

Guest Lecture  
Sound and Music Computing (CS4347)  
National University of Singapore

# Nonnegative Autoencoders with Applications to Music Audio Decomposing

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# Meinard Müller



- Mathematics (Diplom/Master)  
Computer Science (PhD)  
Information Retrieval (Habilitation)
- Since 2012: Professor  
Semantic Audio Processing
- Former President of the International Society for  
Music Information Retrieval (MIR)
- IEEE Fellow for contributions to  
Music Signal Processing



# Meinard Müller: Research Group

## Semantic Audio Processing



- Sebastian Rosenzweig
- Michael Krause
- Yigitcan Özer
- Peter Meier (external)
- Christof Weiß



- Frank Zalkow
- Christian Dittmar
- Stefan Balke
- Jonathan Driedger
- Thomas Prätzlich
- ...



# International Audio Laboratories Erlangen



- Fraunhofer Institute for Integrated Circuits IIS
- Largest Fraunhofer institute with  $\approx 1000$  members
- Applied research for sensor, audio, and media technology



**AUDIO  
LABS**



- Friedrich-Alexander Universität Erlangen-Nürnberg (FAU)
- One of Germany's largest universities with  $\approx 40,000$  students
- Strong Technical Faculty

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# International Audio Laboratories Erlangen



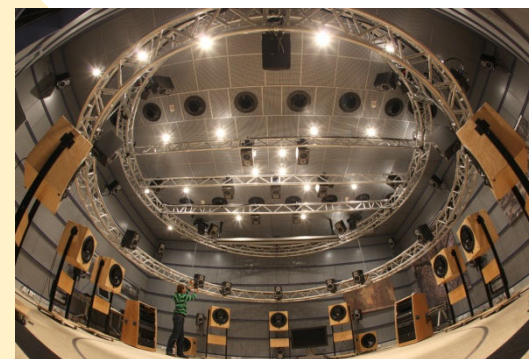
**Audio**

# International Audio Laboratories Erlangen

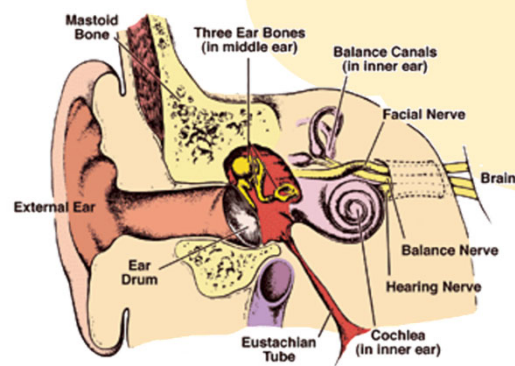
Audio Coding



3D Audio



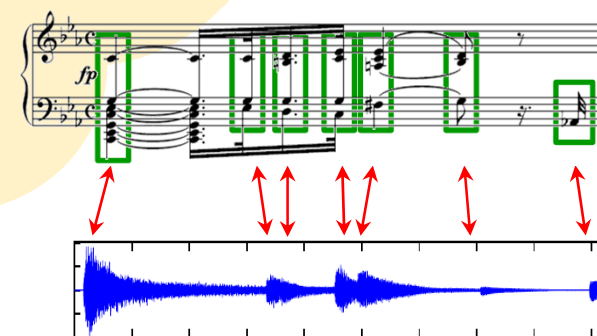
## Audio



Psychoacoustics



Internet of Things



Music Processing



# International Audio Laboratories Erlangen

- Prof. Dr. Jürgen Herre  
Audio Coding
- Prof. Dr. Bernd Edler  
Audio Signal Analysis
- Prof. Dr. Meinard Müller  
Semantic Audio Processing
- Prof. Dr. Emanuël Habets  
Spatial Audio Signal Processing
- Prof. Dr. Nils Peters  
Audio Signal Processing
- Dr. Stefan Turowski  
Coordinator AudioLabs-FAU



# Source Separation

- Decomposition of audio stream into different sound sources
- Central task in digital signal processing
- “Cocktail party effect”



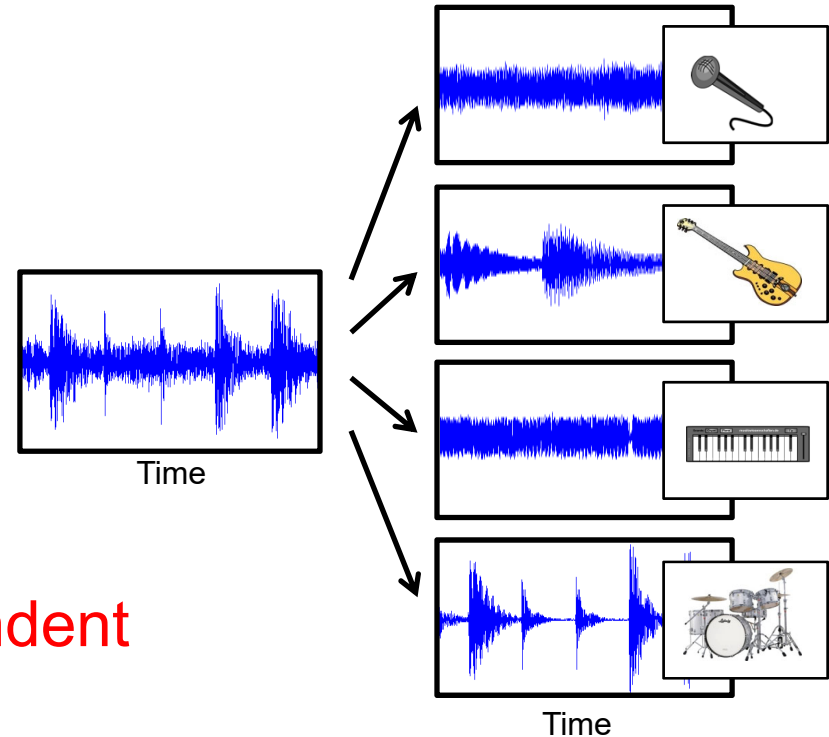


# Source Separation

- Decomposition of audio stream into different sound sources
- Central task in digital signal processing
- “Cocktail party effect”
- Several input signals
- Sources are assumed to be statistically independent

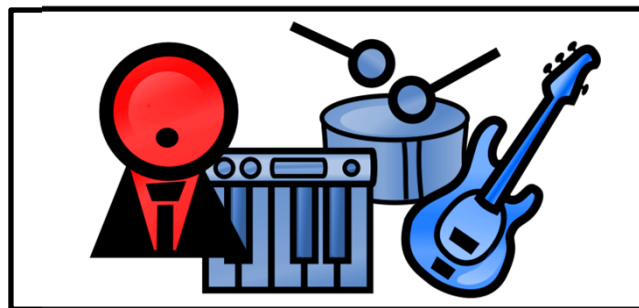
# Source Separation (Music)

- Main melody, accompaniment, drum track
- Instrumental voices
- Individual note events
- Only mono or stereo
- Sources are often highly dependent



# Singing Voice Extraction

Original Recording



Singing voice



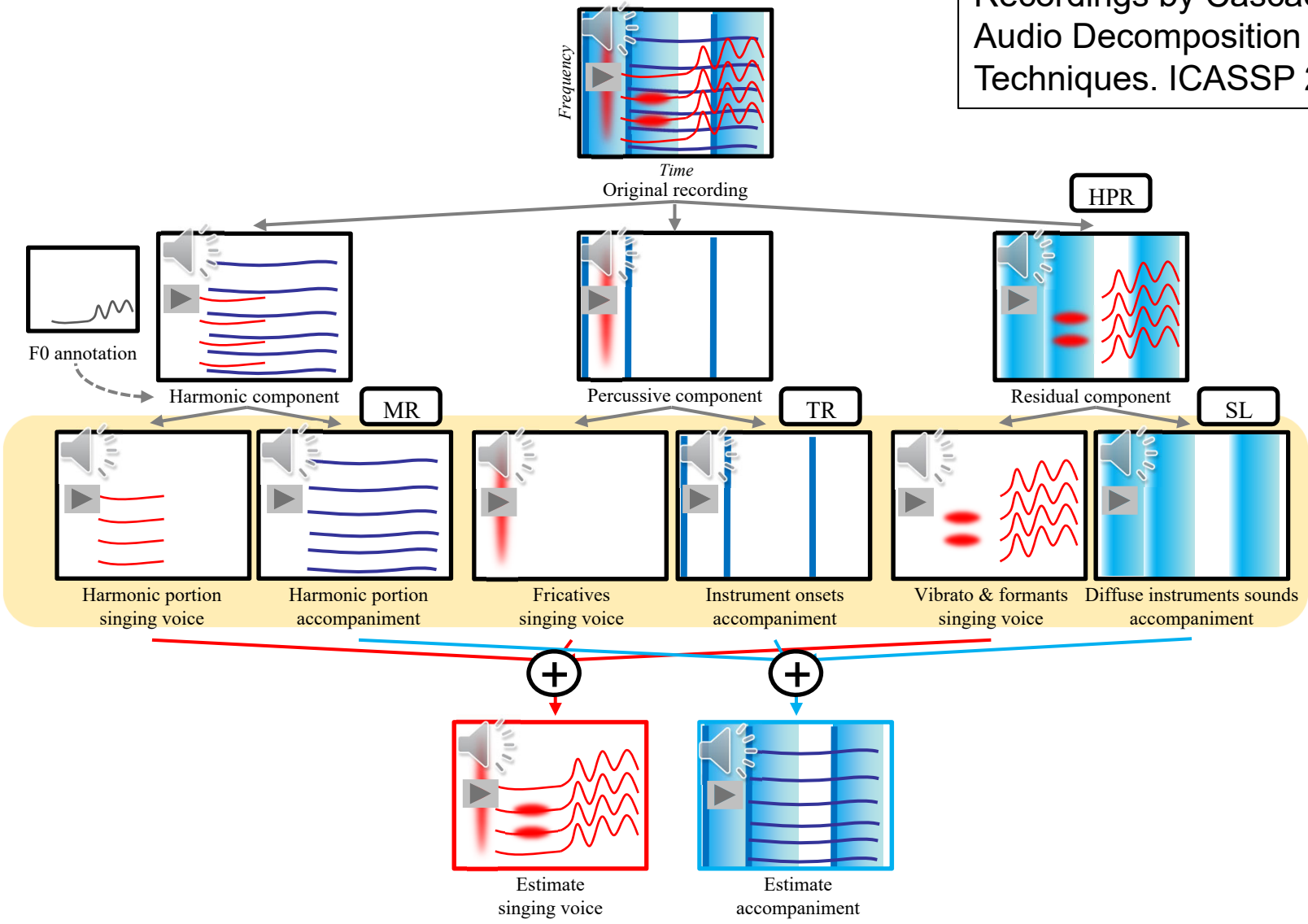
Accompaniment



# Singing Voice Extraction

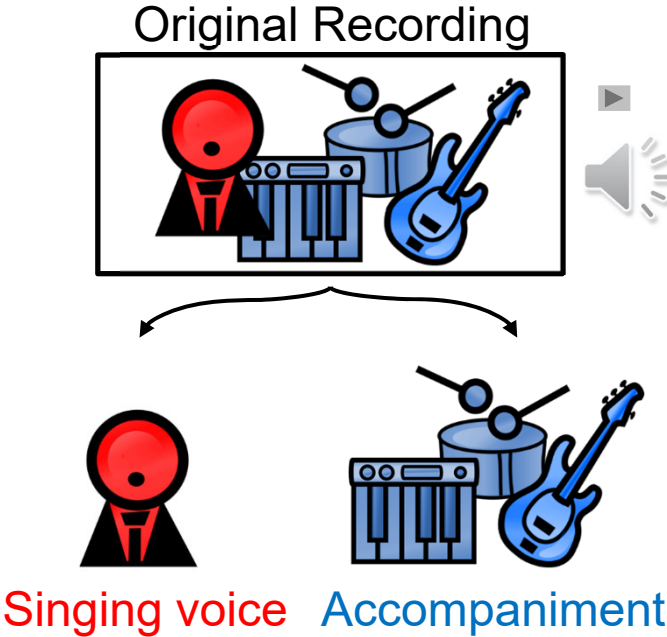
## Traditional Approach

Driedger, Müller: Extracting Singing Voice from Music Recordings by Cascading Audio Decomposition Techniques. ICASSP 2015.



# Singing Voice Extraction

**Deep learning  
has lead to  
breakthrough**



## **DL-Based Approach**

Stöter, Uhlich Luitkus,  
Mitsufuji: Open-Unmix – A  
Reference Implementation  
for Music Source  
Separation. JOSS 2019.

Reference voices:



Engineering approach:



Deep learning approach:



