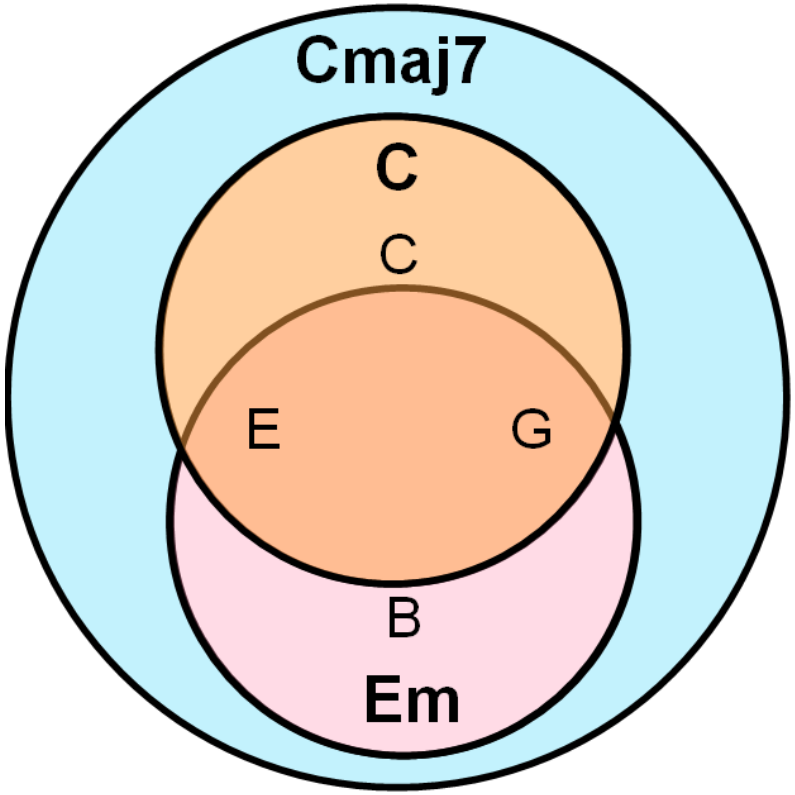
Fig. 5.16



5.2 Template-Based Chord Recognition

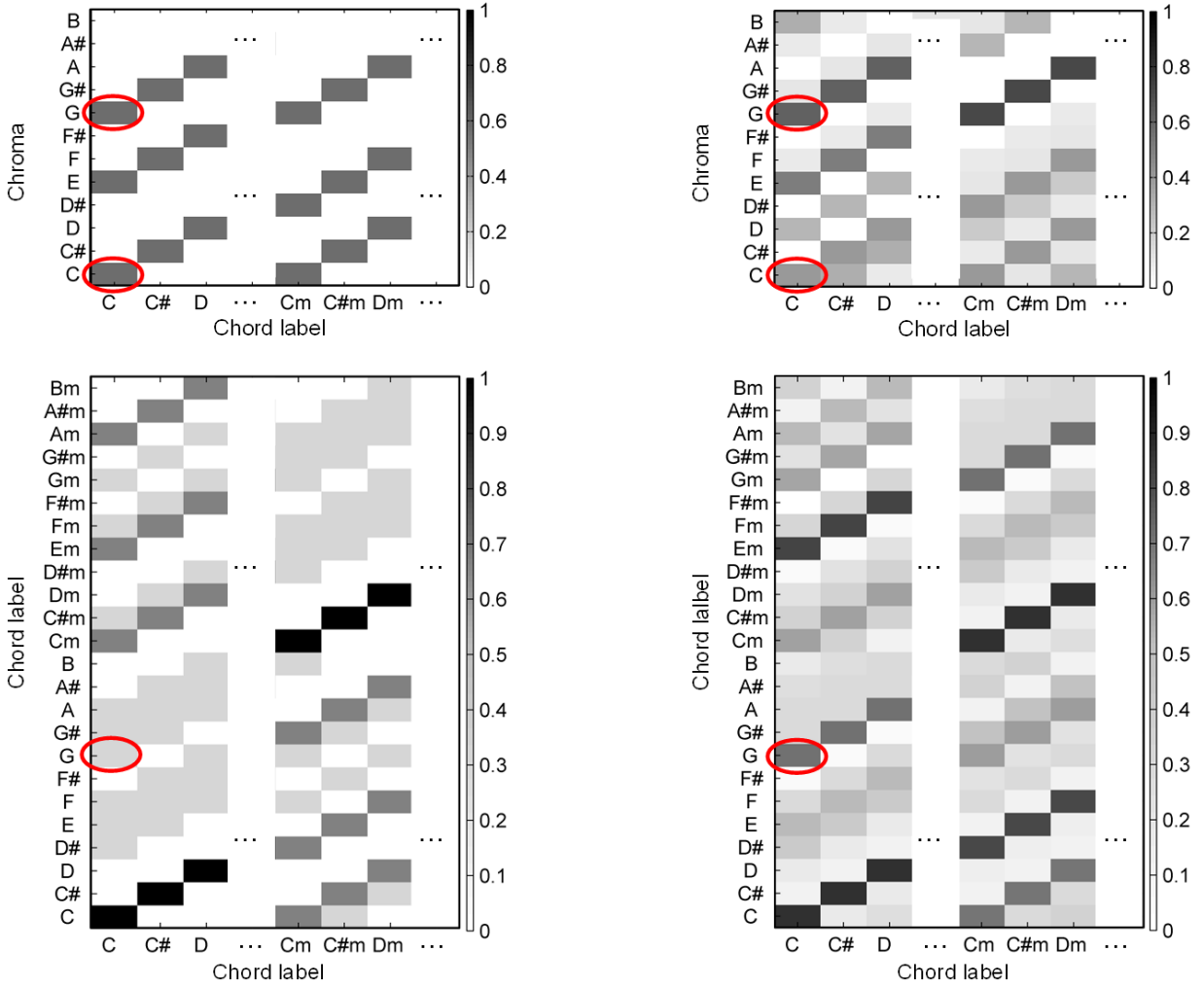
Fig. 5.17





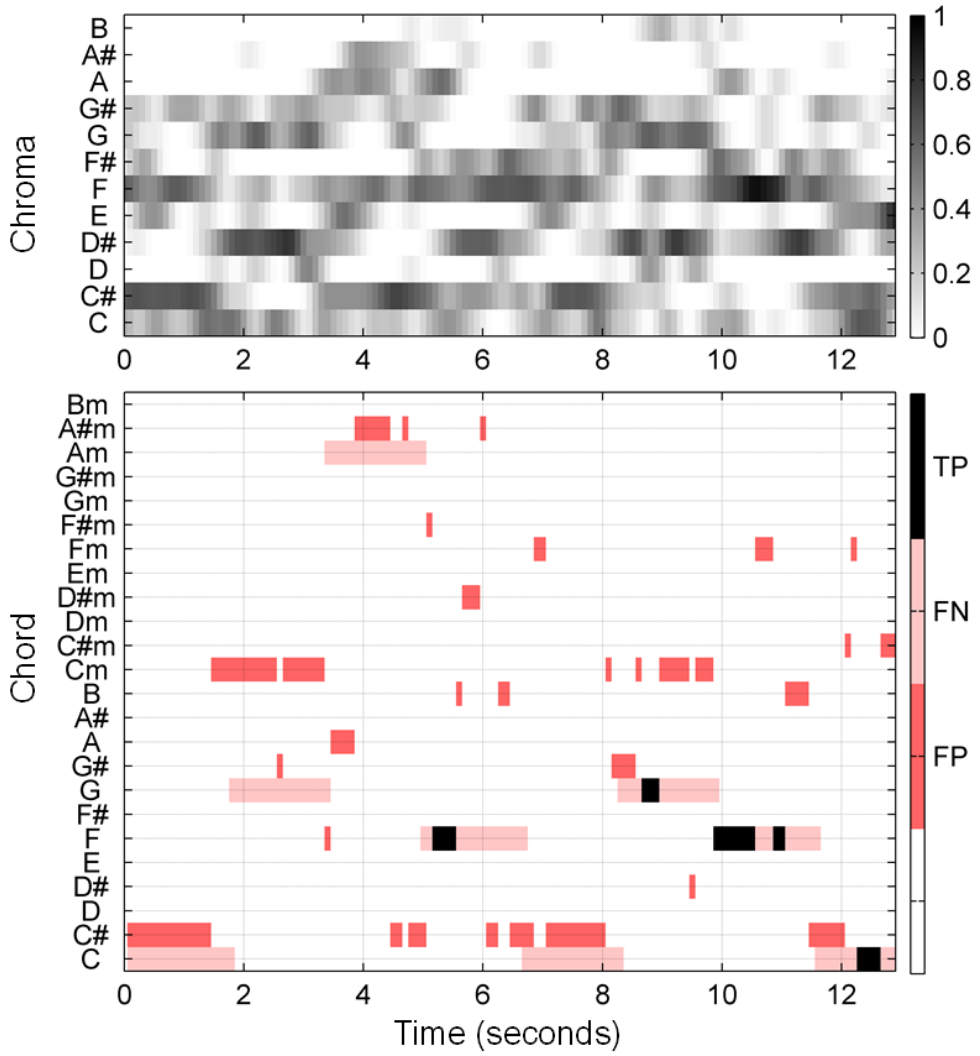
5.2 Template-Based Chord Recognition

Fig. 5.18



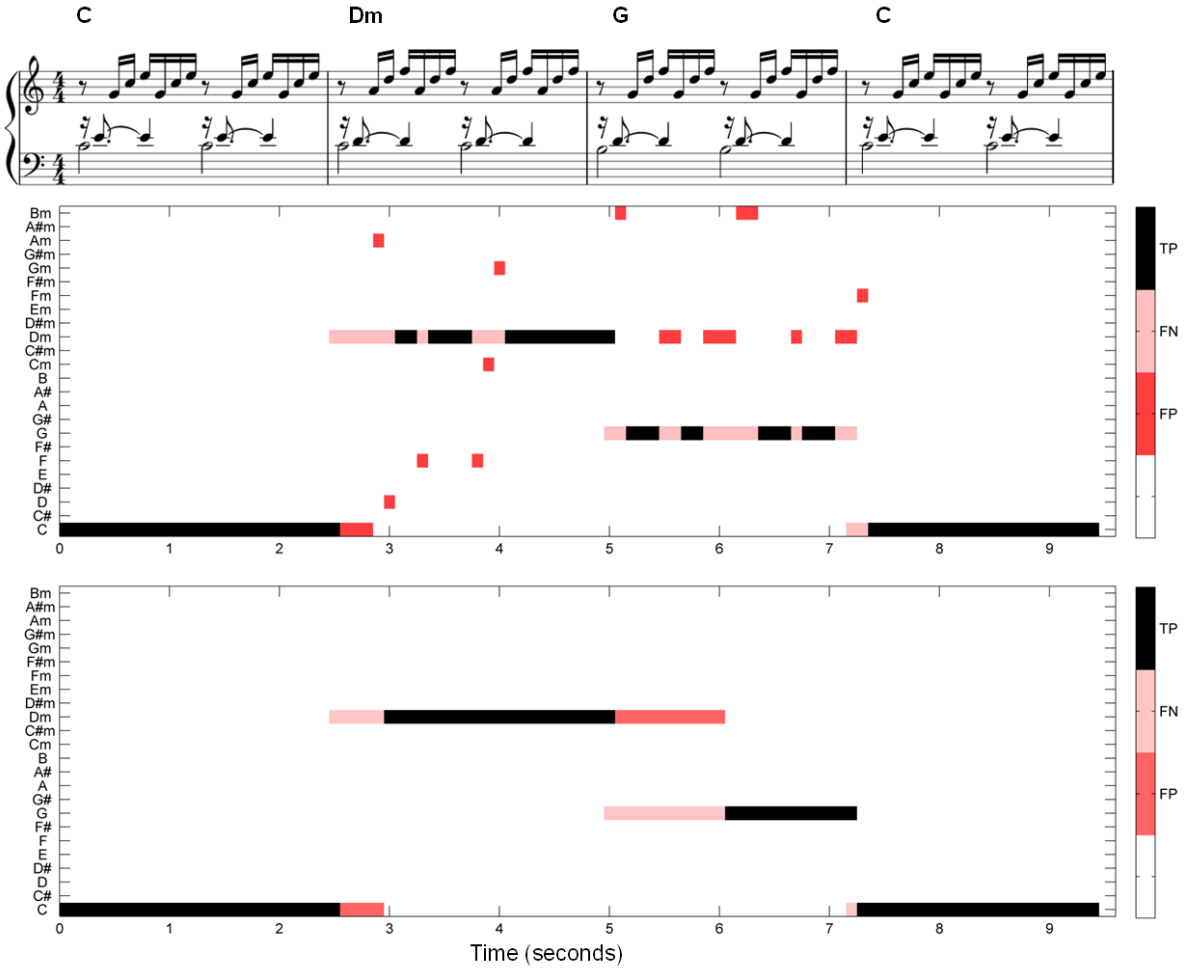
5.2 Template-Based Chord Recognition