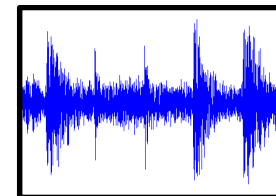
# Score-Informed Source Separation

Exploit musical score to support decomposition process

Musical  
Information



Audio  
Signal

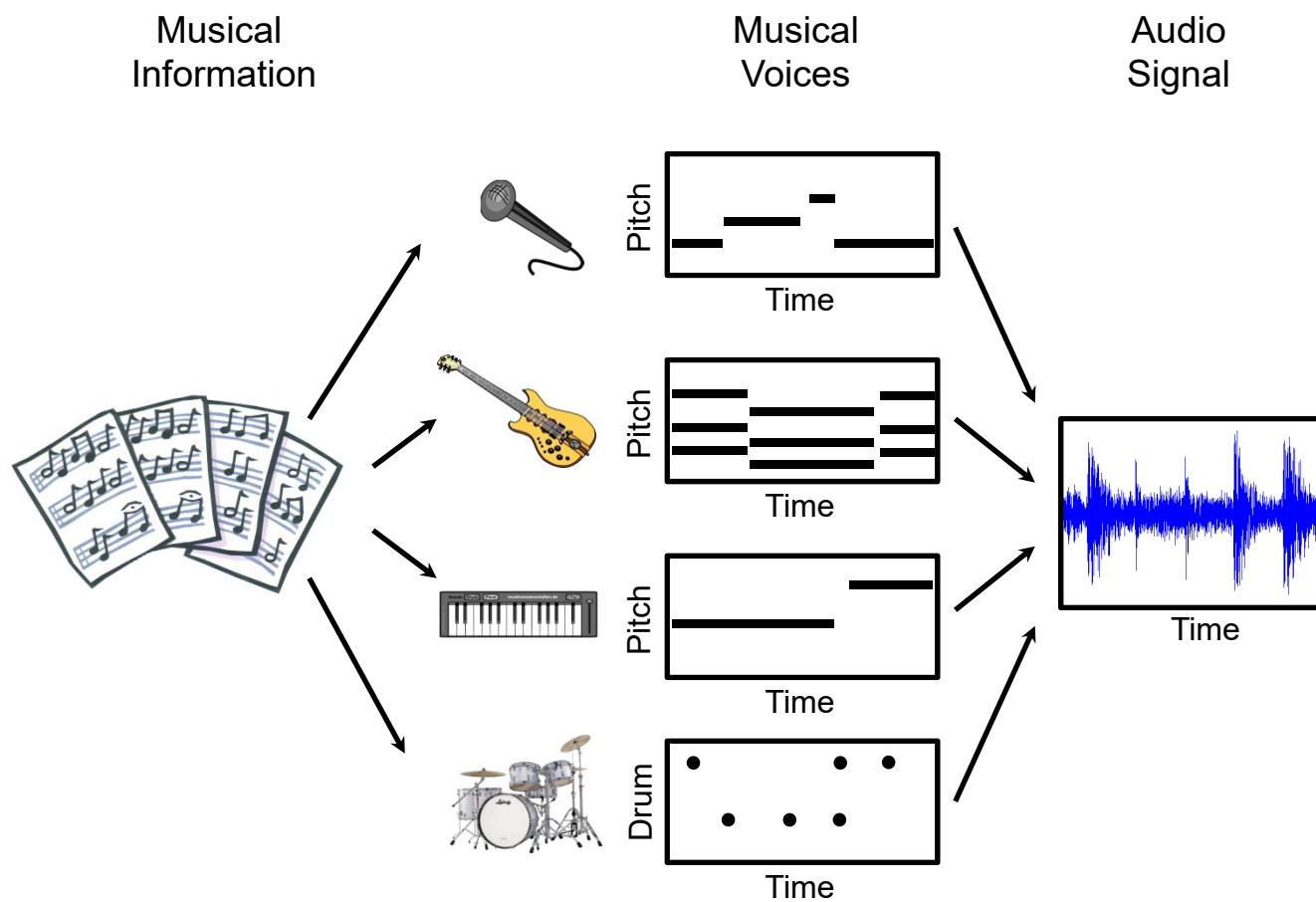


Time



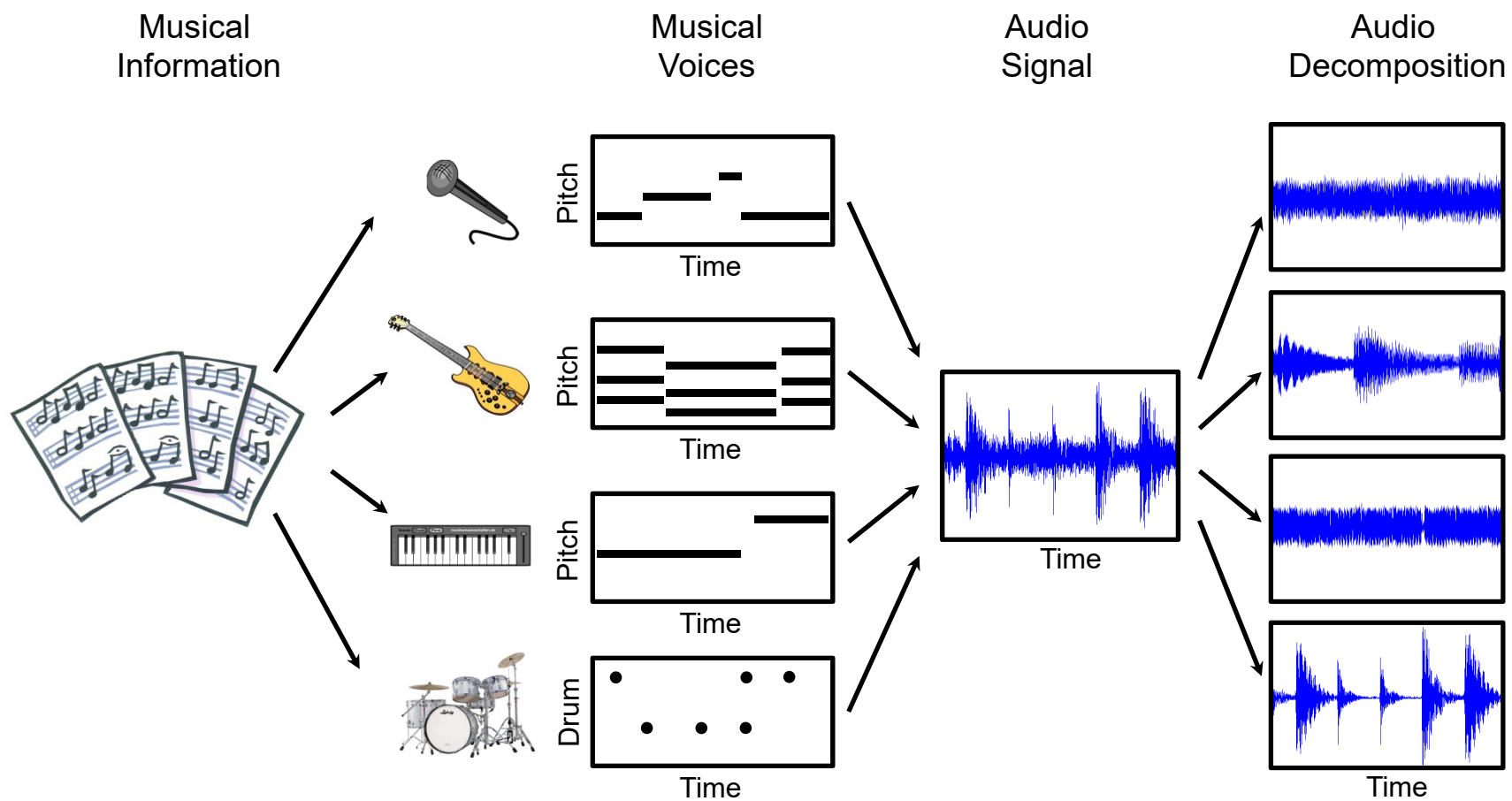
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Exploit musical score to support decomposition process



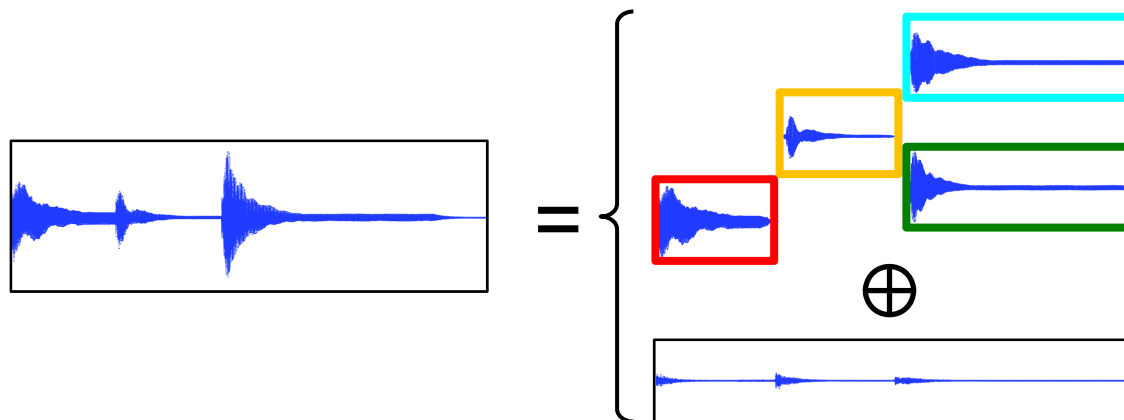
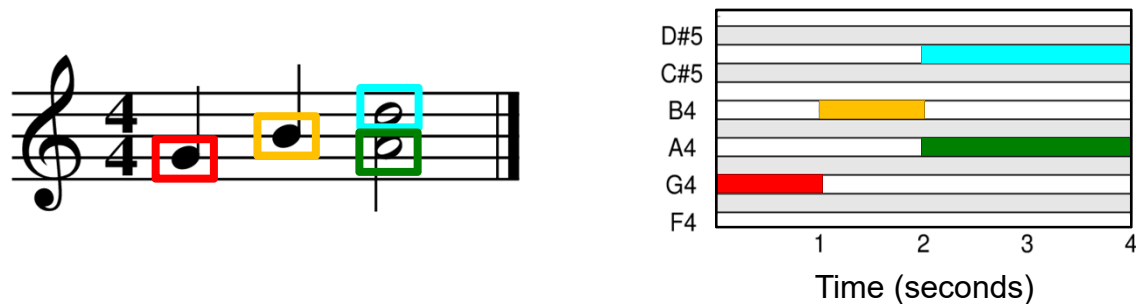
# Score-Informed Source Separation

Exploit musical score to support decomposition process



# Score-Informed Audio Decomposition

## Notewise decomposition

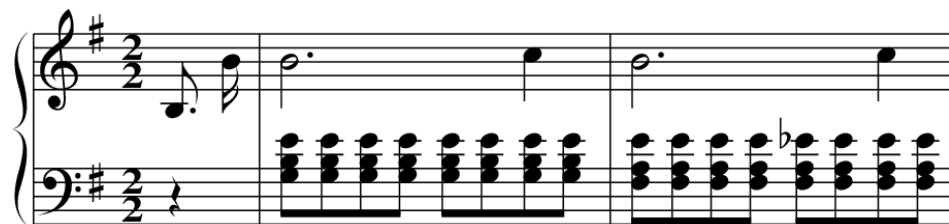


### Prior Knowledge

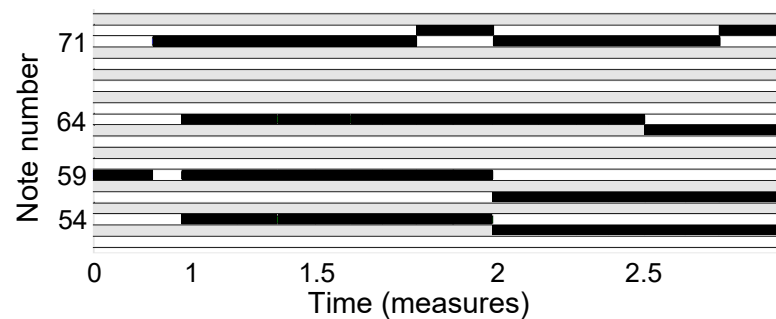
Ewert, Pardo, Müller, Plumbley: Score-Informed Source Separation for Musical Audio Recordings. IEEE SPM, 2014.

# Score-Informed Audio Decomposition

Sheet music

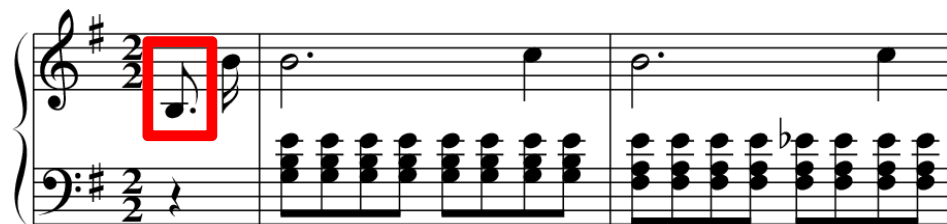


Piano roll



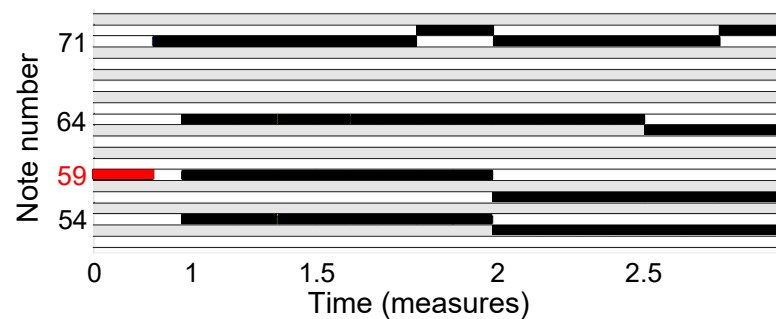
# Score-Informed Audio Decomposition

Sheet music



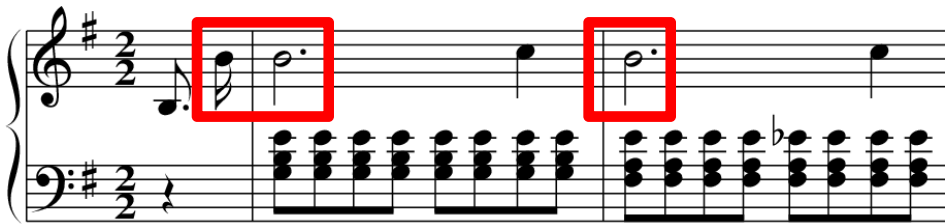
$p = 59$

Piano roll



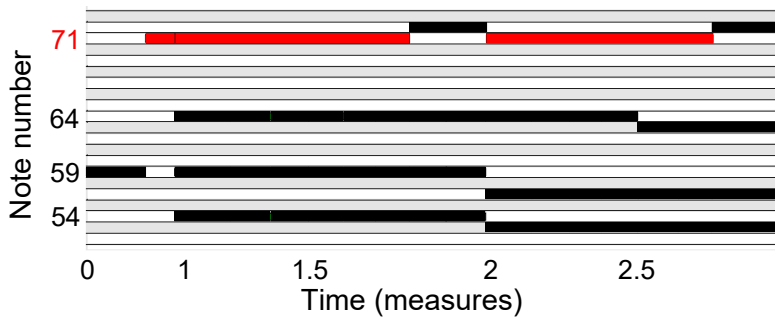
# Score-Informed Audio Decomposition

Sheet music



$p = 71$

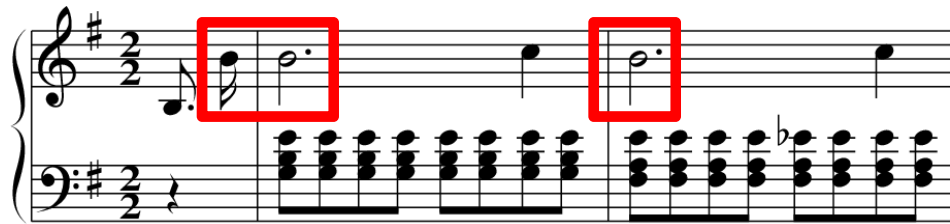
Piano roll





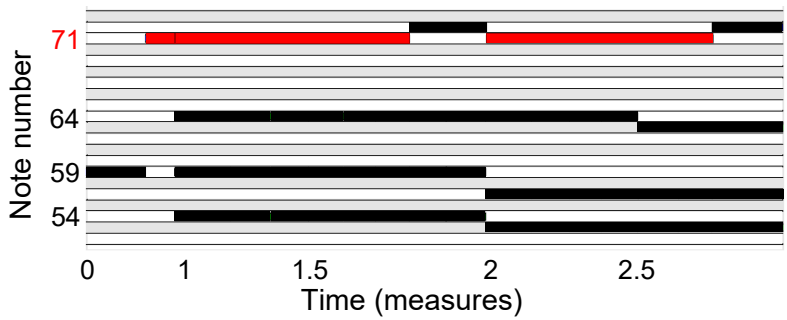
# Score-Informed Audio Decomposition

Sheet music

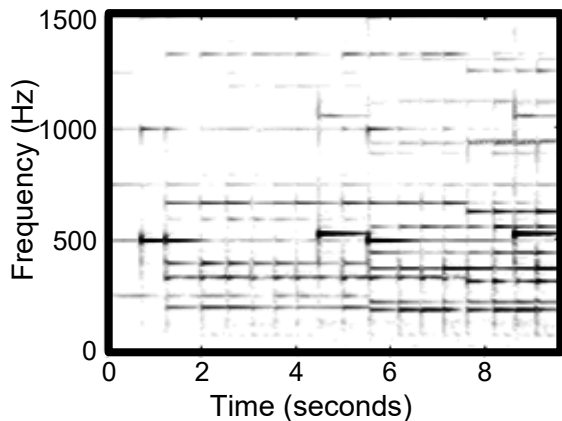


*p = 71*

Piano roll

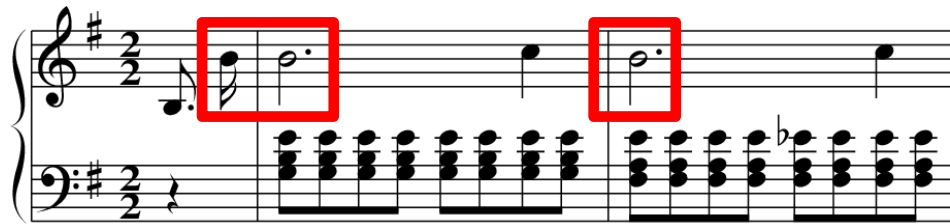


Spectrogram



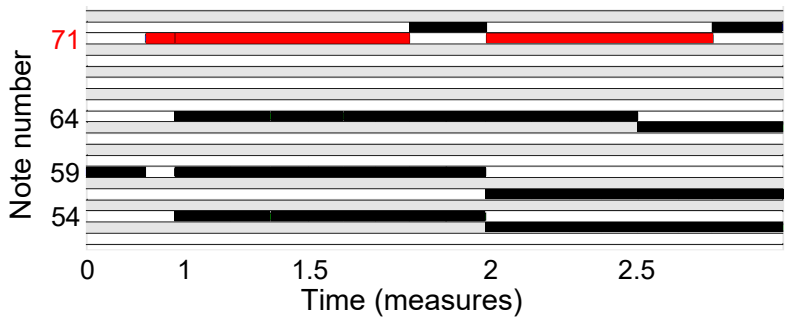
# Score-Informed Audio Decomposition

Sheet music

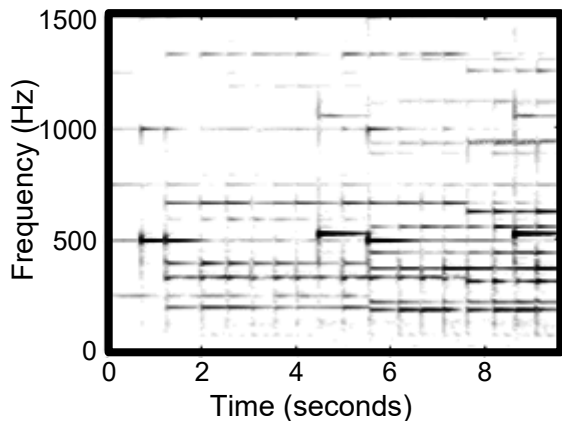


$p = 71$

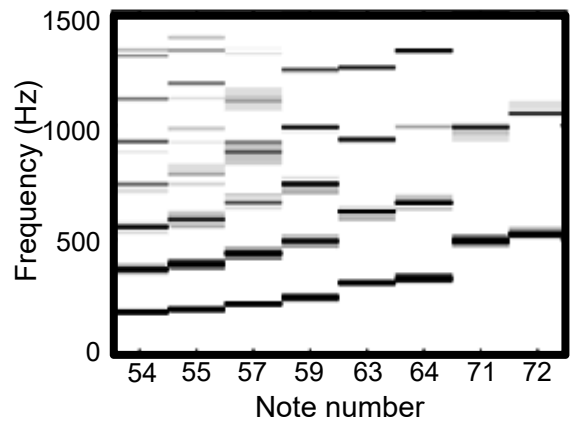
Piano roll



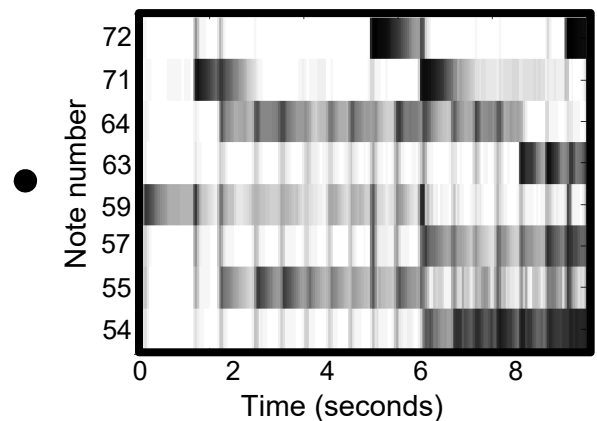
Spectrogram



Spectral patterns

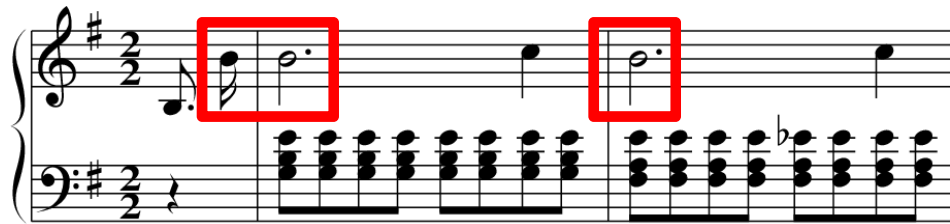


Activity patterns



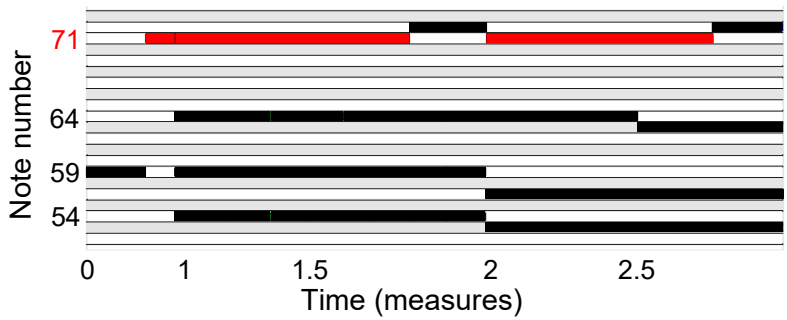
# Score-Informed Audio Decomposition

Sheet music

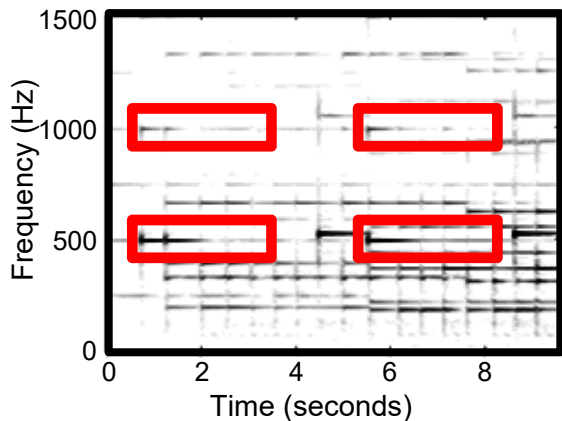


$p = 71$

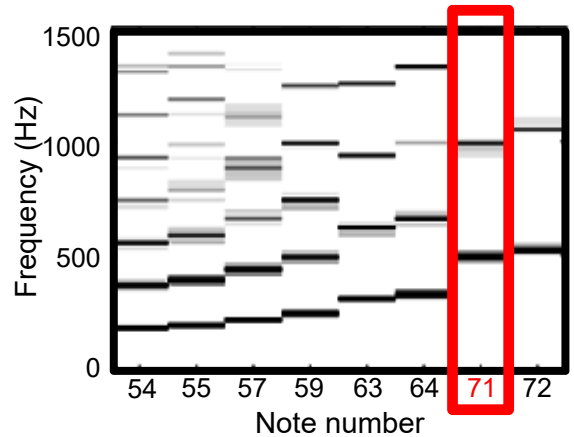
Piano roll



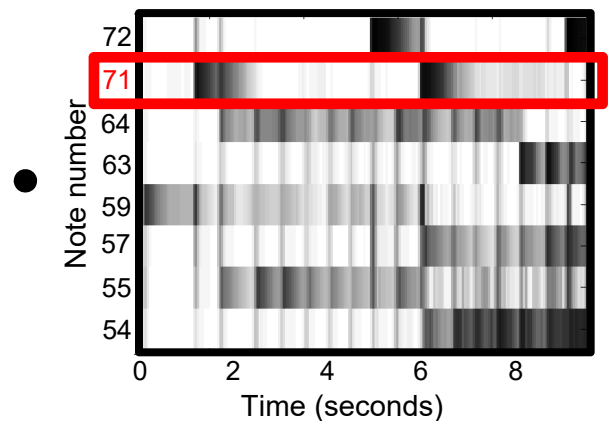
Spectrogram



Spectral patterns

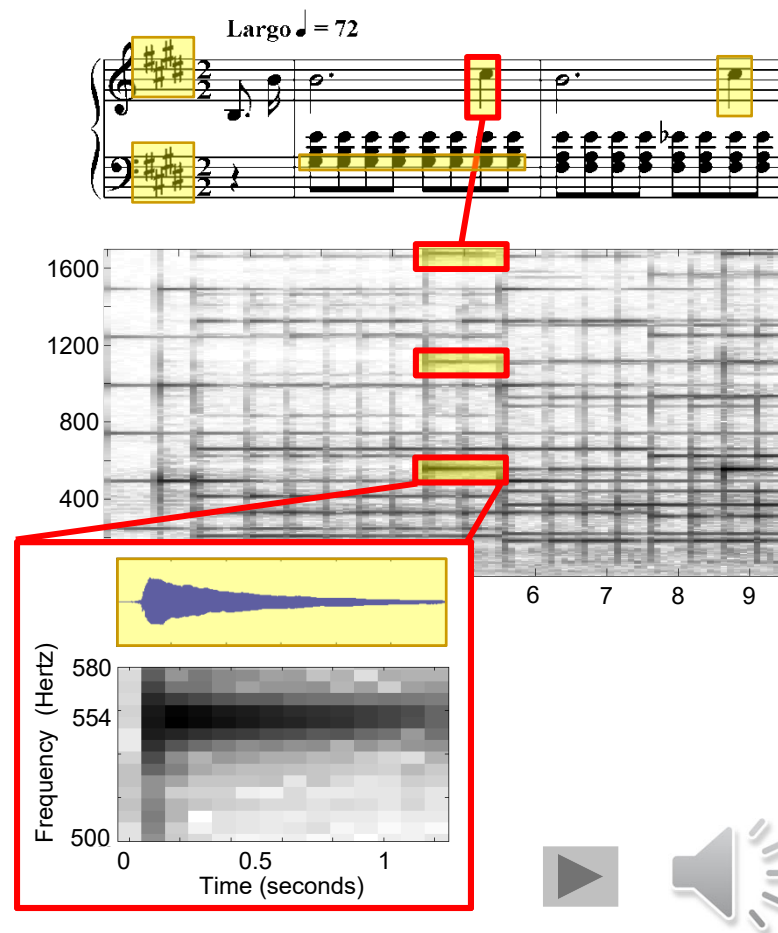
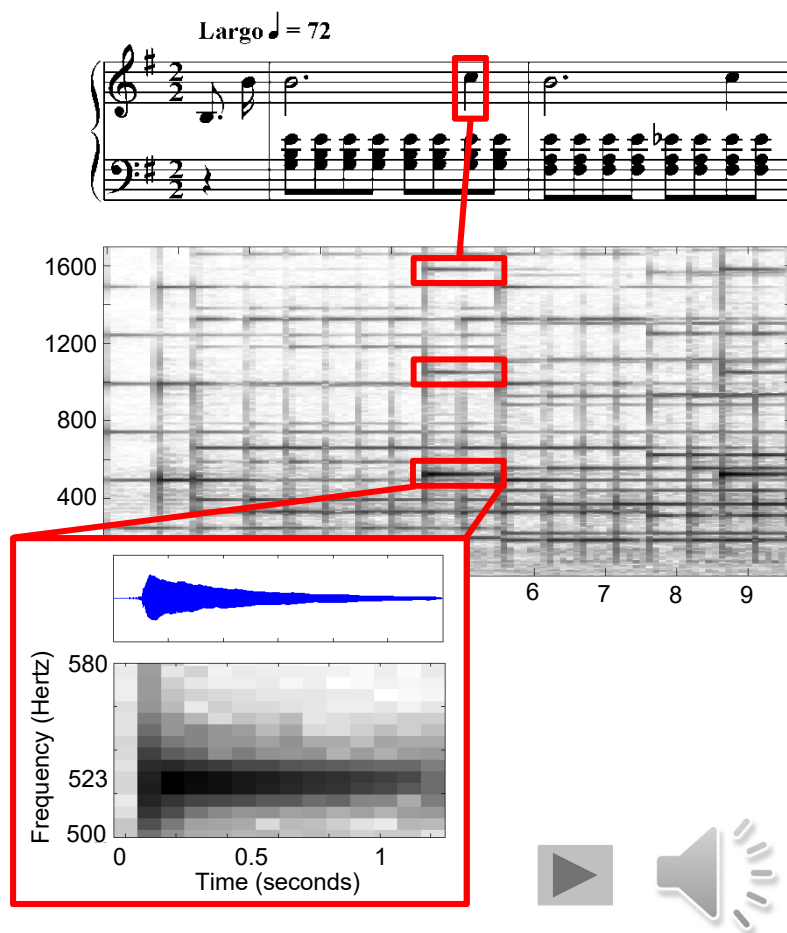


Activity patterns

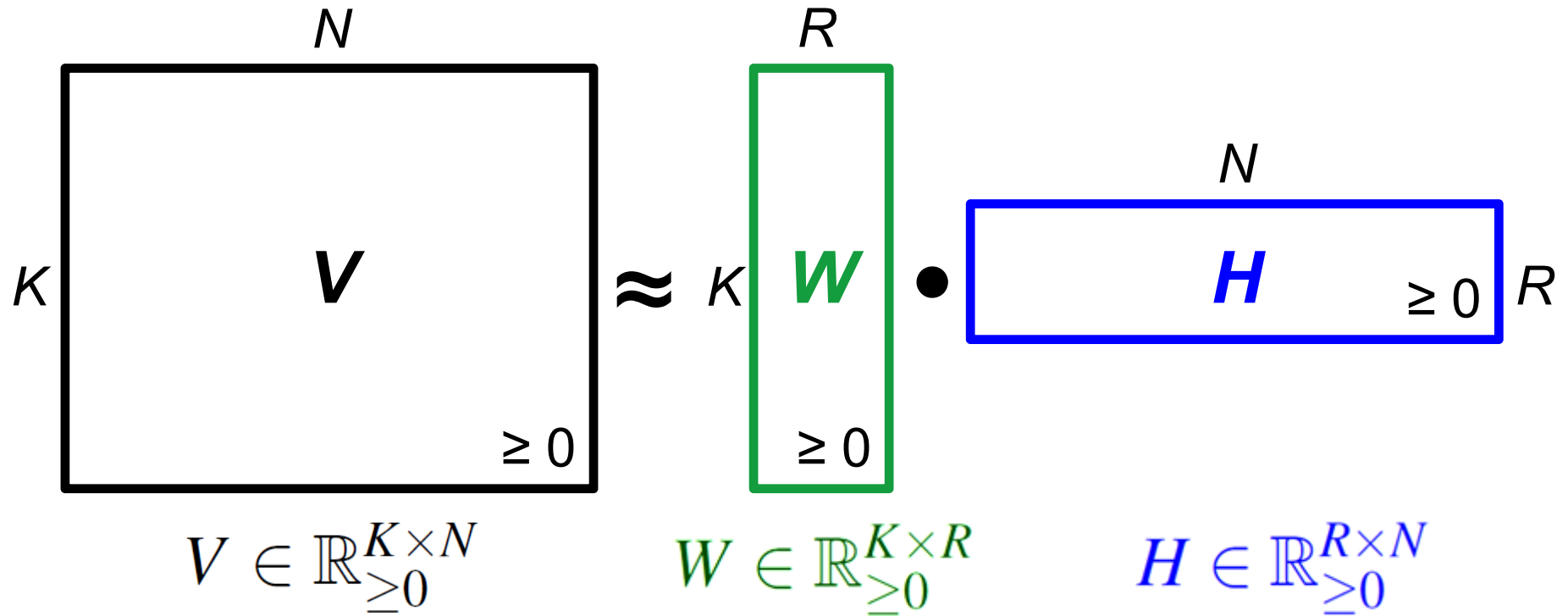


# Score-Informed Audio Decomposition

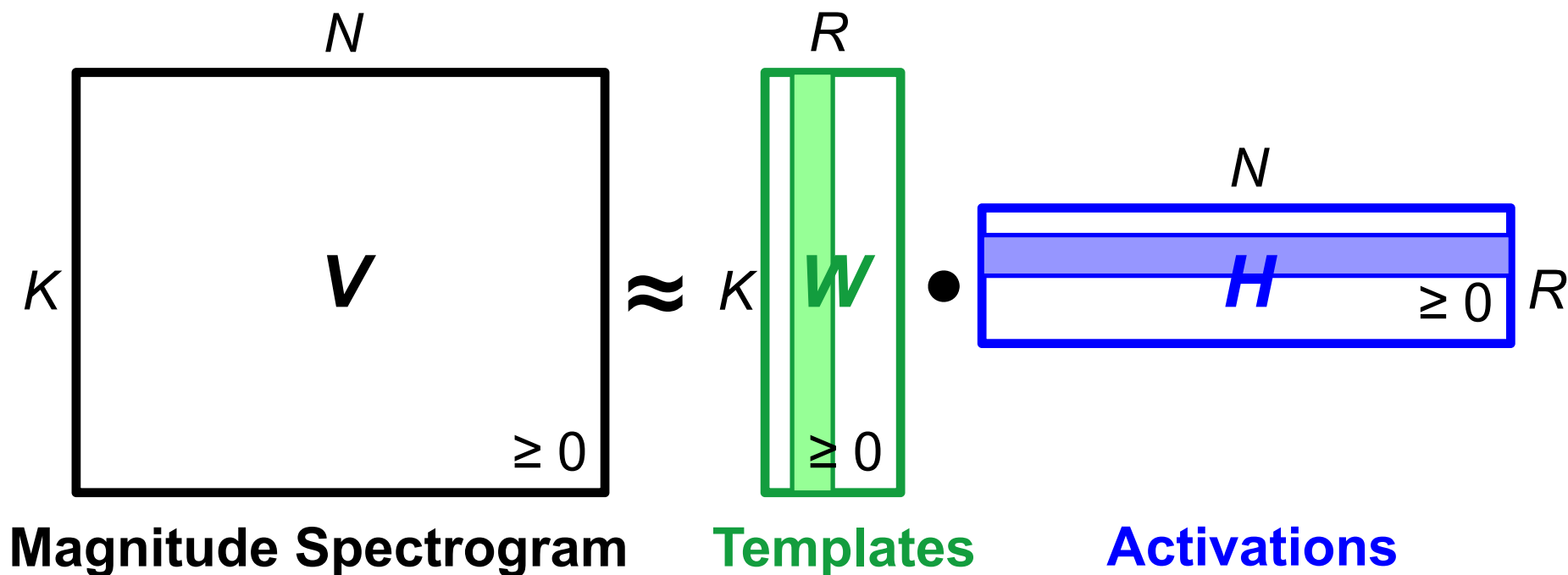
Application: Audio editing



# Nonnegative Matrix Factorization (NMF)



# Nonnegative Matrix Factorization (NMF)



**Templates:** Pitch + Timbre

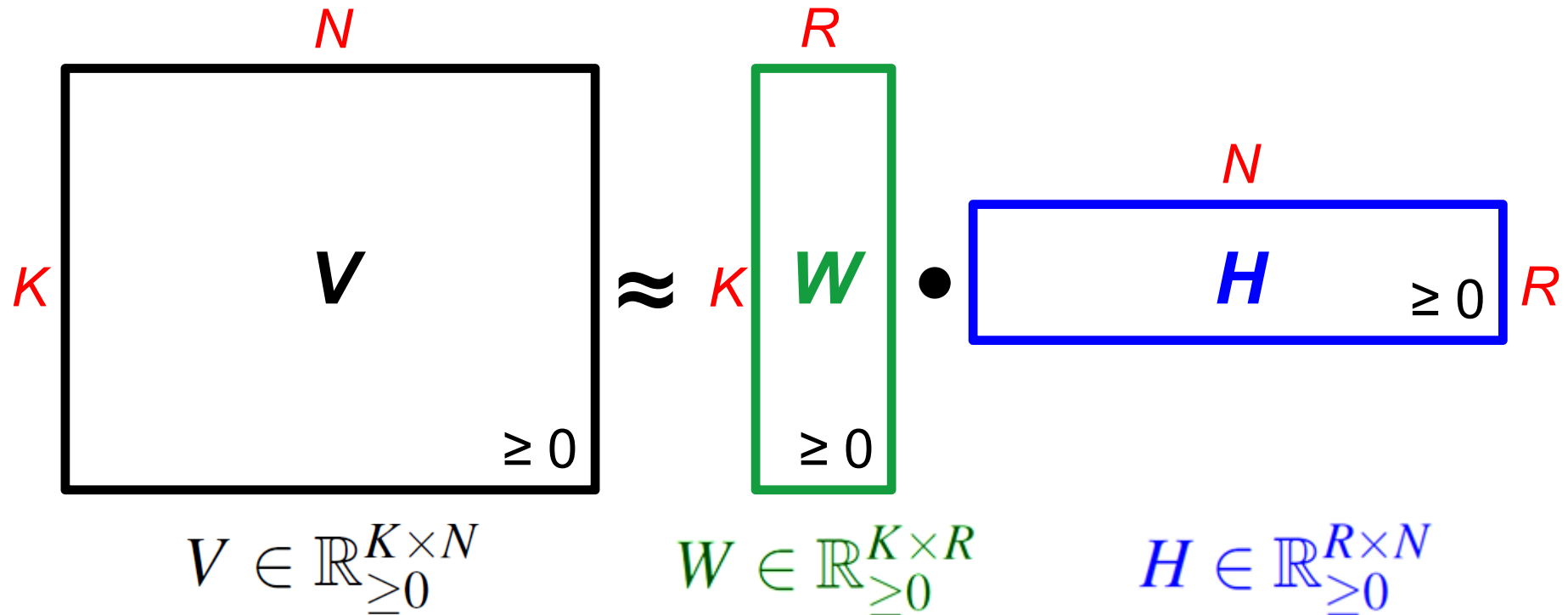
“How does it sound”

**Activations:** Onset time + Duration

“When does it sound”