Fig. 5.19



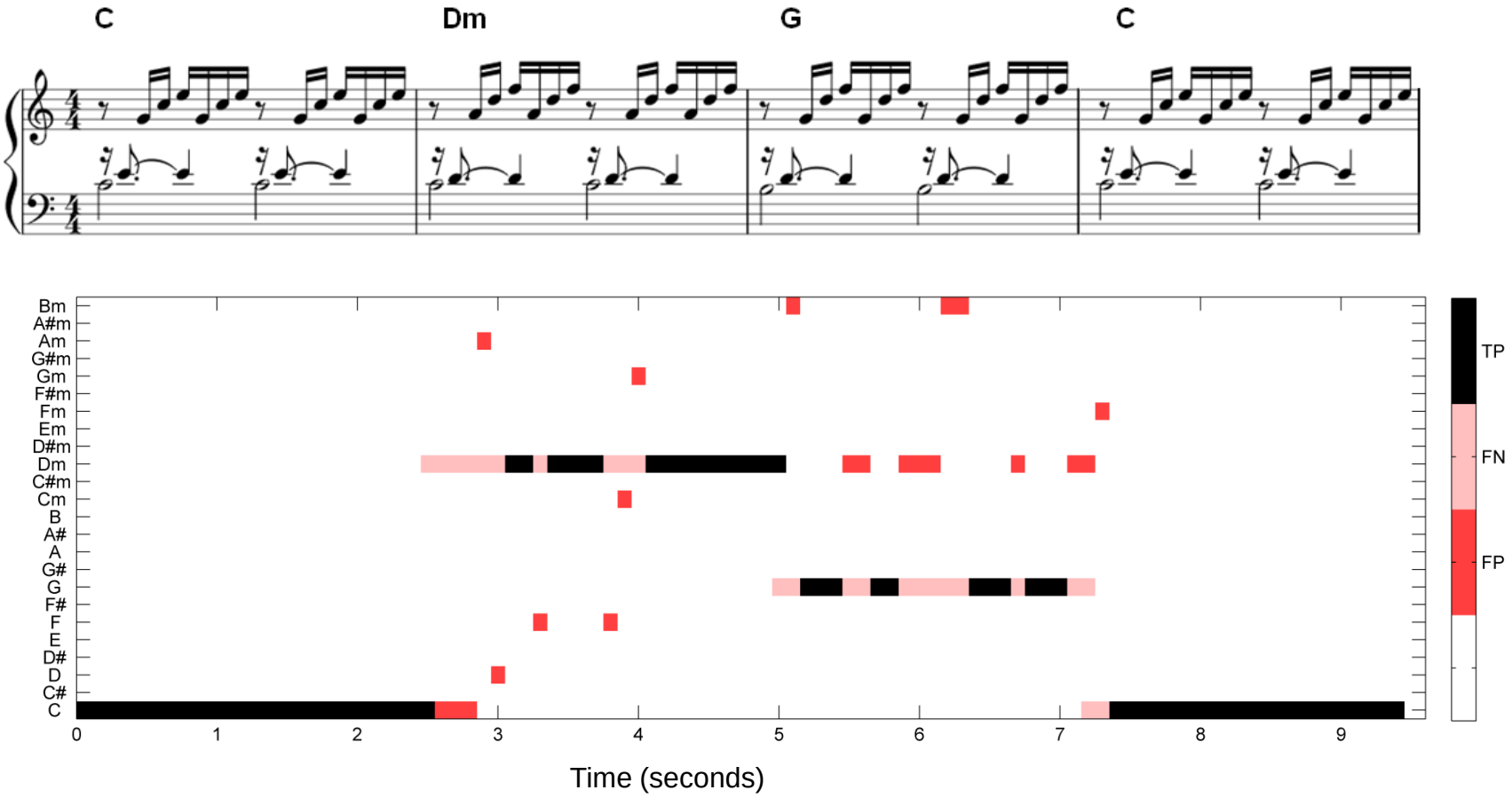
5.2 Template-Based Chord Recognition

Fig. 5.20



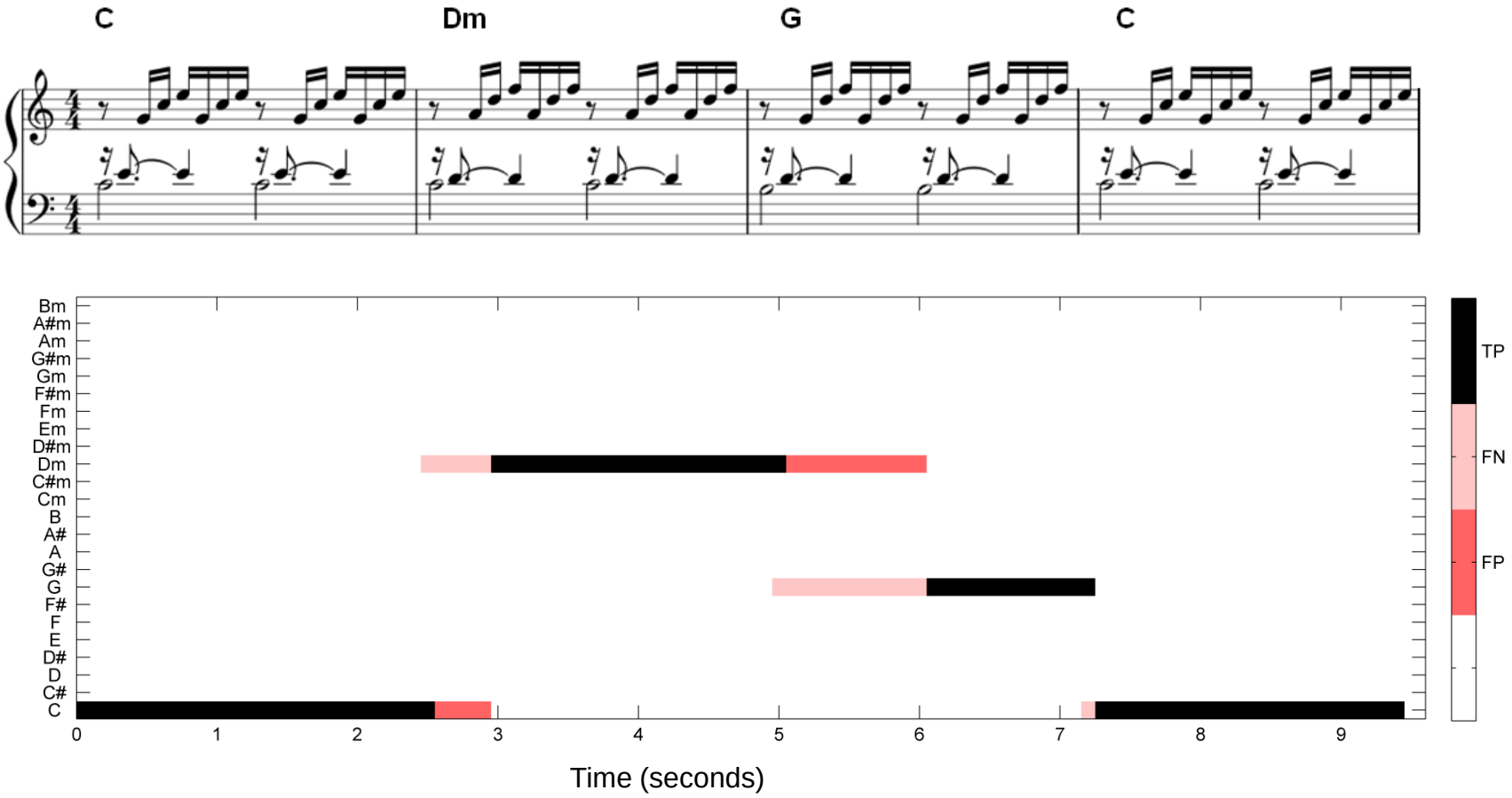
5.2 Template-Based Chord Recognition

Fig. 5.20



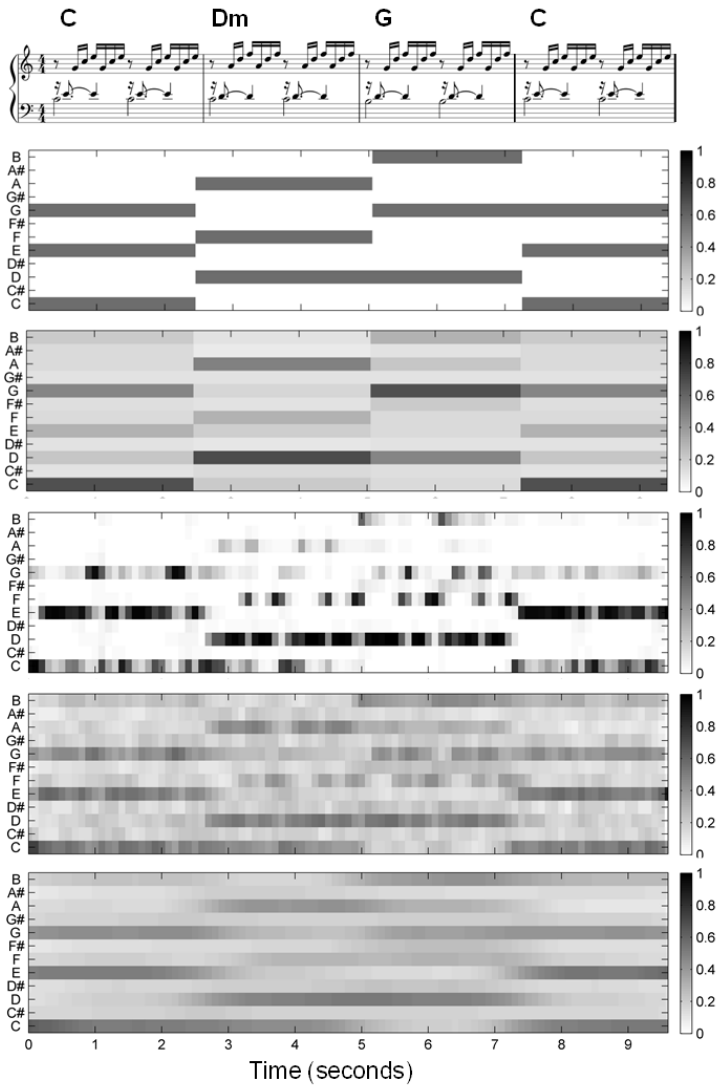
5.2 Template-Based Chord Recognition

Fig. 5.20



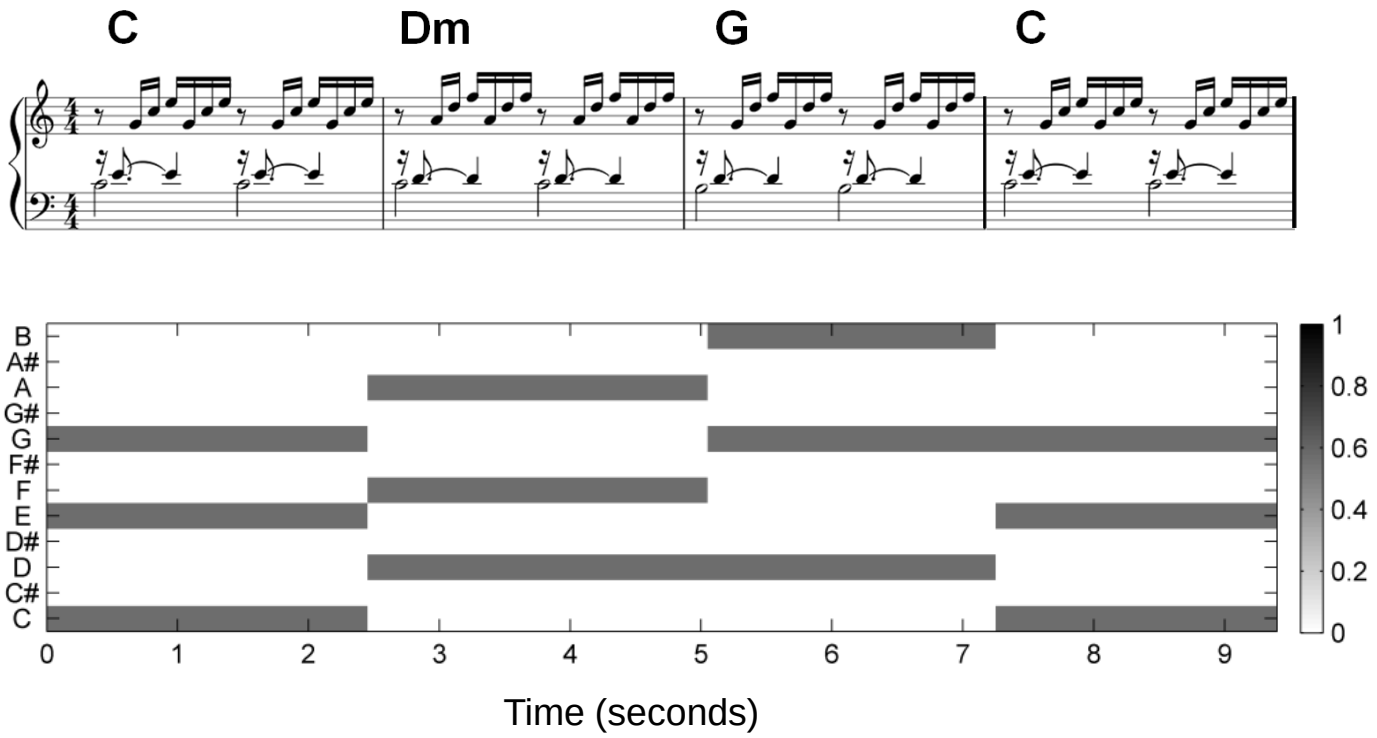