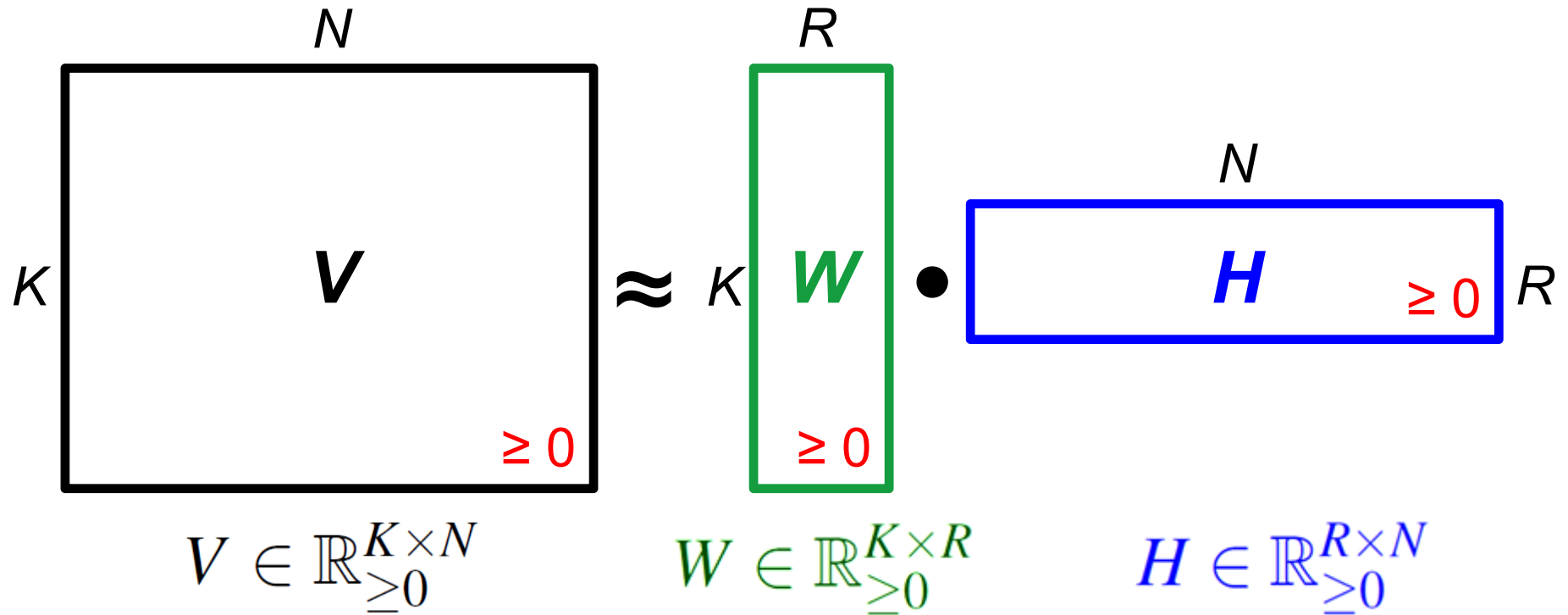
# Nonnegative Matrix Factorization (NMF)



## Dimensionality reduction

- $K, N$  typically much larger than  $R$  (maximal rank)
- Example:  $N = 1000, K = 500, R = 20$   
 $K \times N = 500,000, \quad K \times R = 10,000, \quad R \times N = 20,000$

# Nonnegative Matrix Factorization (NMF)



## Nonnegativity:

- Prevents mutual cancellation of template vectors
- Encourages semantically meaningful decomposition

# NMF Optimization

Optimization problem:

Given  $V \in \mathbb{R}_{\geq 0}^{K \times N}$  and rank parameter  $R$  minimize

$$\|V - WH\|^2$$

with respect to  $W \in \mathbb{R}_{\geq 0}^{K \times R}$  and  $H \in \mathbb{R}_{\geq 0}^{R \times N}$ .

Optimization not easy:

- Nonnegativity constraints
- Nonconvexity when jointly optimizing  $W$  and  $H$

Strategy: Iteratively optimize  $W$  and  $H$  via gradient descent

# NMF Optimization

Computation of gradient with respect to  $H$  (fixed  $W$ )

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

$$\varphi^W(H) := \|V - WH\|^2$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

$$H_{\rho v}$$

$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

# NMF Optimization

Computation of gradient with respect to  $H$  (fixed  $W$ )

$$\begin{aligned} D &:= RN \\ \varphi^W &: \mathbb{R}^D \rightarrow \mathbb{R} \\ \varphi^W(H) &:= \|V - WH\|^2 \end{aligned} \quad \frac{\partial \varphi^W}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K \sum_{n=1}^N (V_{kn} - \sum_{r=1}^R W_{kr} H_{rn})^2 \right)}{\partial H_{\rho v}}$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

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# NMF Optimization

## Computation of gradient with respect to $H$ (fixed $W$ )

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$$= \frac{\partial \left( \sum_{k=1}^K (V_{kv} - \sum_{r=1}^R W_{kr} H_{rv})^2 \right)}{\partial H_{\rho v}}$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

$$H_{\rho v}$$

$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

Summand that does not depend on  $H_{\rho v}$  must be zero

# NMF Optimization

## Computation of gradient with respect to $H$ (fixed $W$ )

$$D := RN$$

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$$= \frac{\partial \left( \sum_{k=1}^K (V_{kv} - \sum_{r=1}^R W_{kr} H_{rv})^2 \right)}{\partial H_{\rho v}}$$

$$= \sum_{k=1}^K 2 \left( V_{kv} - \sum_{r=1}^R W_{kr} H_{rv} \right) \cdot (-W_{k\rho})$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

$$H_{\rho v}$$

$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

Apply chain rule  
from calculus

# NMF Optimization

## Computation of gradient with respect to $H$ (fixed $W$ )

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

$$\varphi^W(H) := \|V - WH\|^2$$

Variables

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$$\begin{aligned} \frac{\partial \varphi^W}{\partial H_{\rho v}} &= \frac{\partial \left( \sum_{k=1}^K \sum_{n=1}^N (V_{kn} - \sum_{r=1}^R W_{kr} H_{rn})^2 \right)}{\partial H_{\rho v}} \\ &= \frac{\partial \left( \sum_{k=1}^K (V_{kv} - \sum_{r=1}^R W_{kr} H_{rv})^2 \right)}{\partial H_{\rho v}} \\ &= \sum_{k=1}^K 2 \left( V_{kv} - \sum_{r=1}^R W_{kr} H_{rv} \right) \cdot (-W_{k\rho}) \\ &= 2 \left( \sum_{r=1}^R \sum_{k=1}^K W_{k\rho} W_{kr} H_{rv} - \sum_{k=1}^K W_{k\rho} V_{kv} \right) \end{aligned}$$

Rearrange  
summands



# NMF Optimization

## Computation of gradient with respect to $H$ (fixed $W$ )

$$D := RN$$

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Introduce  
transposed  $W^\top$

# NMF Optimization

## Computation of gradient with respect to $H$ (fixed $W$ )

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

$$\varphi^W(H) := \|V - WH\|^2$$

Variables

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$$= \sum_{k=1}^K 2 \left( V_{kv} - \sum_{r=1}^R W_{kr} H_{rv} \right) \cdot (-W_{k\rho})$$

$$= 2 \left( \sum_{r=1}^R \sum_{k=1}^K W_{k\rho} W_{kr} H_{rv} - \sum_{k=1}^K W_{k\rho} V_{kv} \right)$$

$$= 2 \left( \sum_{r=1}^R \left( \sum_{k=1}^K W_{\rho k}^\top W_{kr} \right) H_{rv} - \sum_{k=1}^K W_{\rho k}^\top V_{kv} \right)$$

$$= 2 \left( (W^\top W H)_{\rho v} - (W^\top V)_{\rho v} \right).$$

# NMF Optimization

## Gradient descent

Initialization  $H^{(0)} \in \mathbb{R}^{R \times N}$

Iteration for  $\ell = 0, 1, 2, \dots$

$$H_{rn}^{(\ell+1)} = H_{rn}^{(\ell)} - \gamma_{rn}^{(\ell)} \cdot \left( (W^\top W H^{(\ell)})_{rn} - (W^\top V)_{rn} \right)$$

with suitable learning rate  $\gamma_{rn}^{(\ell)} \geq 0$

# NMF Optimization

## Gradient descent

Initialization  $H^{(0)} \in \mathbb{R}^{R \times N}$

Iteration for  $\ell = 0, 1, 2, \dots$

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with suitable learning rate  $\gamma_{rn}^{(\ell)} \geq 0$

Issues:

- How to do the initialization?
- How to choose the learning rate?
- How to ensure nonnegativity?

# NMF Optimization

## Gradient descent

Initialization  $H^{(0)} \in \mathbb{R}^{R \times N}$

Iteration for  $\ell = 0, 1, 2, \dots$

Choose adaptive learning rate:

$$\gamma_{rn}^{(\ell)} := \frac{H_{rn}^{(\ell)}}{(W^T W H^{(\ell)})_{rn}}$$

$$\begin{aligned} H_{rn}^{(\ell+1)} &= H_{rn}^{(\ell)} - \gamma_{rn}^{(\ell)} \cdot \left( (W^T W H^{(\ell)})_{rn} - (W^T V)_{rn} \right) \\ &= H_{rn}^{(\ell)} \cdot \frac{(W^T V)_{rn}}{(W^T W H^{(\ell)})_{rn}} \end{aligned}$$

Issues:

- How to do the initialization?
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# NMF Optimization

## Gradient descent

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Issues:

- How to do the initialization?
- How to choose the learning rate?
- How to ensure nonnegativity?

Choose adaptive learning rate:

$$\gamma_{rn}^{(\ell)} := \frac{H_{rn}^{(\ell)}}{(W^\top W H^{(\ell)})_{rn}}$$

- Update rule become multiplicative
- Nonnegative values stay nonnegative

# NMF Optimization

**Algorithm:** NMF ( $V \approx WH$ )

**Input:** Nonnegative matrix  $V$  of size  $K \times N$   
Rank parameter  $R \in \mathbb{N}$   
Threshold  $\varepsilon$  used as stop criterion

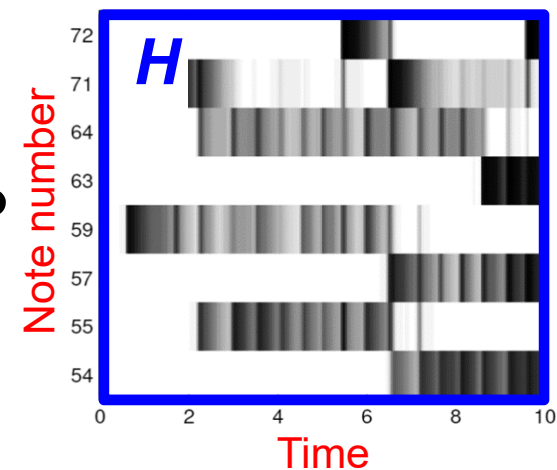
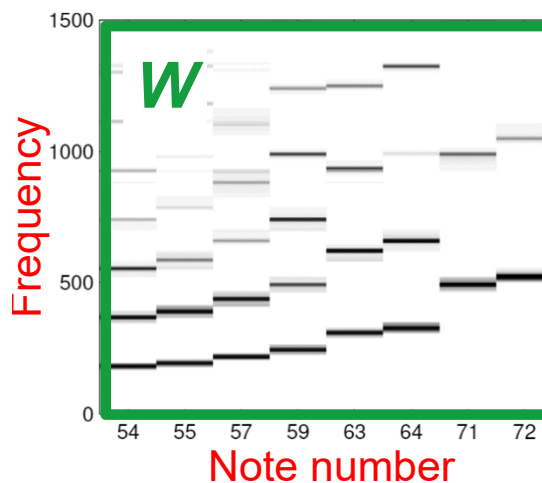
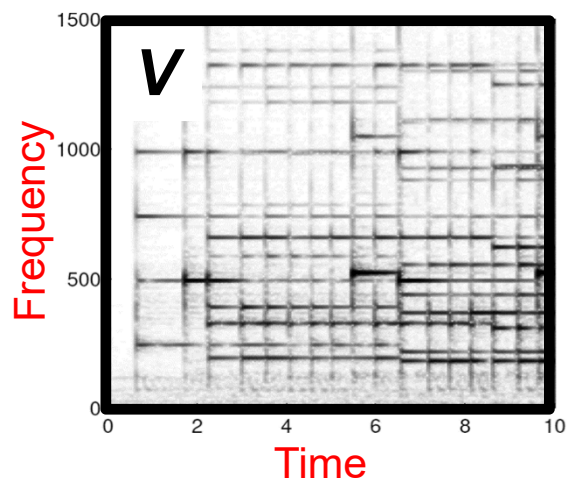
**Output:** Nonnegative template matrix  $W$  of size  $K \times R$   
Nonnegative activation matrix  $H$  of size  $R \times N$

**Procedure:** Define nonnegative matrices  $W^{(0)}$  and  $H^{(0)}$  by some random or informed initialization. Furthermore set  $\ell = 0$ . Apply the following update rules (written in matrix notation):

- (1)  $H^{(\ell+1)} = H^{(\ell)} \odot (((W^{(\ell)})^\top V) \oslash ((W^{(\ell)})^\top W^{(\ell)} H^{(\ell)}))$
- (2)  $W^{(\ell+1)} = W^{(\ell)} \odot ((V(H^{(\ell+1)})^\top) \oslash (W^{(\ell)} H^{(\ell+1)} (H^{(\ell+1)})^\top))$
- (3) Increase  $\ell$  by one.

Repeat the steps (1) to (3) until  $\|H^{(\ell)} - H^{(\ell-1)}\| \leq \varepsilon$  and  $\|W^{(\ell)} - W^{(\ell-1)}\| \leq \varepsilon$  (or until some other stop criterion is fulfilled). Finally, set  $H = H^{(\ell)}$  and  $W = W^{(\ell)}$ .

# NMF-based Spectrogram Decomposition



Templates: Pitch + Timbre

“How does it sound”

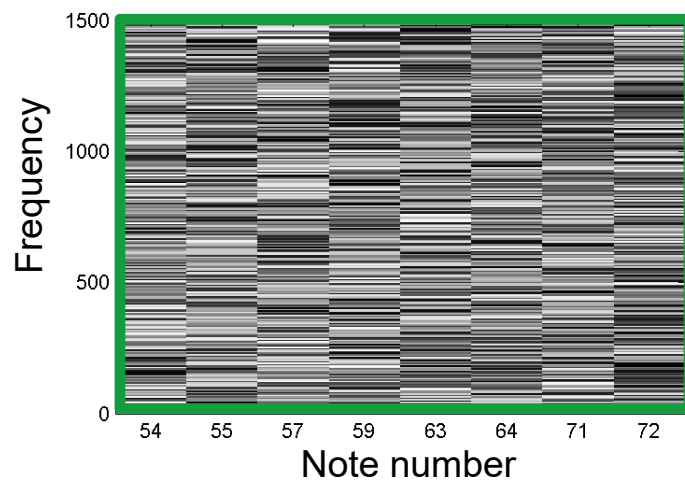
Activations: Onset time + Duration

“When does it sound”

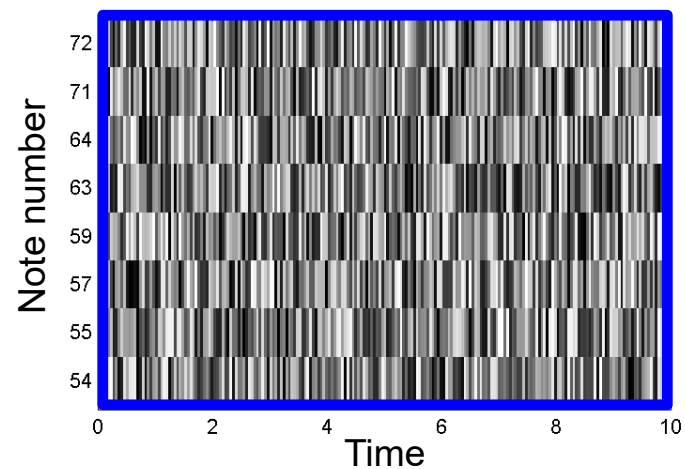


# NMF-based Spectrogram Decomposition

Template initialization



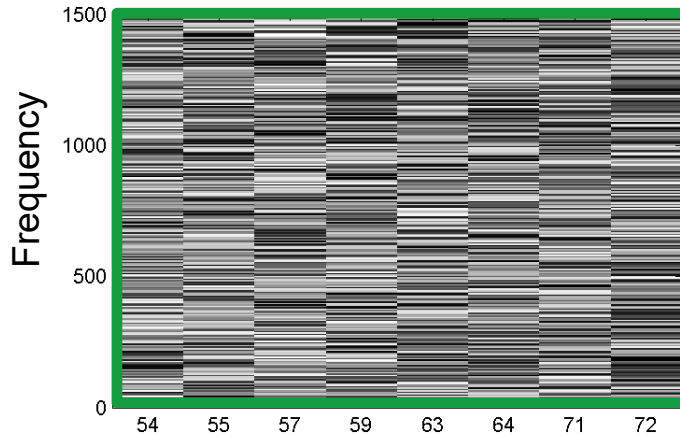
Activation initialization



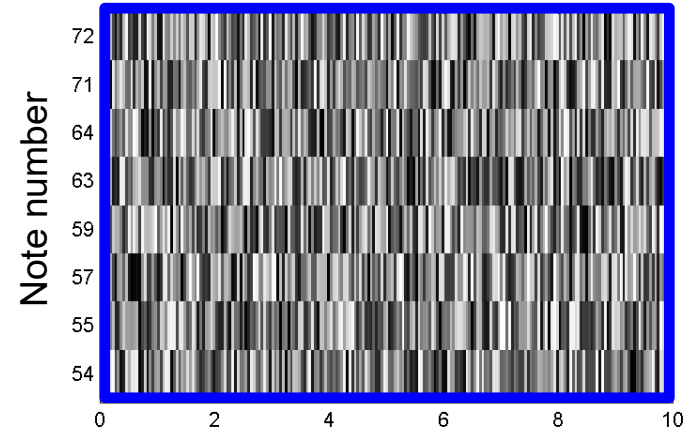
Random initialization

# NMF-based Spectrogram Decomposition

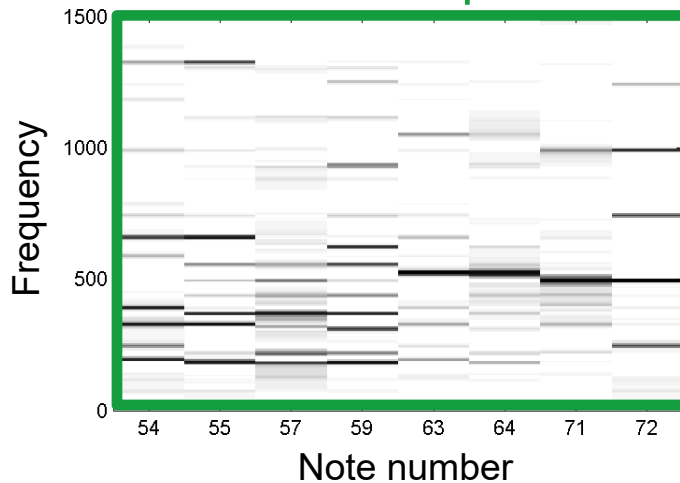
Template initialization



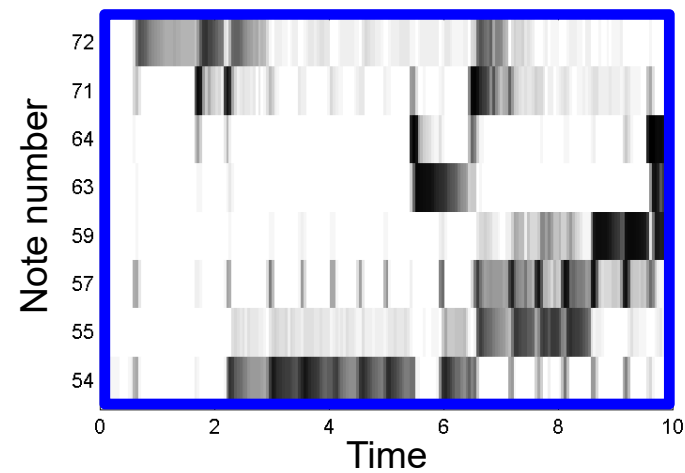
Activation initialization



Learnt templates



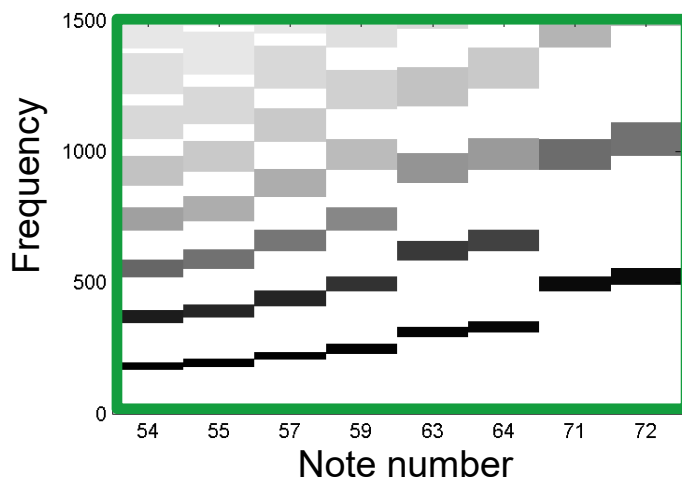
Learnt activations



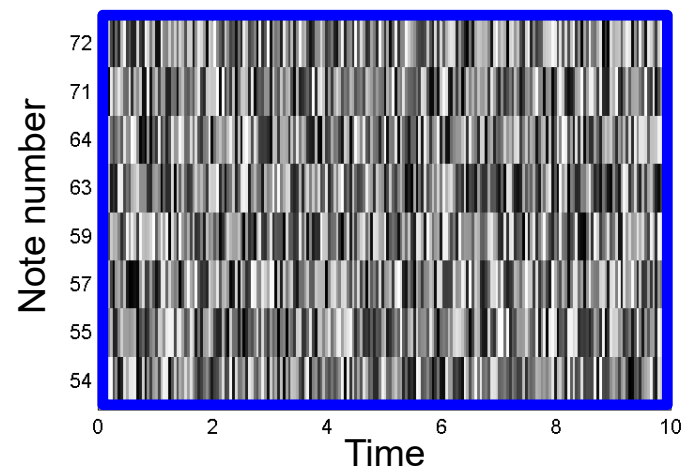
Random initialization → No semantic meaning

# Constrained NMF: Templates

Template initialization



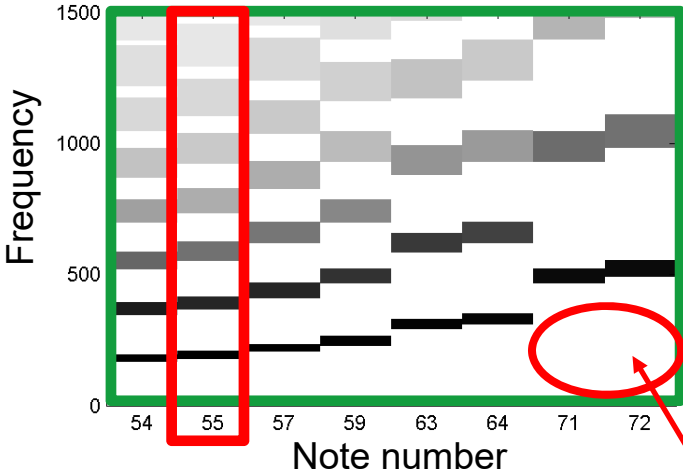
Activation initialization



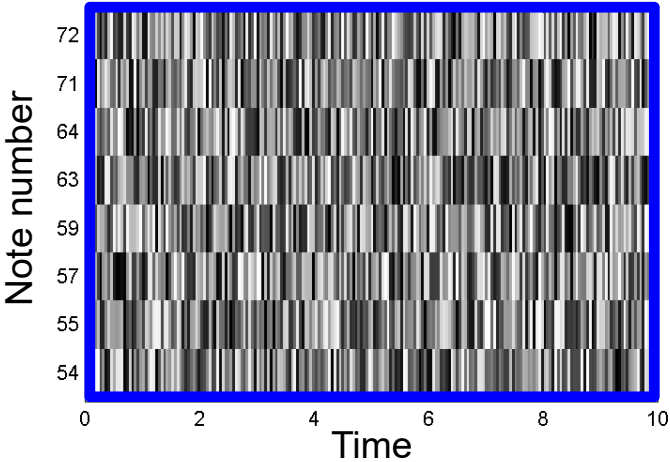
Enforce harmonic structure with zero-valued entries

# Constrained NMF: Templates

Template initialization



Activation initialization

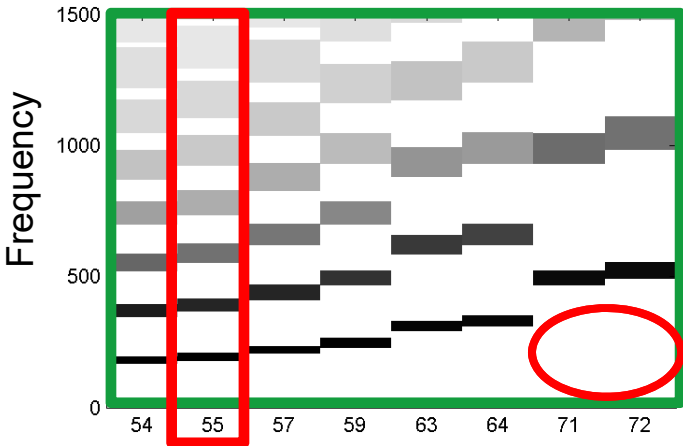


Template constraint for  $p=55$

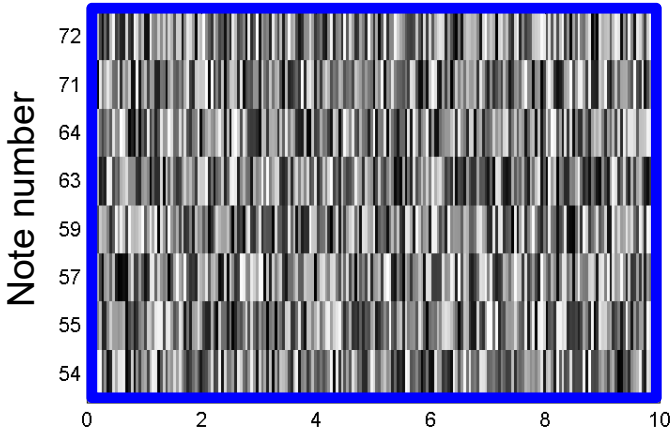
Enforce harmonic structure with zero-valued entries

# Constrained NMF: Templates

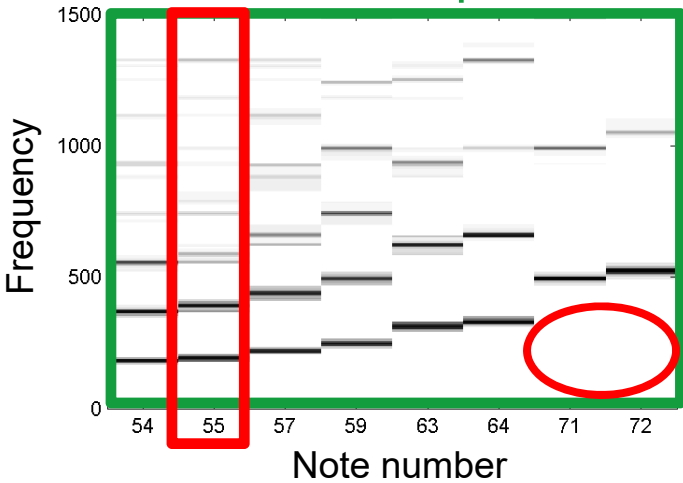
Template initialization



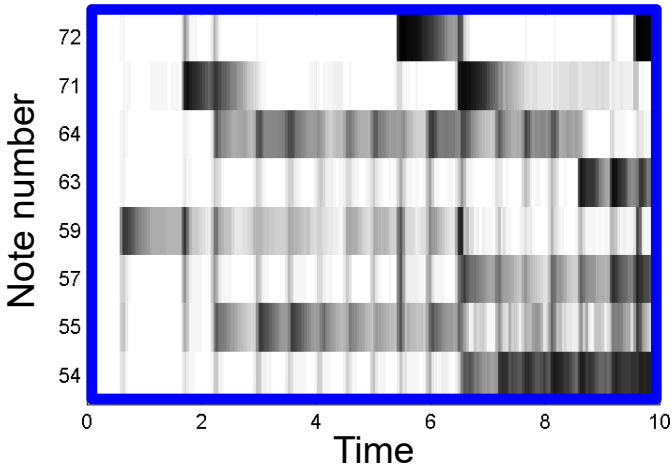
Activation initialization



Learnt templates



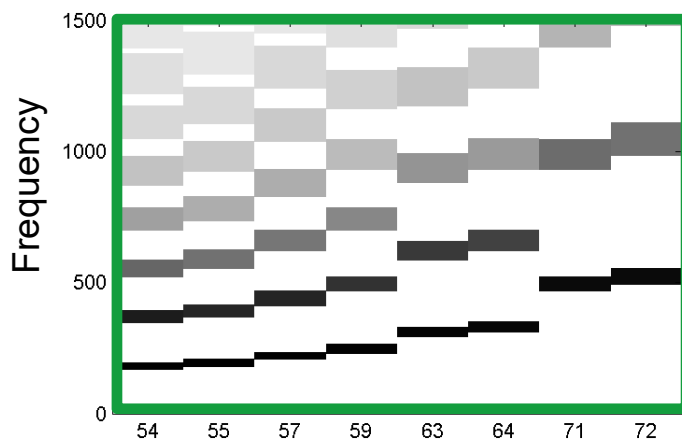
Learnt activations



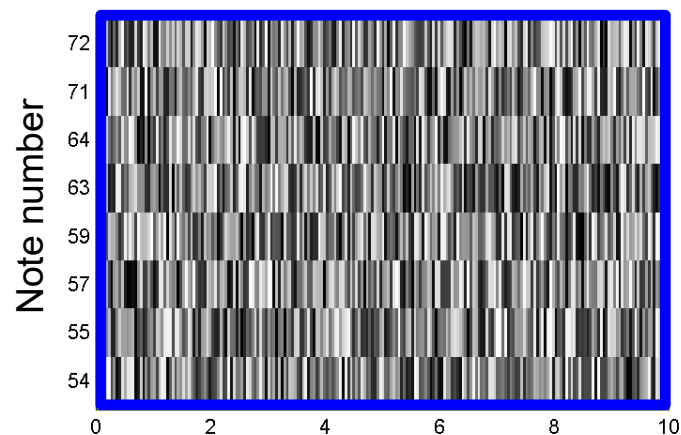
Zero-valued entries remain zero-valued entries!

# Constrained NMF: Templates

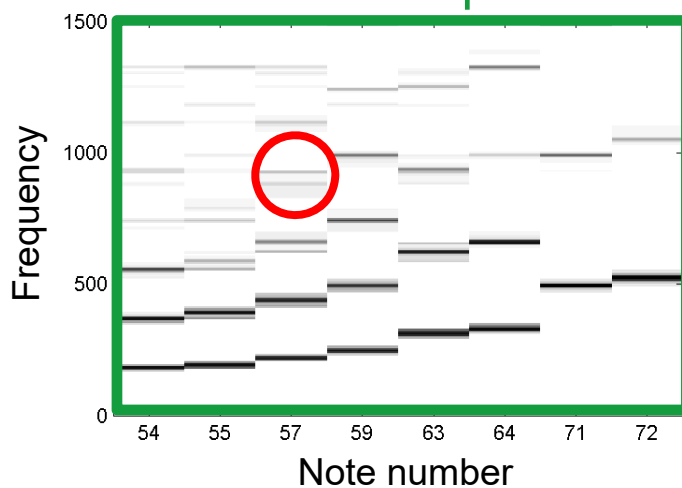
Template initialization



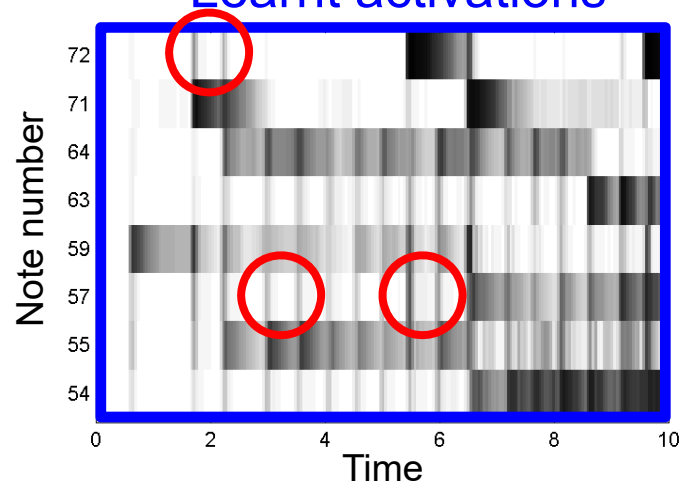
Activation initialization



Learnt templates



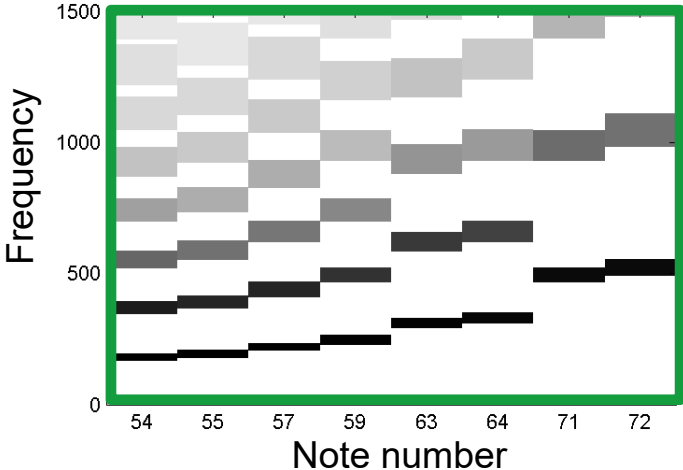
Learnt activations



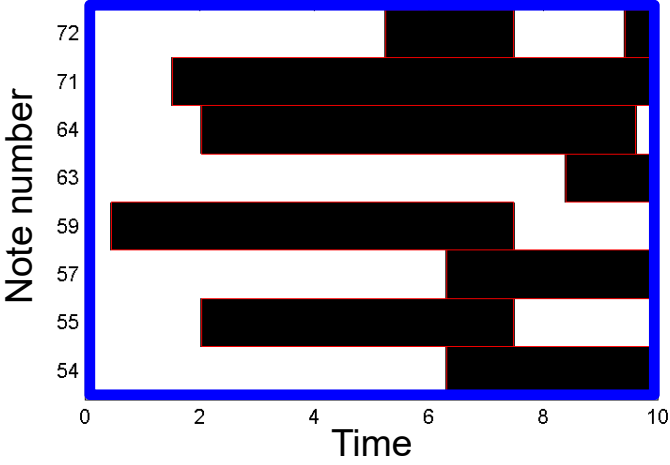
Pitch templates misused to represent onsets