5.2 Template-Based Chord Recognition

Fig. 5.21



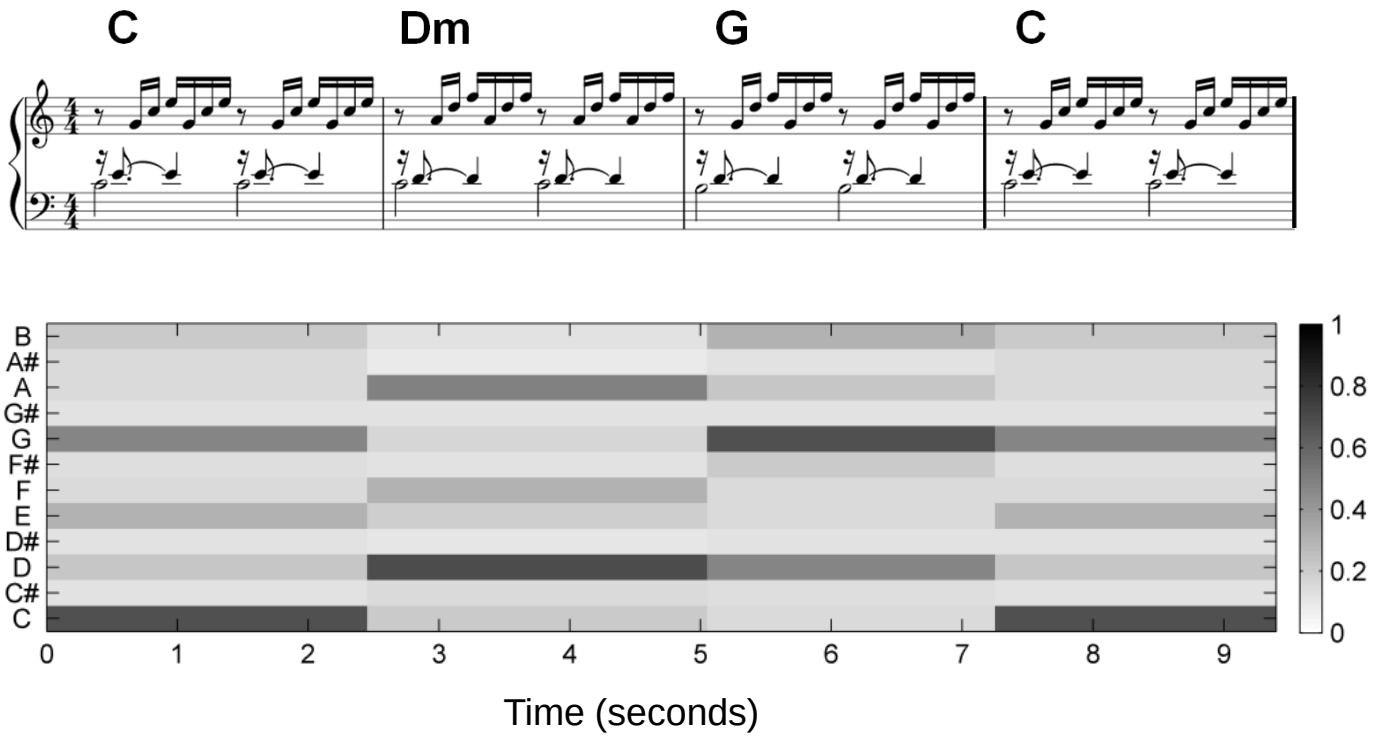
5.2 Template-Based Chord Recognition

Fig. 5.21



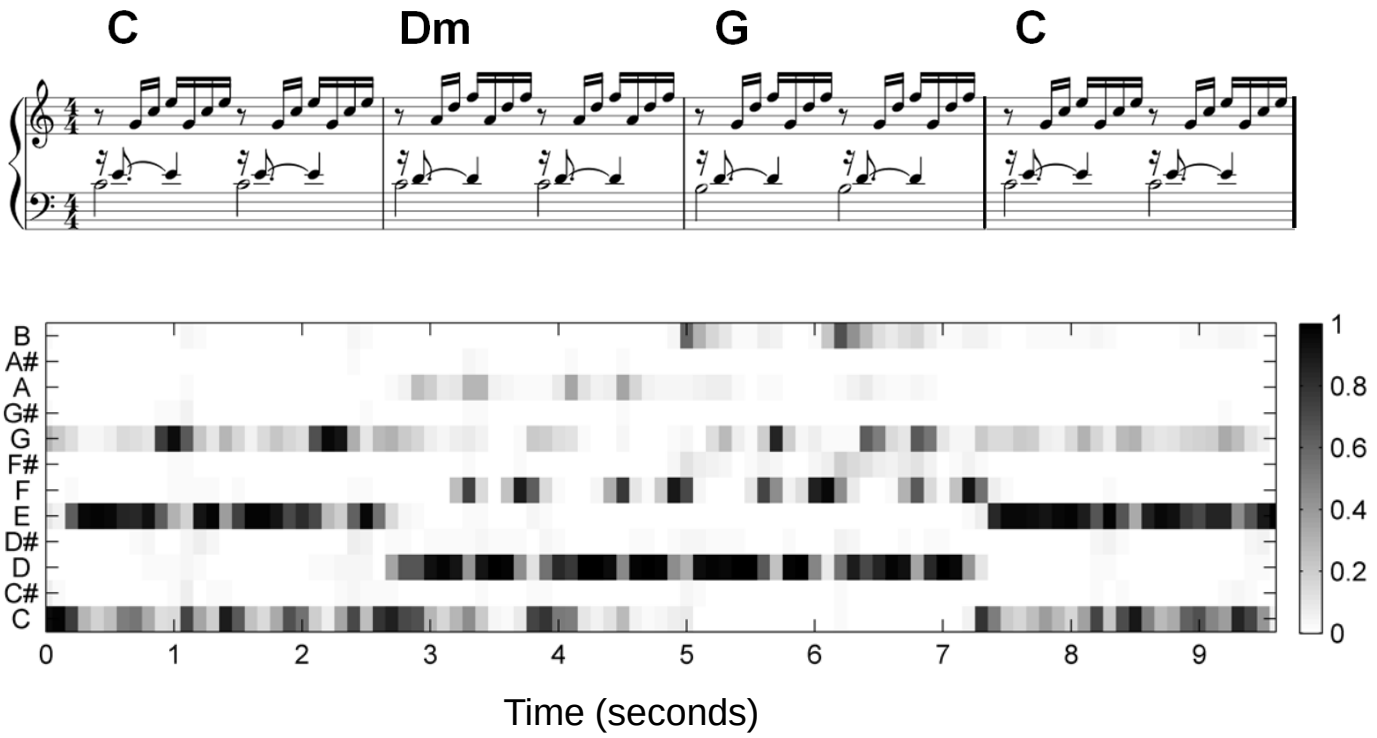
5.2 Template-Based Chord Recognition

Fig. 5.21



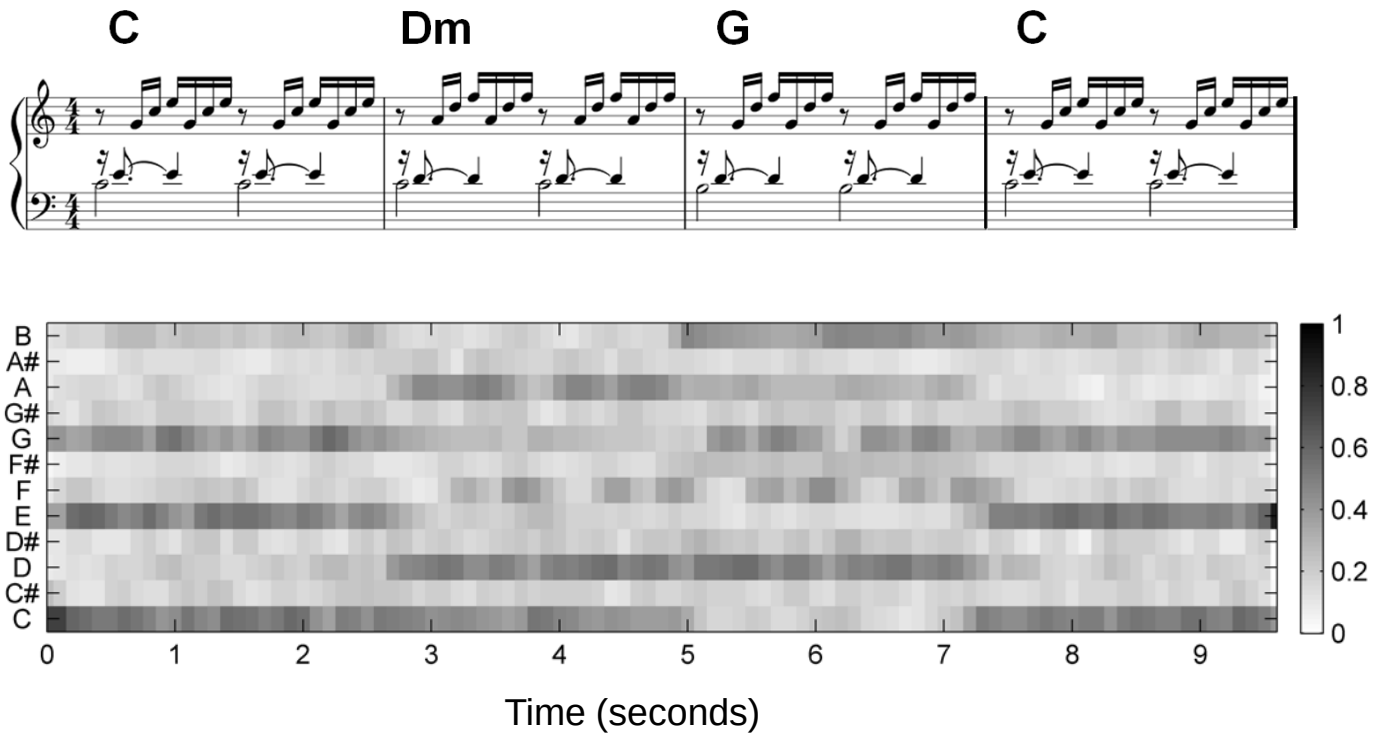
5.2 Template-Based Chord Recognition

Fig. 5.21



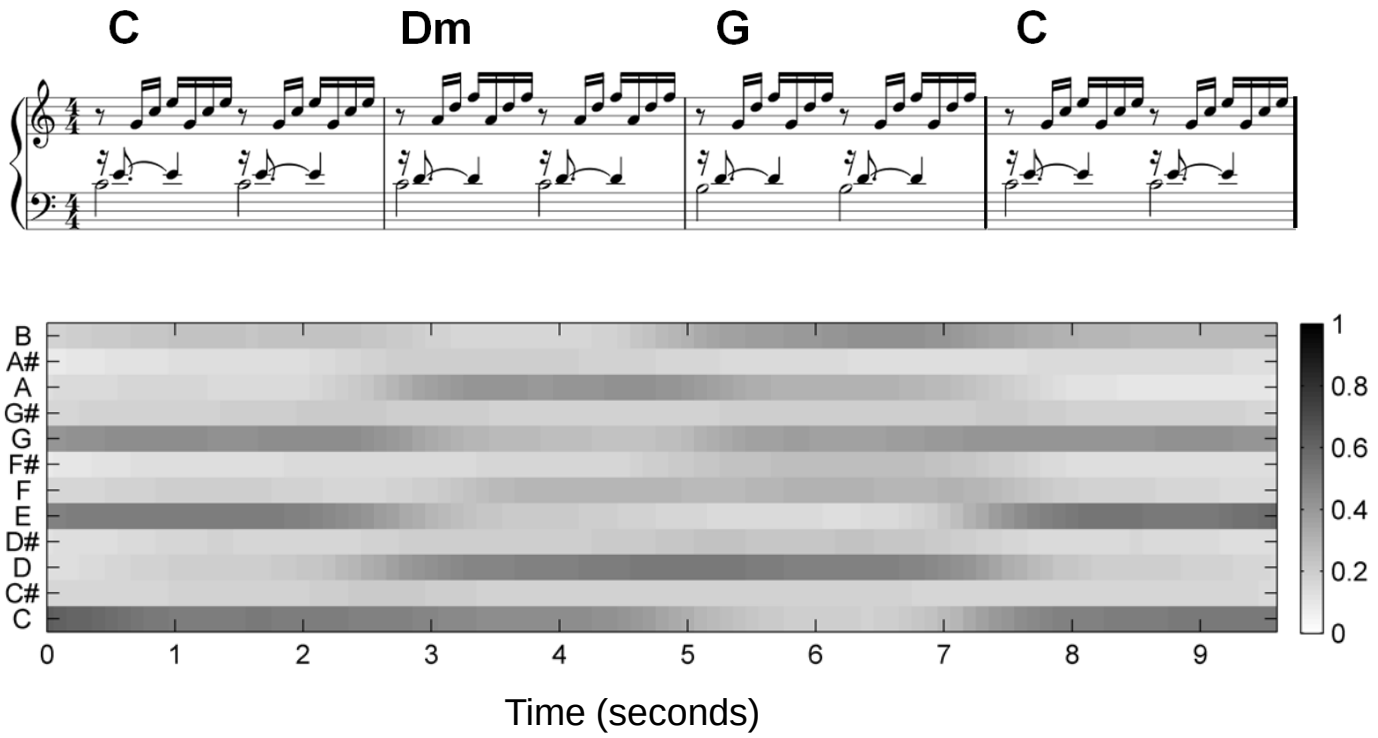
5.2 Template-Based Chord Recognition

Fig. 5.21



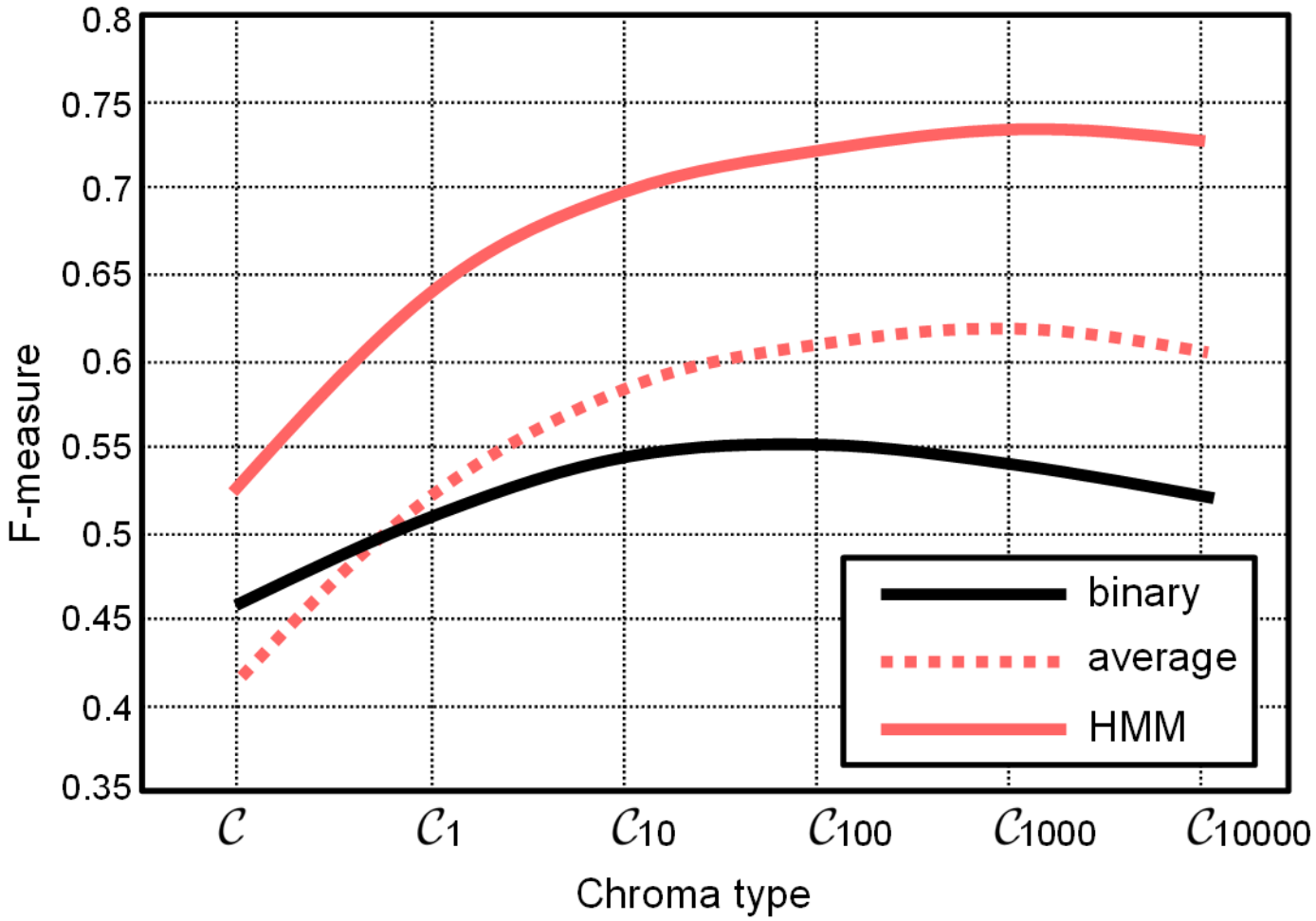
5.2 Template-Based Chord Recognition

Fig. 5.21



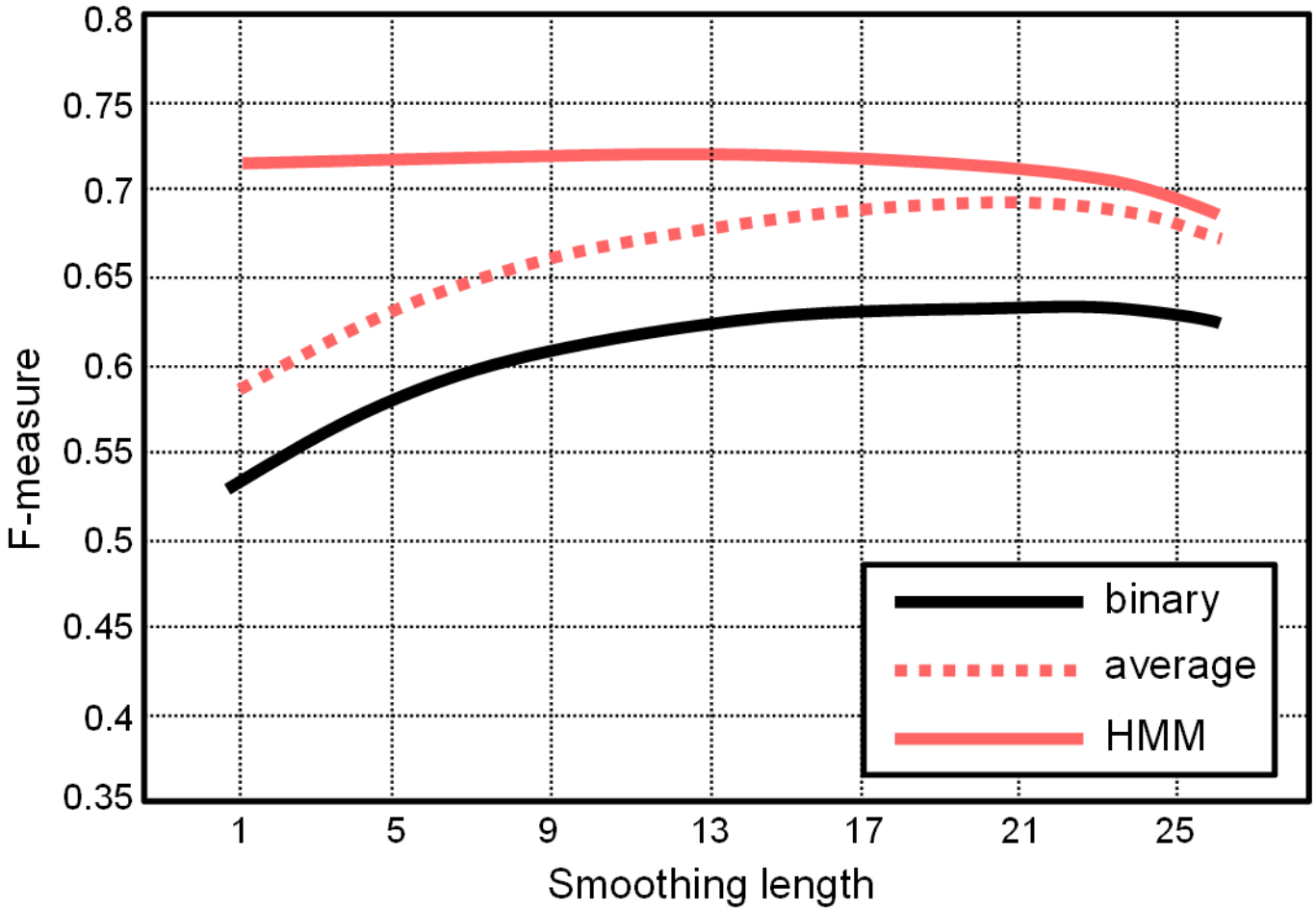
5.2 Template-Based Chord Recognition

Fig. 5.22



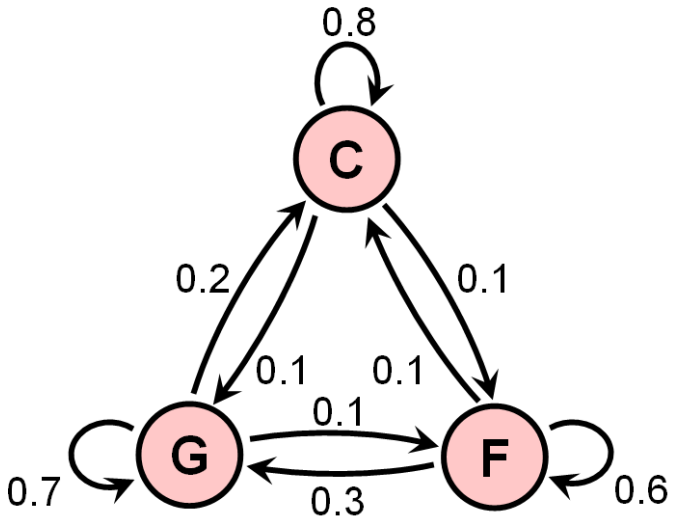