# Constrained NMF: Double Constraints

Template initialization

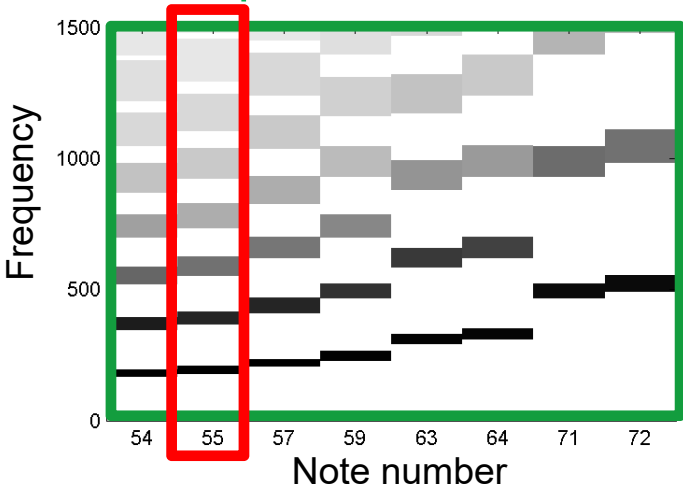


Activation initialization



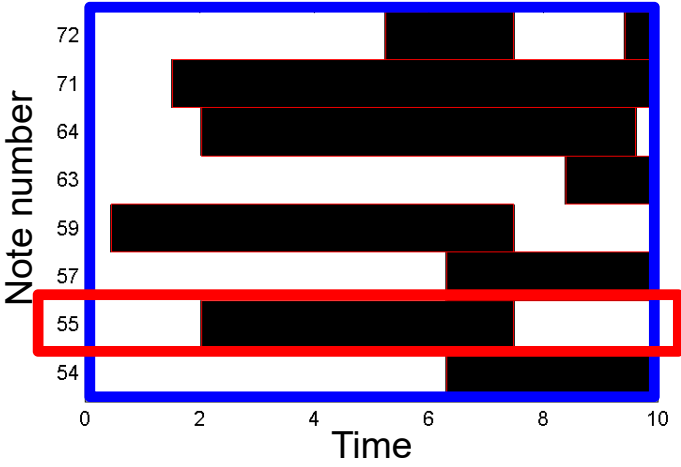
# Constrained NMF: Double Constraints

Template initialization



Template constraint for  $p=55$

Activation initialization

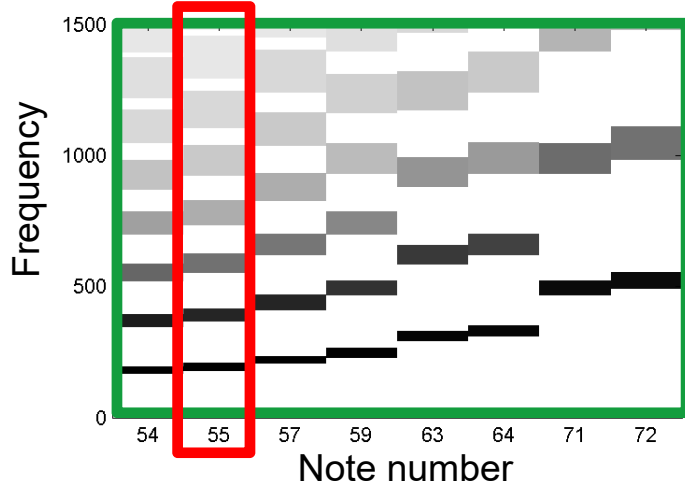


Activation constraints for  $p=55$



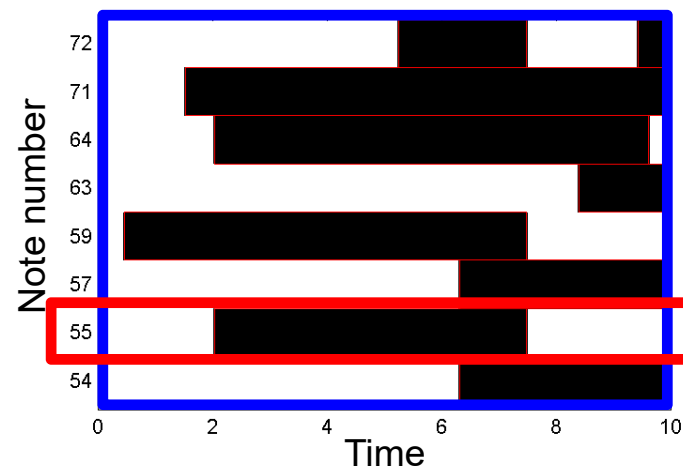
# Constrained NMF: Double Constraints

Template initialization



Template constraint for  $p=55$

Activation initialization

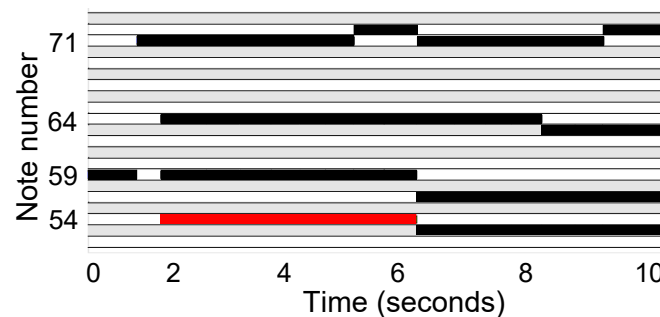
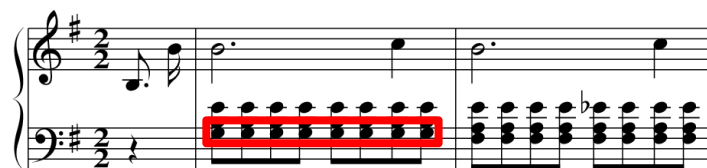


Activation constraints for  $p=55$

Such information may come from a synchronized score

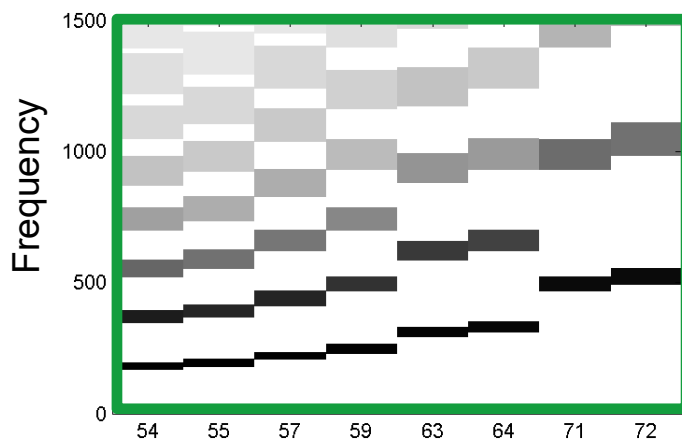


Sheet music

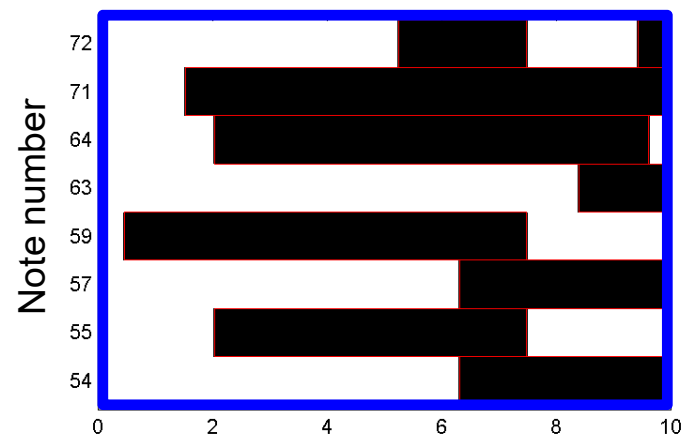


# Constrained NMF: Double Constraints

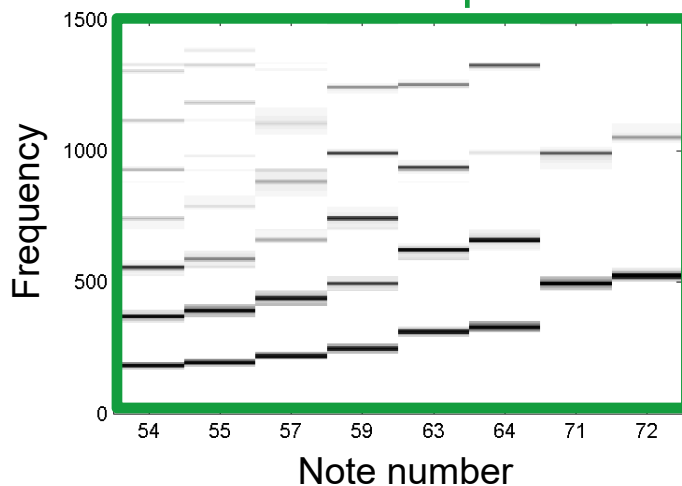
Template initialization



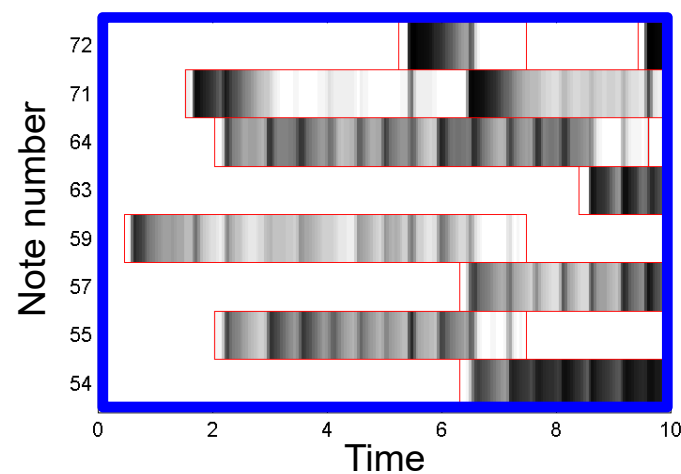
Activation initialization



Learnt templates



Learnt activations



Original



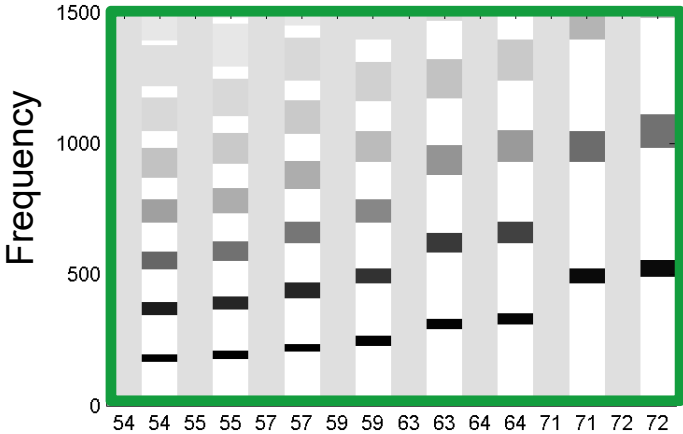
Model



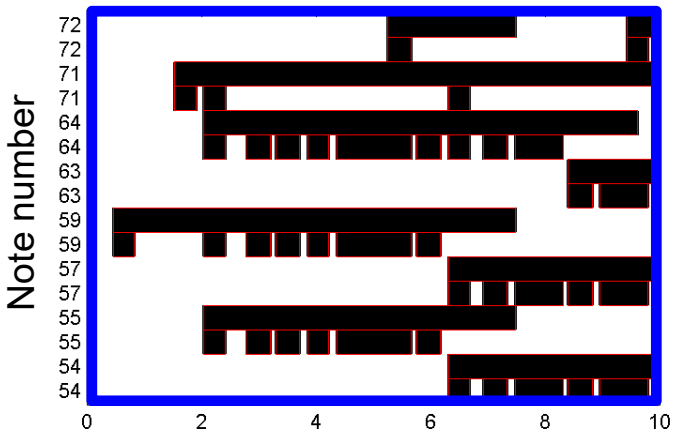
Significant gain in structure, but onsets are missing

# Constrained NMF: Onset Templates

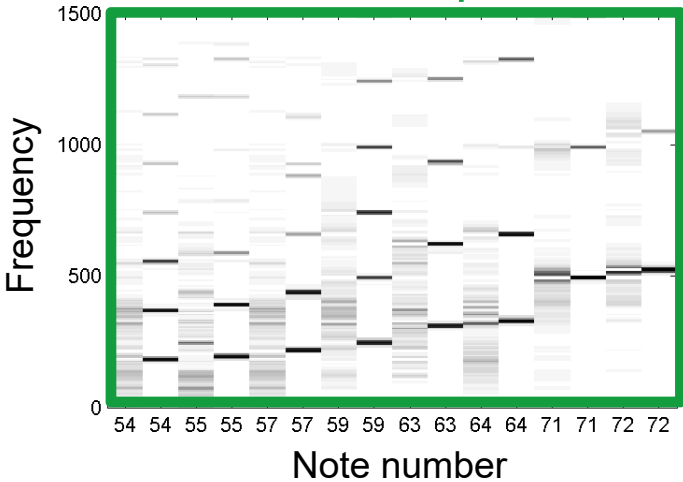
Template initialization



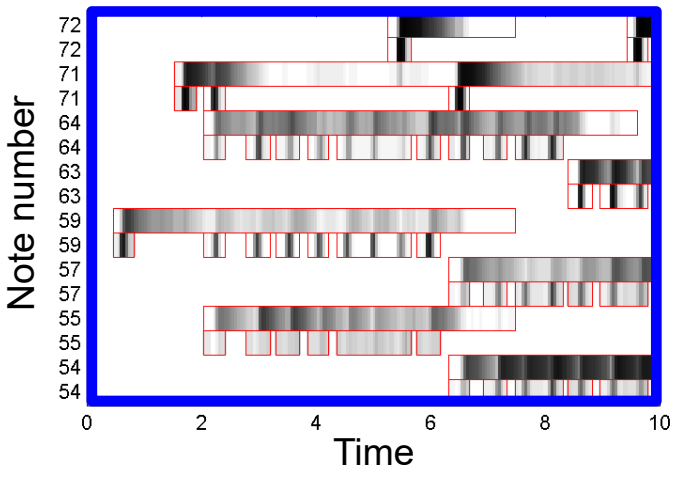
Activation initialization



Learnt templates



Learnt activations



Original



Model Onset



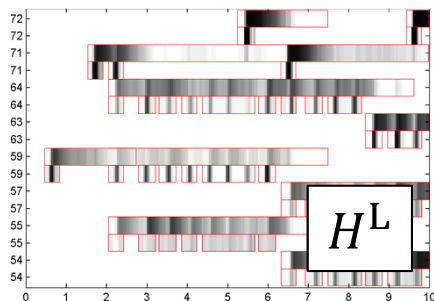
# Score-Informed Audio Decomposition

Application: Separating left and right hands for piano



1. Split activation matrix

$$H^R$$

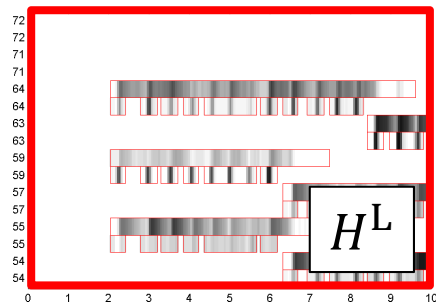
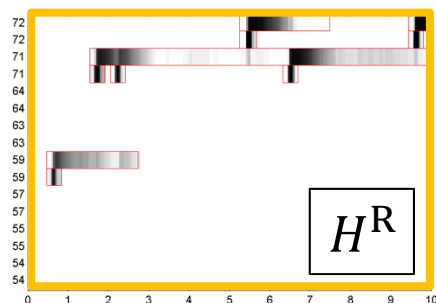


# Score-Informed Audio Decomposition

Application: Separating left and right hands for piano



1. Split activation matrix

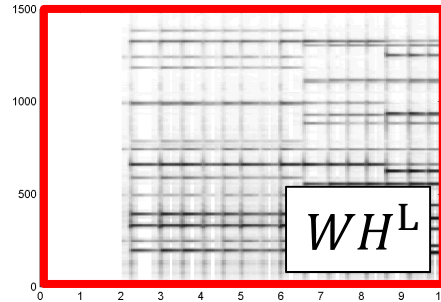
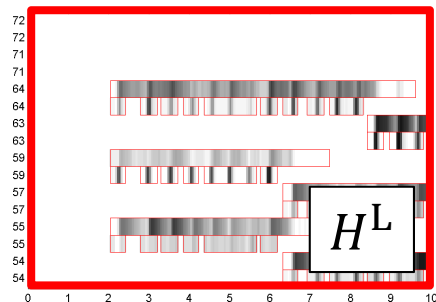
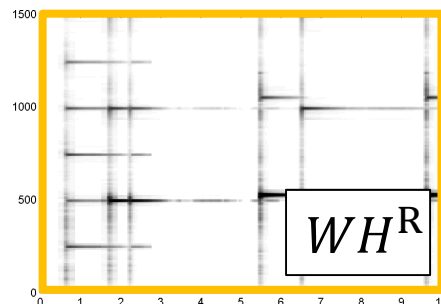
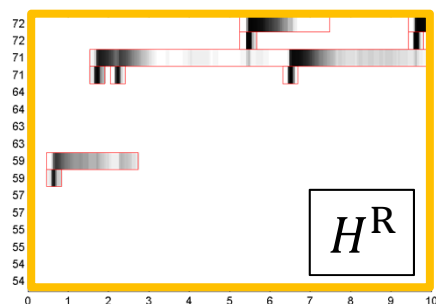