5.2 Template-Based Chord Recognition

Fig. 5.23



5.3 HMM-Based Chord Recognition

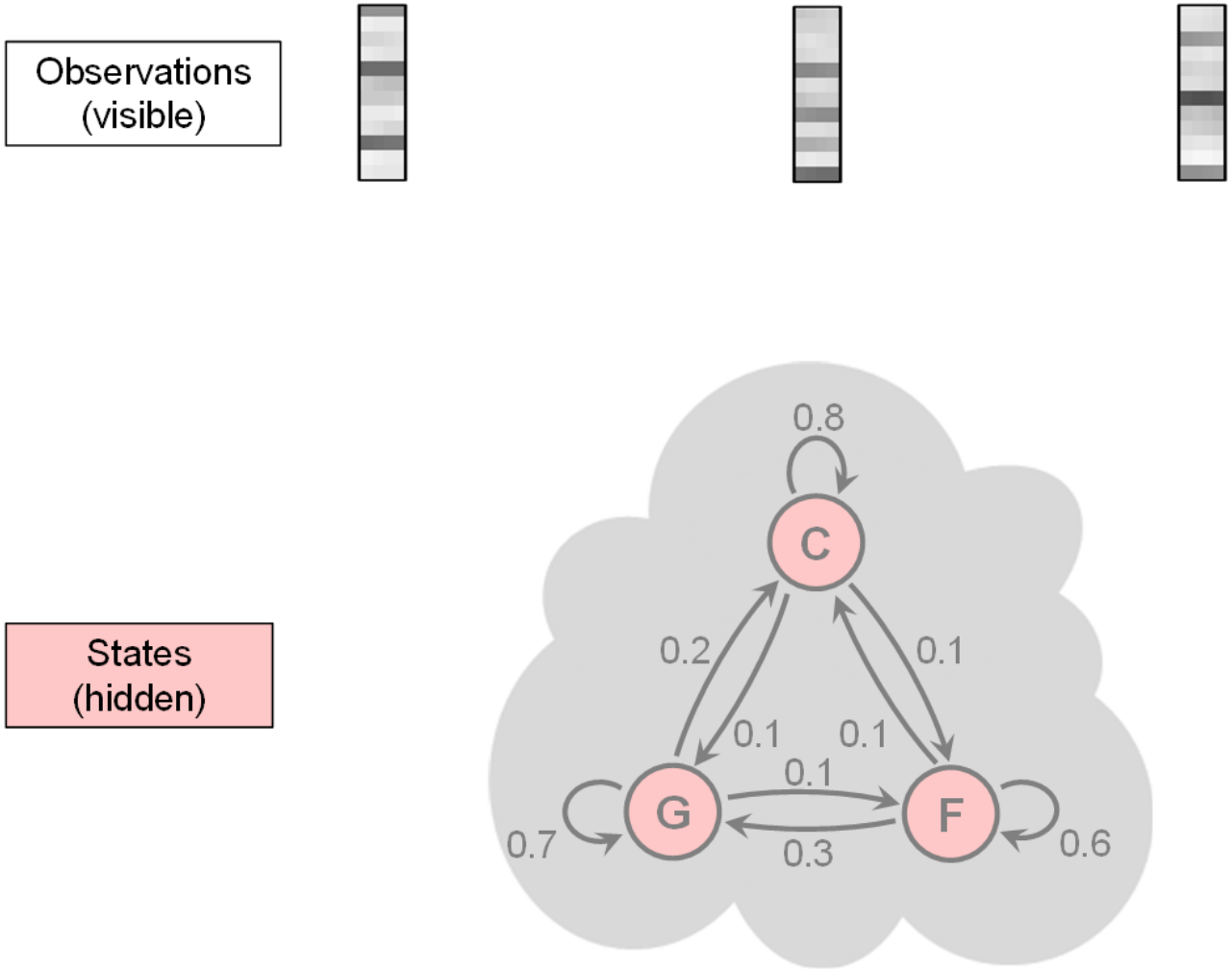
Fig. 5.24



	α_1 = C	α_2 = G	α_3 = F
α_1 = C	$a_{11} = 0.8$	$a_{12} = 0.1$	$a_{13} = 0.1$
α_2 = G	$a_{21} = 0.2$	$a_{22} = 0.7$	$a_{23} = 0.1$
α_3 = F	$a_{31} = 0.1$	$a_{32} = 0.3$	$a_{33} = 0.6$

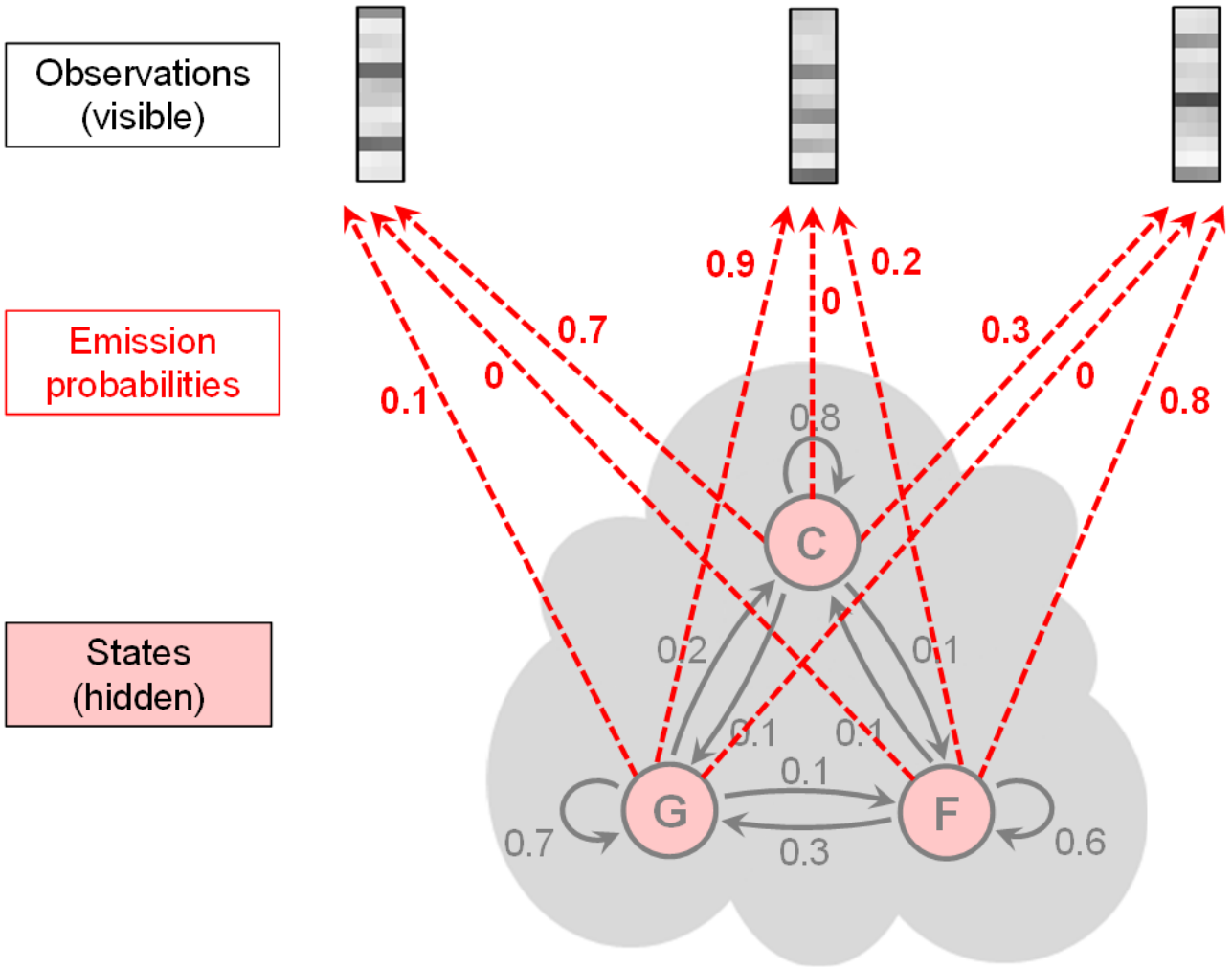
5.3 HMM-Based Chord Recognition

Fig. 5.25



5.3 HMM-Based Chord Recognition

Fig. 5.25



5.3 HMM-Based Chord Recognition

Fig. 5.26

Component	Meaning	Reference
$\mathcal{A}$	Set of states α_i for $i \in [1:I]$	(5.18)
A	State transition probabilities a_{ij} for $i, j \in [1:I]$	(5.20)
C	Initial state probabilities c_i for $i \in [1:I]$	(5.22)
$\mathcal{B}$	Set of observation symbols β_k for $k \in [1:K]$	(5.25)
B	Emission probabilities b_{ik} for $i \in [1:I]$ and $k \in [1:K]$	(5.26)

5.3 HMM-Based Chord Recognition

Table 5.1

Algorithm: HMM-based generation of observations

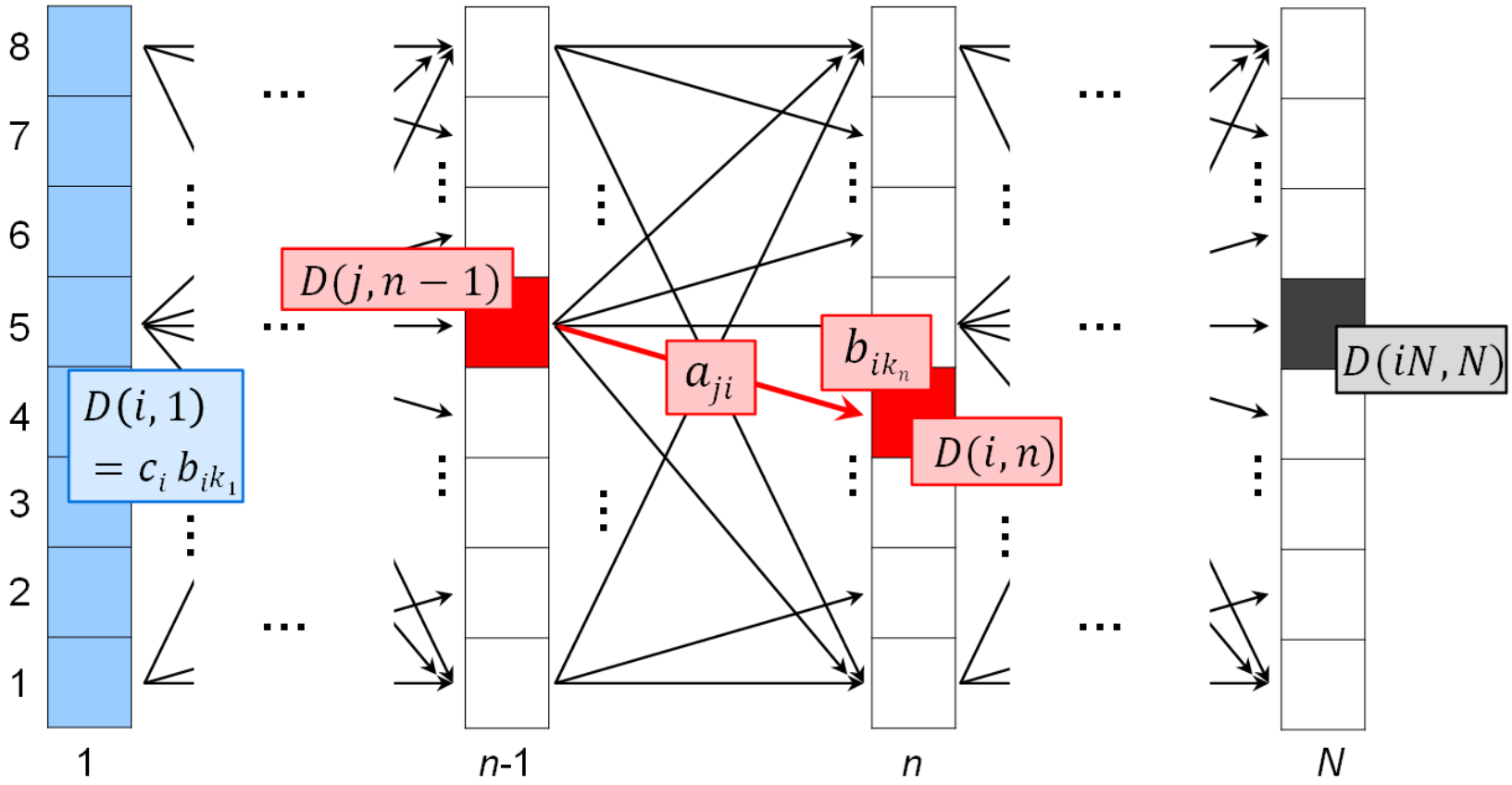
Input: HMM specified by $(\mathcal{A}, A, C, \mathcal{B}, B)$
Output: Observation sequence $O = (o_1, o_2, \ldots, o_N)$

Procedure:

(1) Choose an initial state $s_1 = \alpha_i$ for some $i \in [1:I]$ according to the initial state distribution C.
(2) Set $n = 1$.
(3) Choose $o_n = \beta_k$ for some $k \in [1:K]$ according to the emission probabilities in state $s_1 = \alpha_i$ specified by the i^{th} row of B.
(4) Transit to the new state $s_{n+1} = \alpha_j$ according to the state transition probability at state α_i specified by the i^{th} row of A.
(5) Set $n = n + 1$ and return to step (3) if $n < N$. Otherwise terminate the procedure.