# Score-Informed Audio Decomposition

## Application: Separating left and right hands for piano



1. Split activation matrix
2. Model spectrogram for left/right

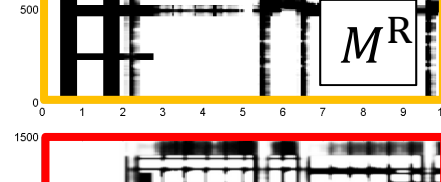
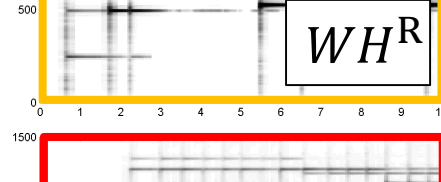
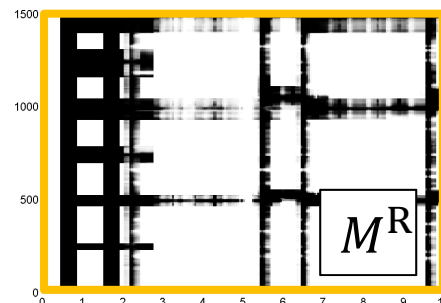
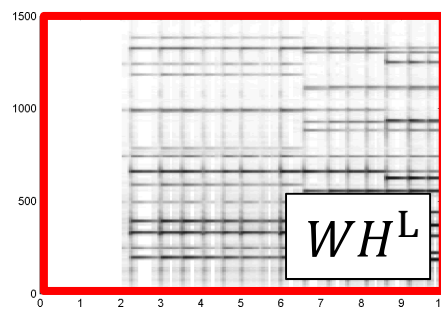
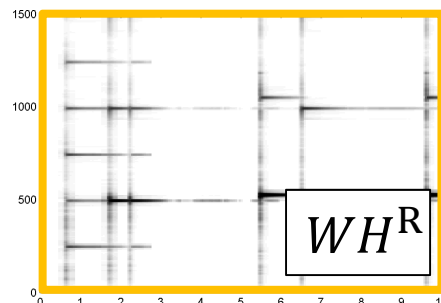
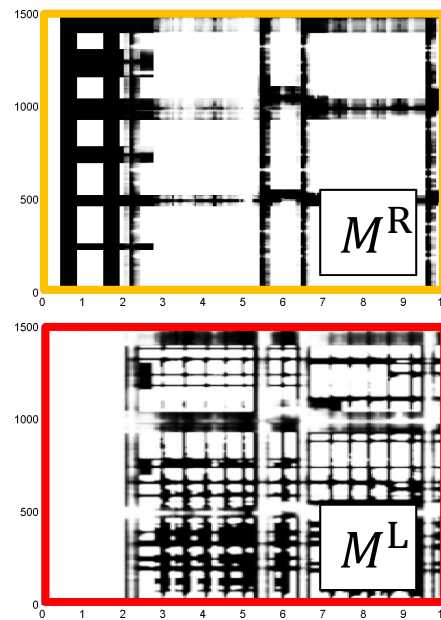
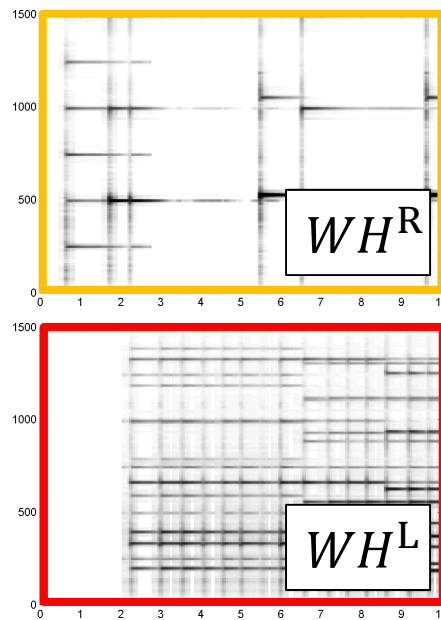
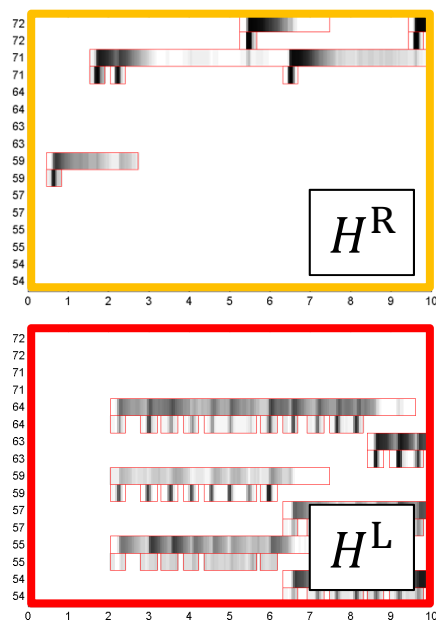


# Score-Informed Audio Decomposition

## Application: Separating left and right hands for piano

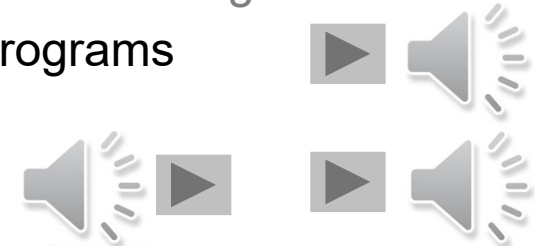


1. Split activation matrix
2. Model spectrogram for left/right
3. Separation masks for left/right

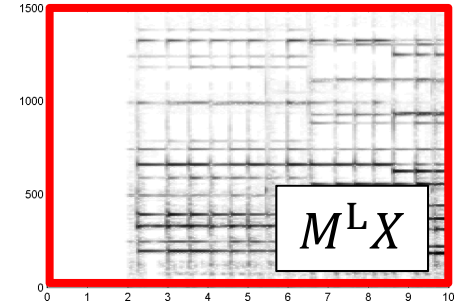
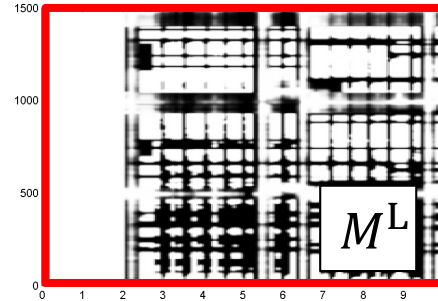
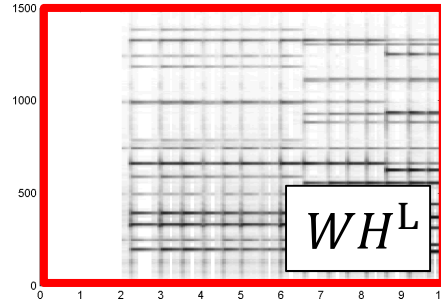
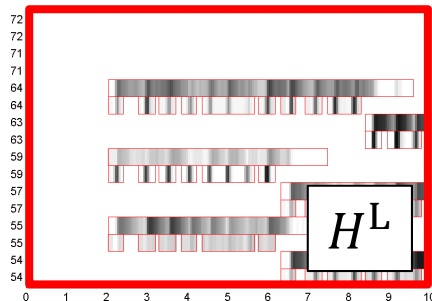
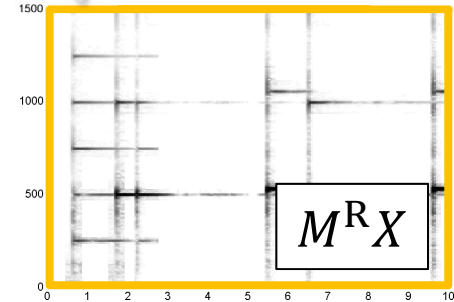
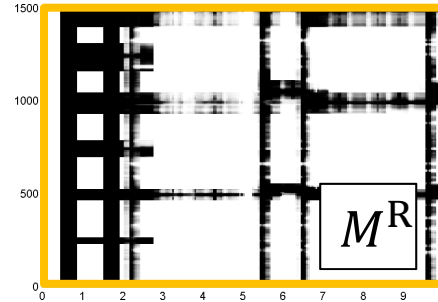
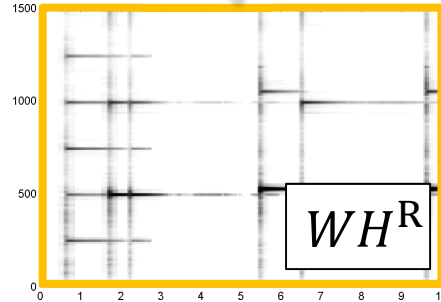
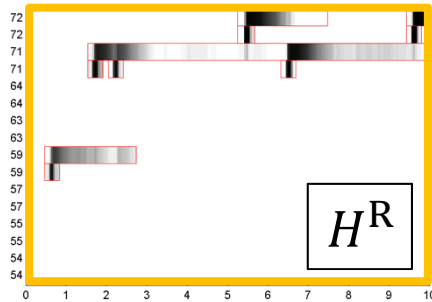


# Score-Informed Audio Decomposition

## Application: Separating left and right hands for piano



1. Split activation matrix
2. Model spectrogram for left/right
3. Separation masks for left/right
4. Estimated spectrograms for left/right





# Score-Informed Audio Decomposition

Application: Separating left and right hands for piano

Chopin, Waltz Op. 64, No. 1

Molto Vivace

leggiero

Original



Ewert, Müller: Using Score-Informed Constraints for NMF-based Source Separation. Proc. ICASSP, 2012.

Further results available at

<http://www.mpi-inf.mpg.de/resources/MIR/ICASSP2012-ScoreInformedNMF/>

# Score-Informed Audio Decomposition

Application: Separating left and right hands for piano

Chopin, Waltz Op. 64, No. 1

Molto Vivace



Original



Left/right hand



Right hand



Left hand



Ewert, Müller: Using Score-Informed Constraints for NMF-based Source Separation. Proc. ICASSP, 2012.

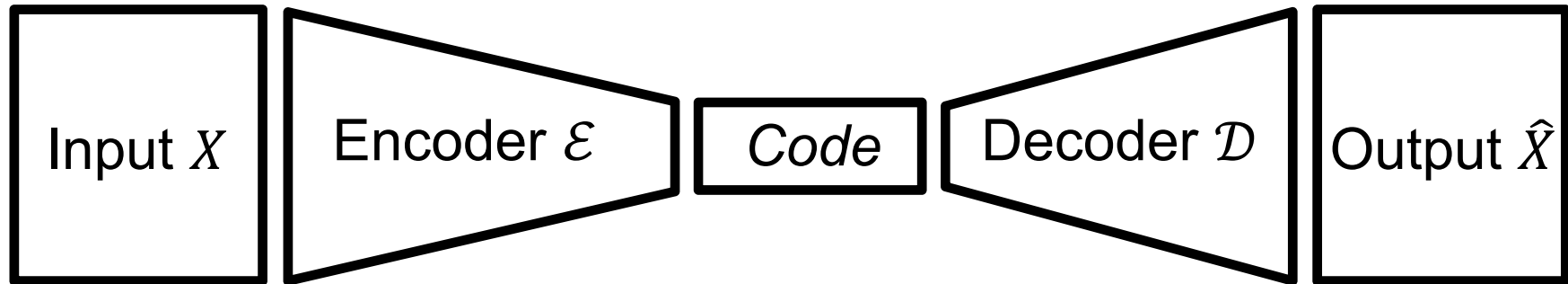
Further results available at

<http://www.mpi-inf.mpg.de/resources/MIR/ICASSP2012-ScoreInformedNMF/>

## Conclusions (NMF)

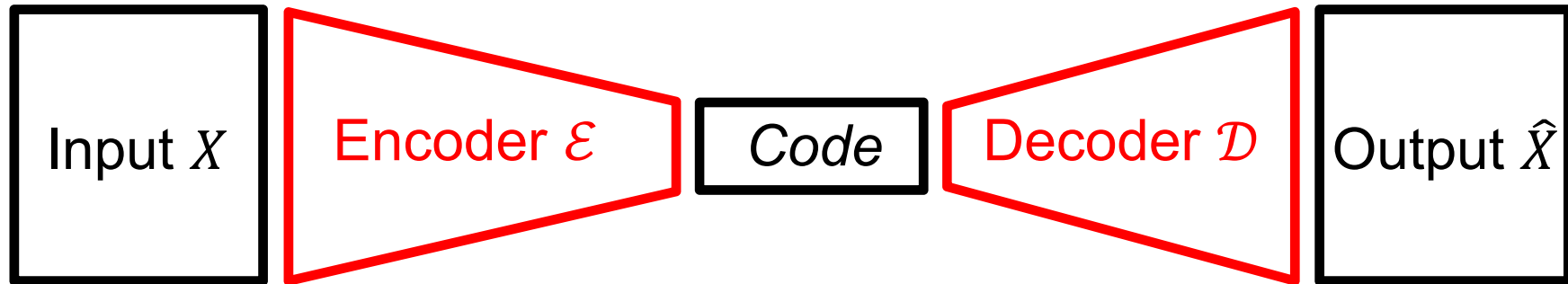
- NMF used for spectrogram decomposition
- Multiplicative update rules make it easy to constrain NMF model via zero initialization
- Exploiting score information to guide separation process (requires score–audio synchronization)
- Application: Separation of arbitrary note groups from given audio recording

# Autoencoder



- Specific type of neural network
- Encoder: Compress input  $X$  into a low-dimensional code
- Decoder: Reconstruct output  $\hat{X}$  from code

# Autoencoder



- Specific type of neural network
- Encoder: Compress input  $X$  into a low-dimensional code
- Decoder: Reconstruct output  $\hat{X}$  from code
- Goal: Learn **parameters** for **encoder** and **decoder** such that output is close to input with respect to some loss function:

$$\mathcal{L}(X, \hat{X}) \approx 0$$

# NMF and Autoencoder (AE)

Smaragdis, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.

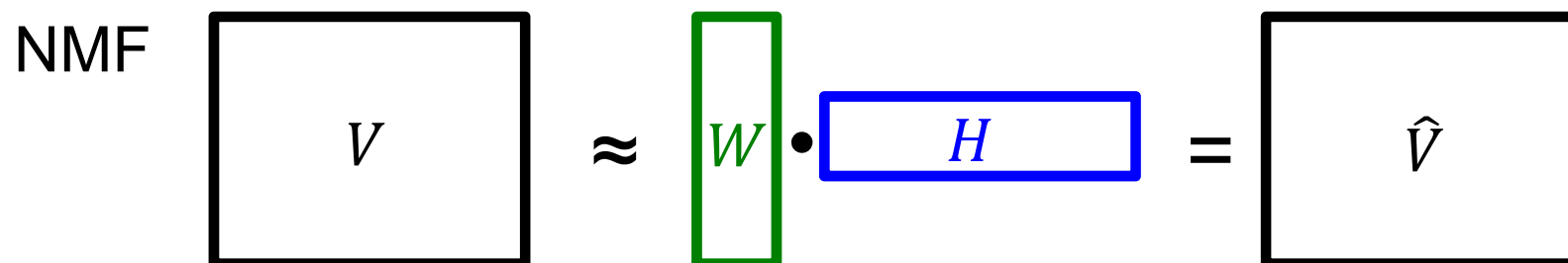
NMF

$$V \approx W \cdot H = \hat{V}$$

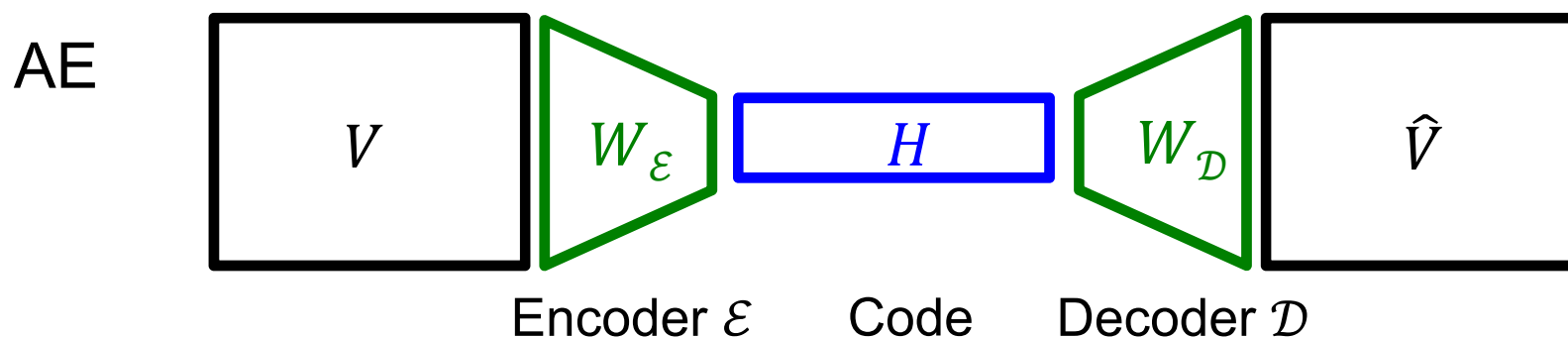
$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$

# NMF and Autoencoder (AE)

Smaragdīs, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.