5.3 HMM-Based Chord Recognition

Fig. 5.27



5.3 HMM-Based Chord Recognition

Table 5.2

Algorithm: VITERBI

Input: HMM specified by $\Theta = (\mathcal{A}, A, C, \mathcal{B}, B)$
Observation sequence $O = (o_1 = \beta_{k_1}, o_2 = \beta_{k_2}, \ldots, o_N = \beta_{k_N})$

Output: Optimal state sequence $S^* = (s_1^*, s_2^*, \ldots, s_N^*)$

Procedure: Initialize the $(I \times N)$ matrix $\mathbf{D}$ by $\mathbf{D}(i,1) = c_i b_{ik_1}$ for $i \in [1:I]$. Then compute in a nested loop for $n = 2, \ldots, N$ and $i = 1, \ldots, I$:

$$\begin{aligned} \mathbf{D}(i,n) &= \max_{j \in [1:I]} \big(a_{ji} \cdot \mathbf{D}(j,n-1)\big) \cdot b_{ik_n} \\ \mathbf{E}(i,n-1) &= \operatorname{argmax}_{j \in [1:I]} \big(a_{ji} \cdot \mathbf{D}(j,n-1)\big) \end{aligned}$$

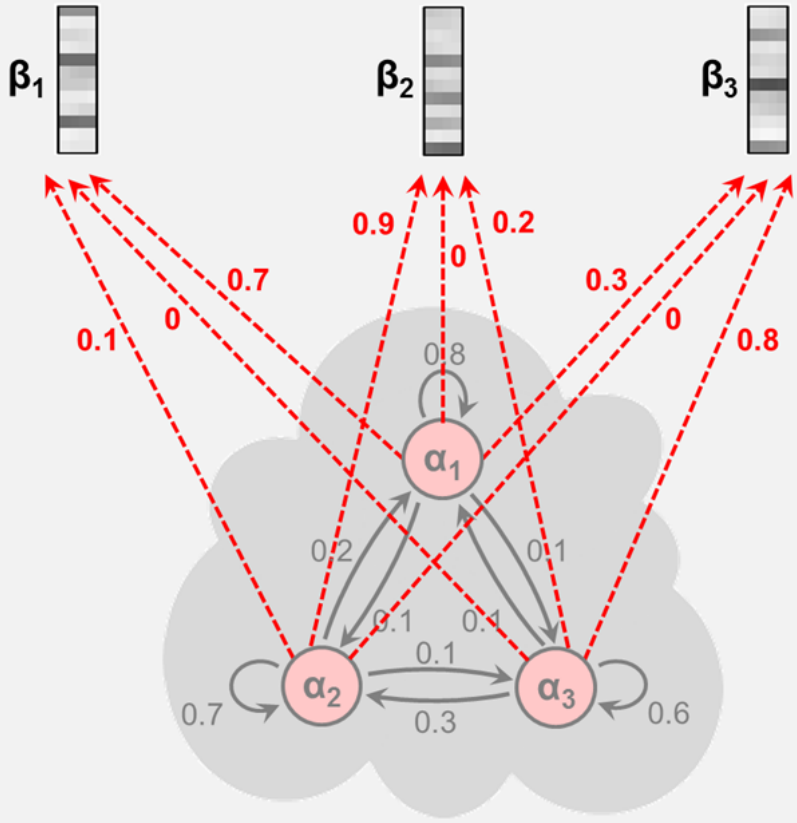
Set $i_N = \operatorname{argmax}_{j \in [1:I]} \mathbf{D}(j,N)$ and compute for decreasing $n = N-1, \ldots, 1$ the maximizing indices

$$i_n = \operatorname{argmax}_{j \in [1:I]} \big(a_{ji_{n+1}} \cdot \mathbf{D}(j,n)\big) = \mathbf{E}(i_{n+1}, n).$$

The optimal state sequence $S^* = (s_1^*, \ldots, s_N^*)$ is defined by $s_n^* = \alpha_{i_n}$ for $n \in [1:N]$.

5.3 HMM-Based Chord Recognition

Fig. 5.28



State transition probabilities

A	α_1	α_2	α_3
α_1	0.8	0.1	0.1
α_2	0.2	0.7	0.1
α_3	0.1	0.3	0.6

Initial state probabilities

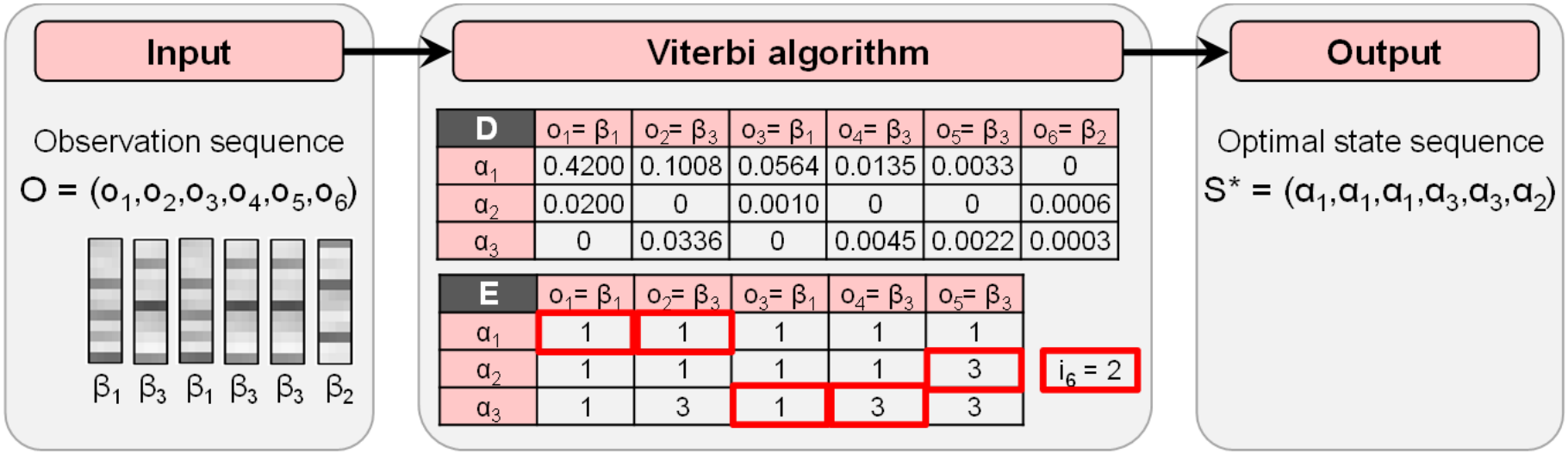
C	α_1	α_2	α_3
	0.6	0.2	0.2

Emission probabilities

B	β_1	β_2	β_3
α_1	0.7	0	0.3
α_2	0.1	0.9	0
α_3	0	0.2	0.8

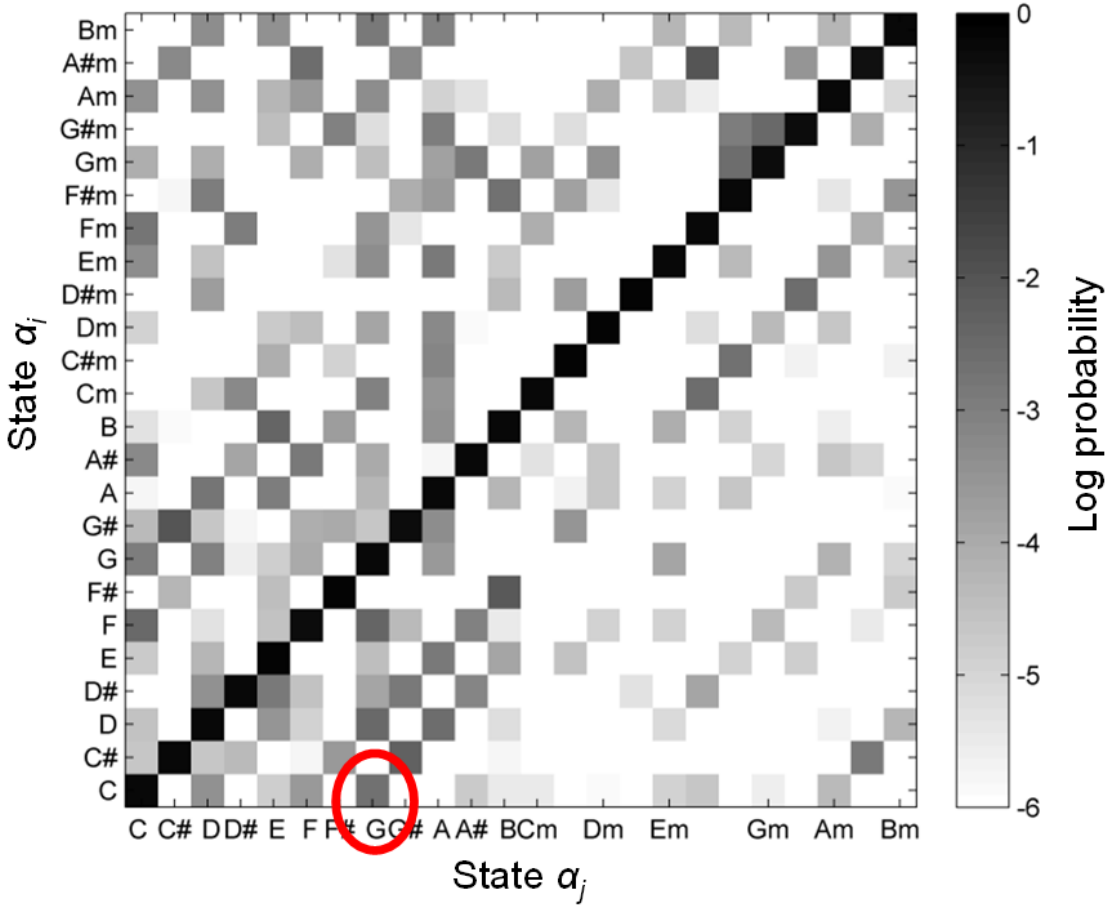
5.3 HMM-Based Chord Recognition

Fig. 5.28



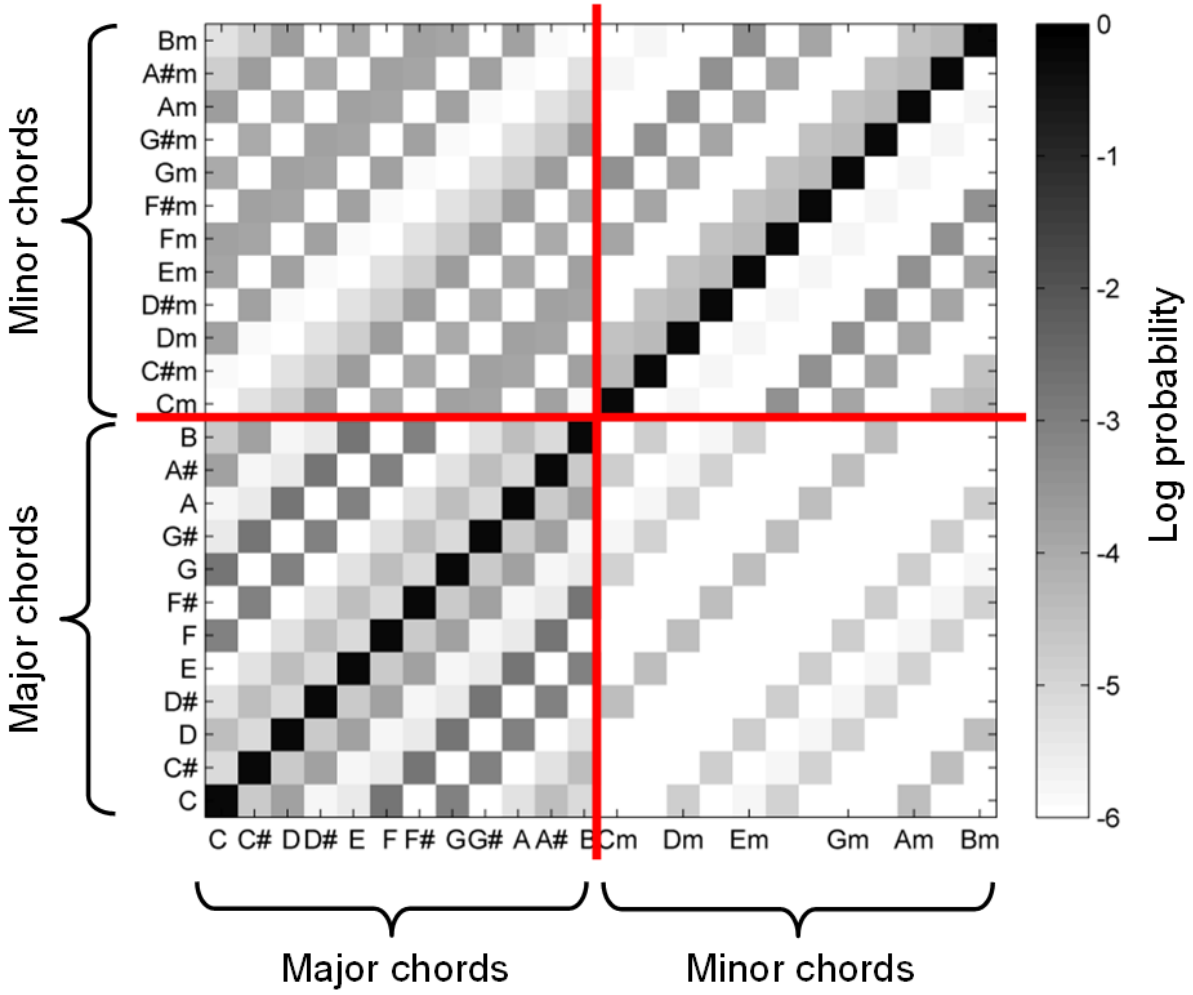
5.3 HMM-Based Chord Recognition

Fig. 5.29



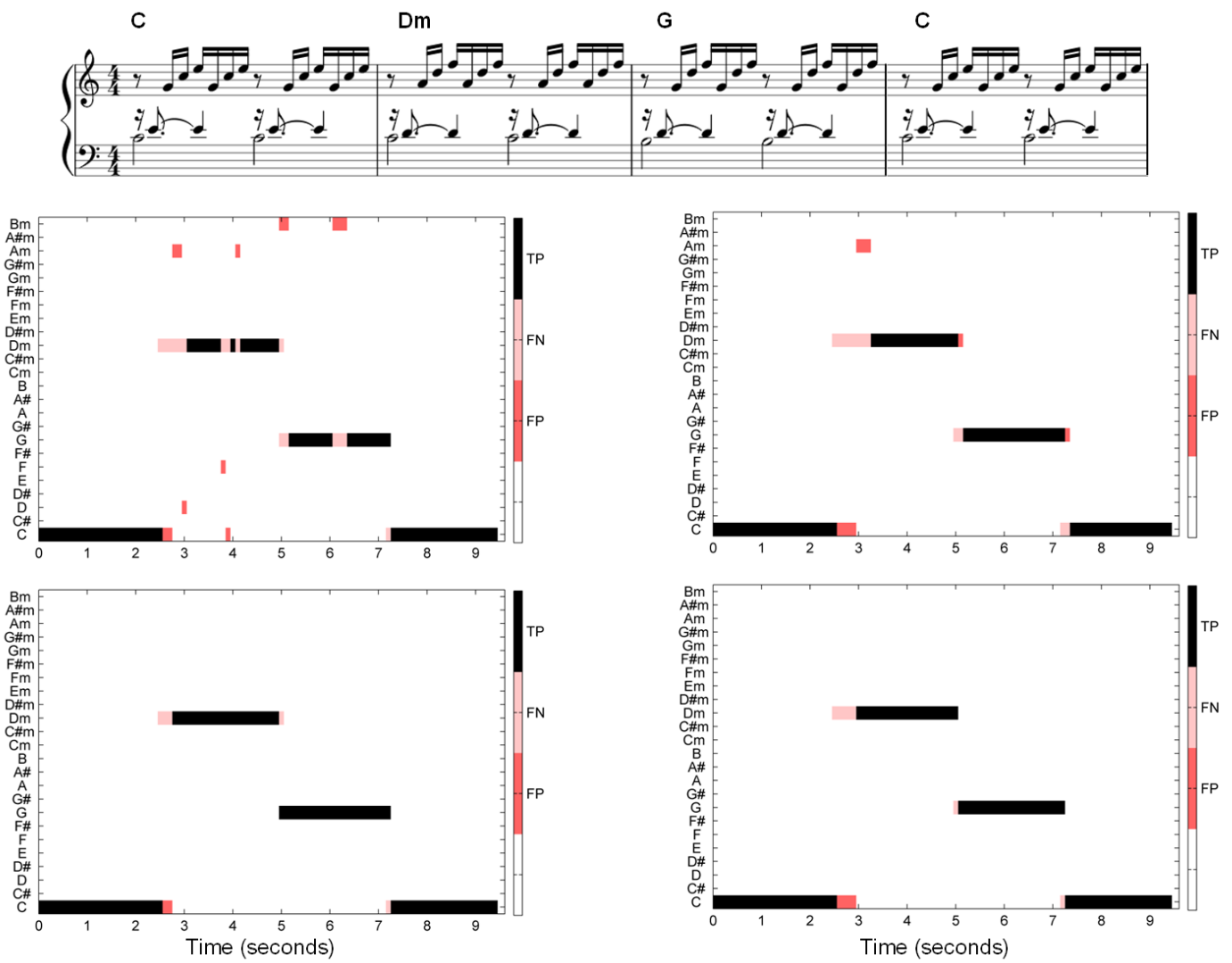
5.3 HMM-Based Chord Recognition

Fig. 5.30



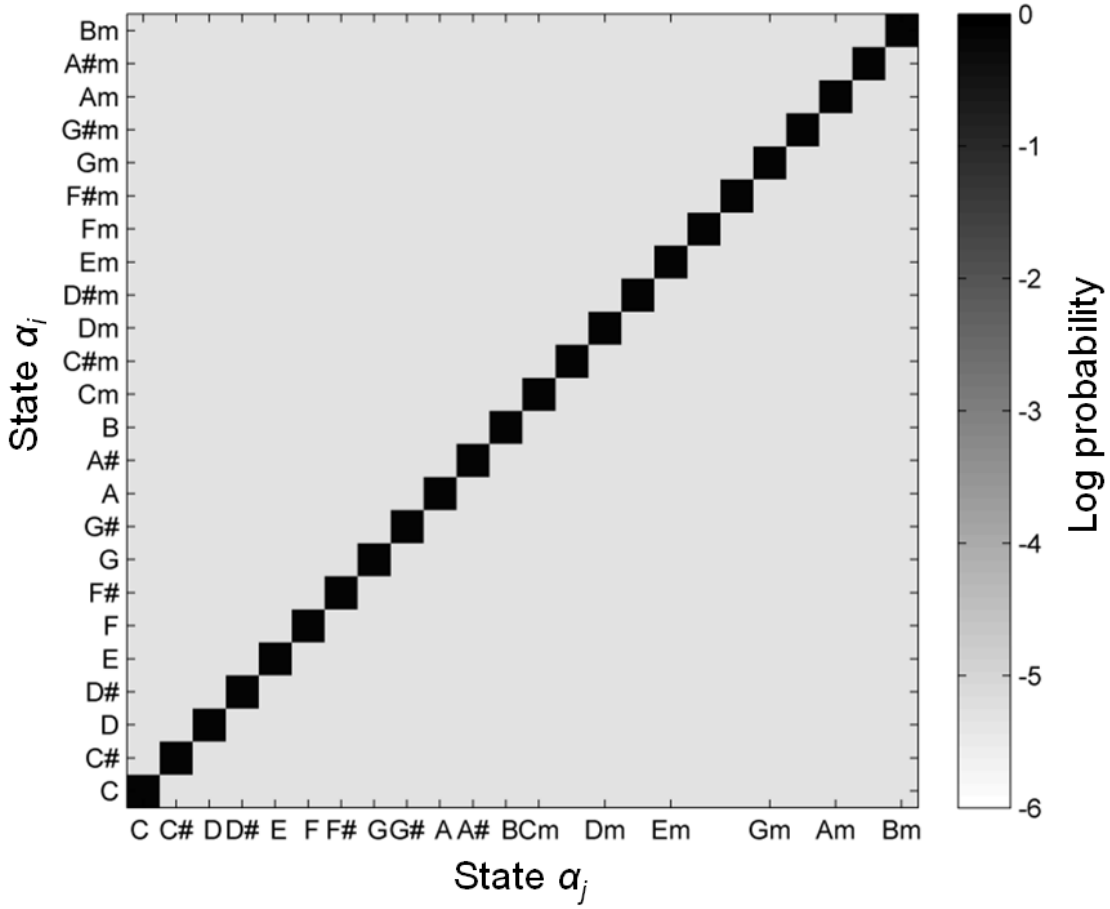
5.3 HMM-Based Chord Recognition

Fig. 5.31



5.3 HMM-Based Chord Recognition

Fig. 5.32



5.4 Further Notes

Fig. 5.33