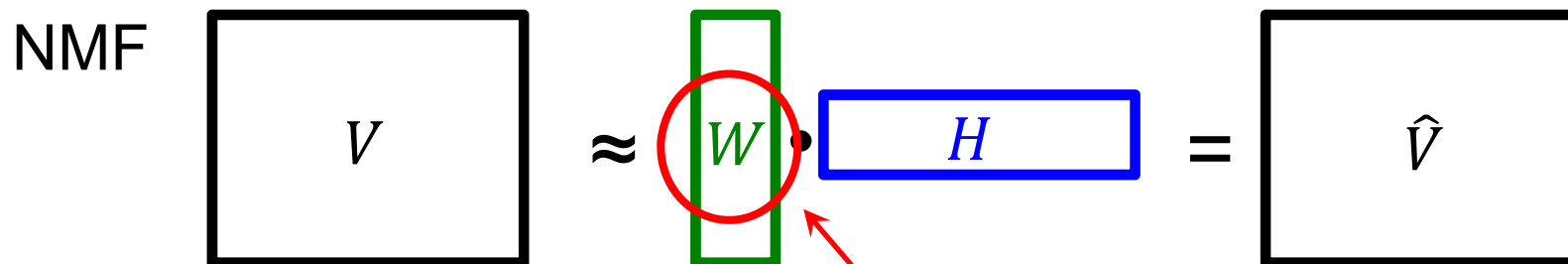
$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$



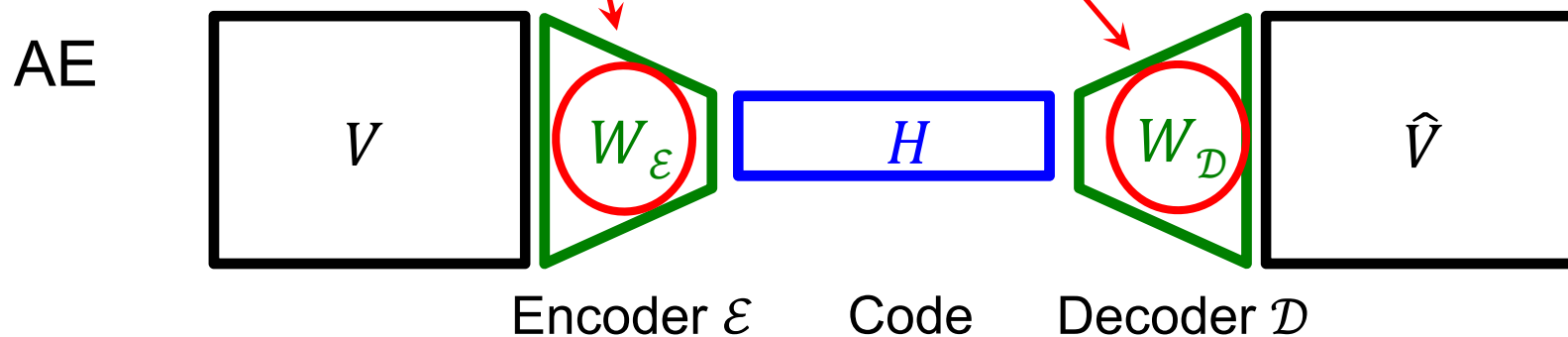
1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

# NMF and Autoencoder (AE)

Smaragdīs, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.



$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$



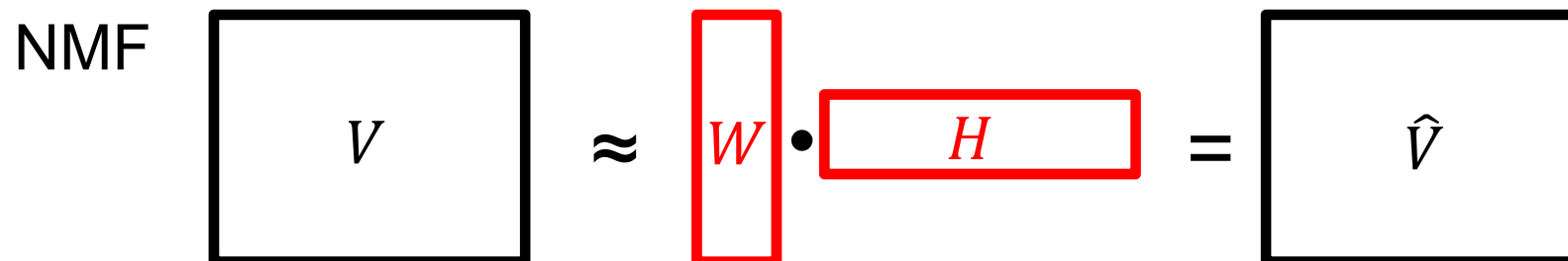
1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

Fully connected network

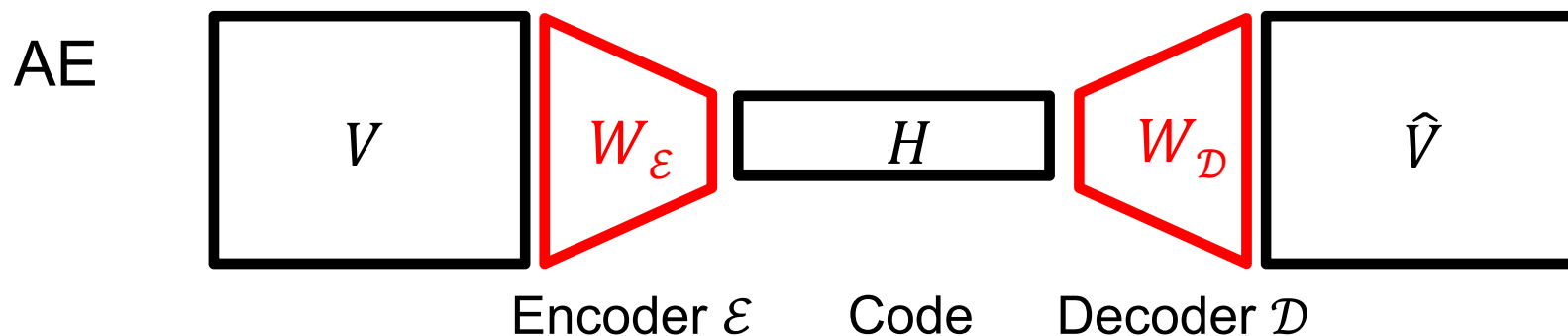


# NMF and Autoencoder (AE)

Smaragdīs, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.



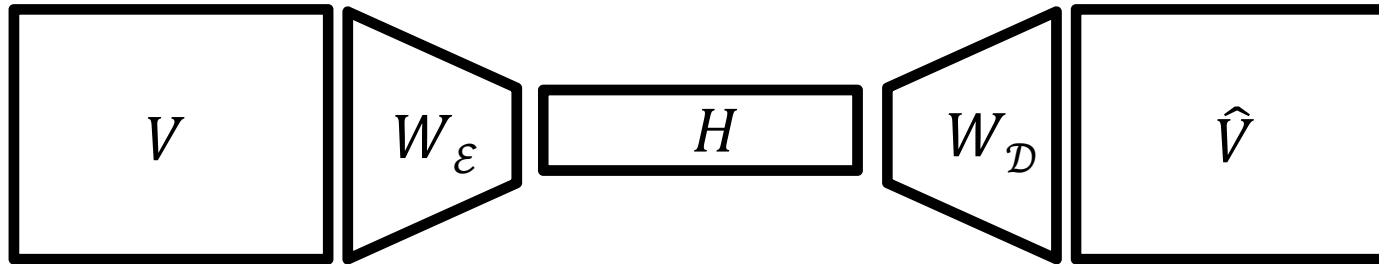
$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$



1. Layer:  $H = W_{\mathcal{E}} V$
2. Layer:  $\hat{V} = W_{\mathcal{D}} H$

NMF: Learn  $H$  and  $W$   
AE: Learn  $W_{\mathcal{E}}$  and  $W_{\mathcal{D}}$

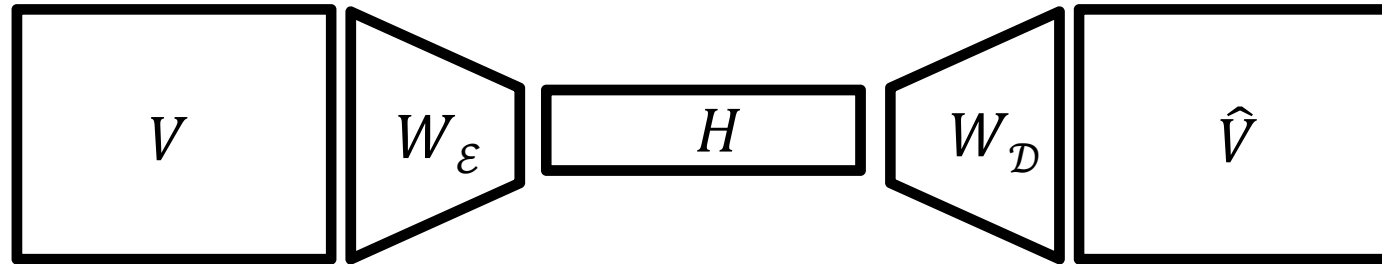
# Nonnegative Autoencoder (NAE)



1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

- How can one adjust the AE to simulate NMF?
- How can one achieve nonnegativity?
- How can one incorporate musical knowledge?
- ...

# Nonnegative Autoencoder (NAE)

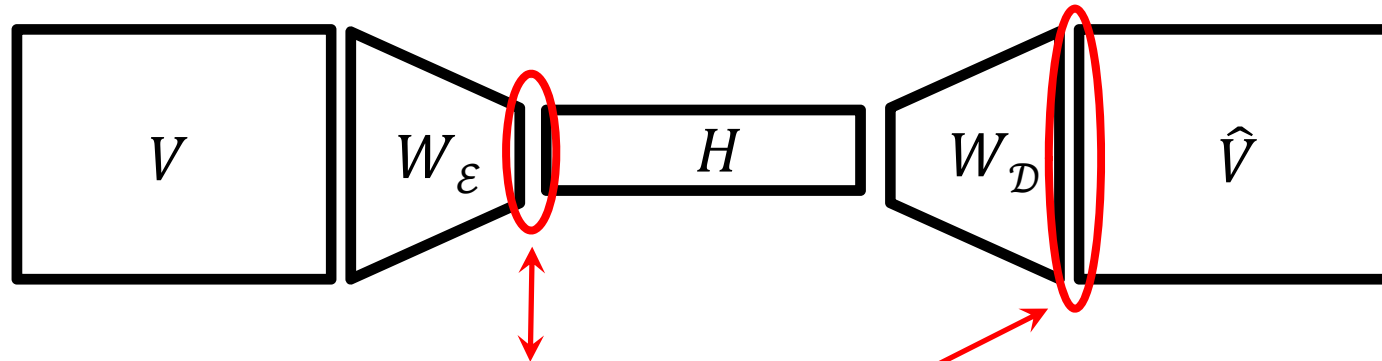


1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

$$\mathcal{L}(V, \hat{V}) = \|V - \hat{V}\|^2$$

- **Loss function:** same as in NMF

# Nonnegative Autoencoder (NAE)

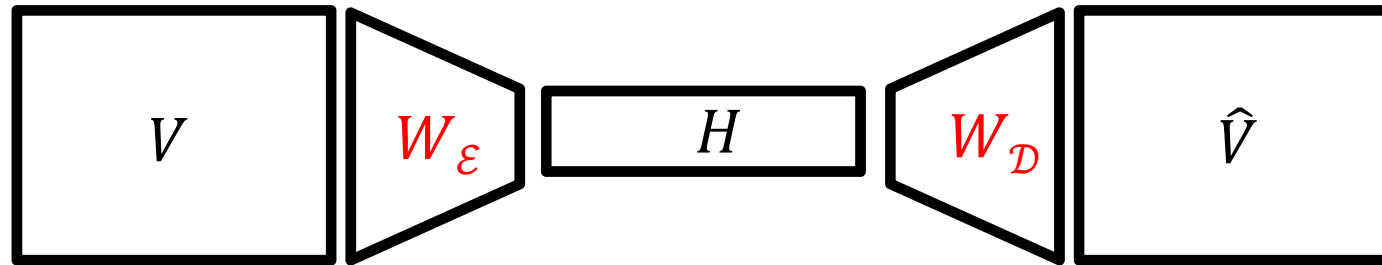


1. Layer:  $H = \max(W_\epsilon V, 0)$
2. Layer:  $\hat{V} = \max(W_D H, 0)$

$$\mathcal{L}(V, \hat{V}) = \|V - \hat{V}\|^2$$

- Loss function: same as in NMF
- Activation function (**ReLU**) makes  $H$  and  $\hat{V}$  nonnegative

# Nonnegative Autoencoder (NAE)

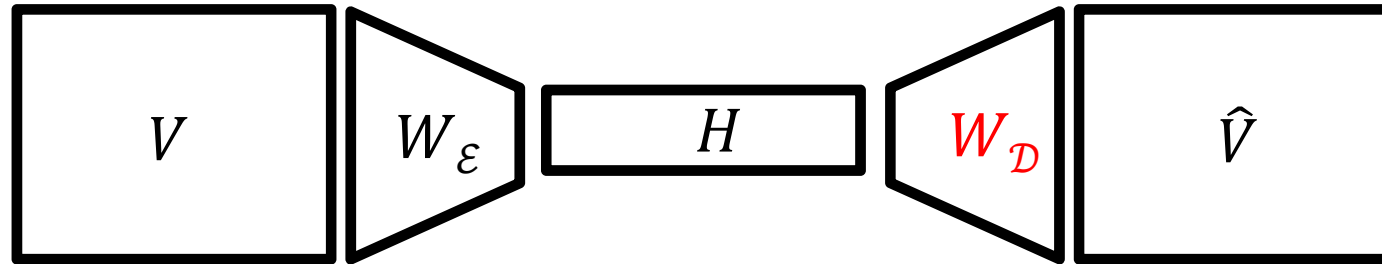


1. Layer:  $H = \max(W_\epsilon V, 0)$
  2. Layer:  $\hat{V} = \max(W_D H, 0)$
- $\mathcal{L}(V, \hat{V}) = \|V - \hat{V}\|^2$

$$W_D \leftarrow \max\left(W_D - \gamma \frac{\partial \mathcal{L}}{\partial W_D}, 0\right)$$

- Loss function: same as in NMF
- Activation function (ReLU) makes  $H$  and  $\hat{V}$  nonnegative
- **Projected gradient descent** can be used to keep  $W_D$  (and  $W_\epsilon$ ) nonnegative

# Musical Constraints



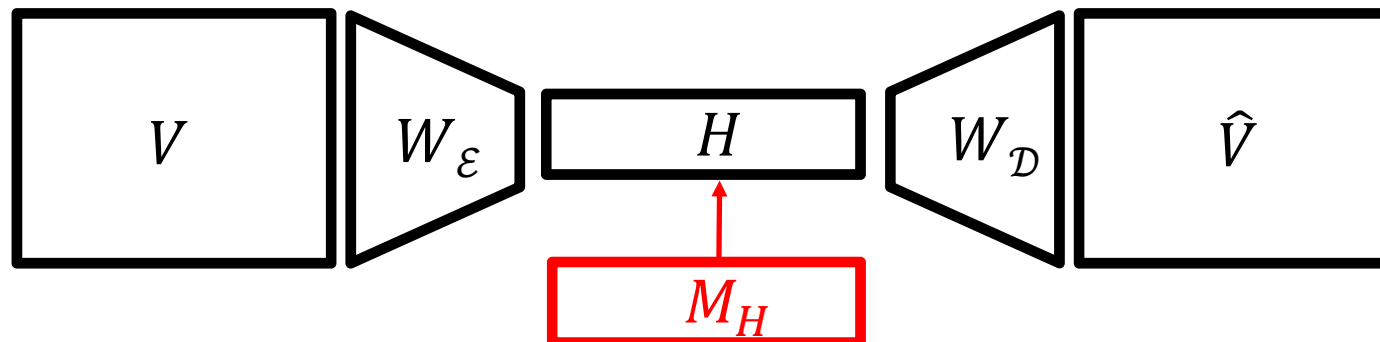
$$H = \max(W_\epsilon V, 0)$$

$$\hat{V} = \max(W_D H, 0)$$

- **Template constraints:** Project certain entries in  $W_D$  to zero values (using projected gradient decent)

# Musical Constraints

Ewert, Sandler: Structured Dropout for Weak Label and Multi-Instance Learning and Its Application to Score-Informed Source Separation. Proc. ICASSP, 2017.



$$H' = H \odot M_H$$

$$\hat{V} = \max(W_D H', 0)$$

- Template constraints: Project certain entries in  $W_D$  to zero values (using projected gradient decent)
- **Activation constraints: Use structured dropout by applying pointwise multiplication with binary mask  $M_H$**

# NAE with Multiplicative Update Rules

- Multiplicative update rules in NMF:
  - Preserve nonnegativity
  - Lead to fast convergence
- Question: Can one introduce multiplicative update rules to train network weights for NAE?
- Use in additive gradient descent

$$W^{(\ell+1)} = W^{(\ell)} - \gamma \cdot \frac{\partial \mathcal{L}}{\partial W}$$

a suitable (adaptive) learning rate  $\gamma$ .



# NAE with Multiplicative Update Rules

- Encoder:

$$H = W_{\mathcal{E}}V$$

- Structured Dropout:

$$H' = H \odot M_H$$

- Decoder:

$$\hat{V} = W_{\mathcal{D}}H'$$

# NAE with Multiplicative Update Rules

- Encoder:

$$H = W_{\mathcal{E}}V$$

$$W_{\mathcal{E},rk}^{(\ell+1)} = W_{\mathcal{E},rk}^{(\ell)} \cdot \frac{\left( \left( (W_{\mathcal{D}}^{\top} V) \odot M_H \right) V^{\top} \right)_{rk}}{\left( \left( (W_{\mathcal{D}}^{\top} W_{\mathcal{D}} H'^{(\ell)}) \odot M_H \right) V^{\top} \right)_{rk}}$$

- Structured Dropout:

$$H' = H \odot M_H$$

- Decoder:

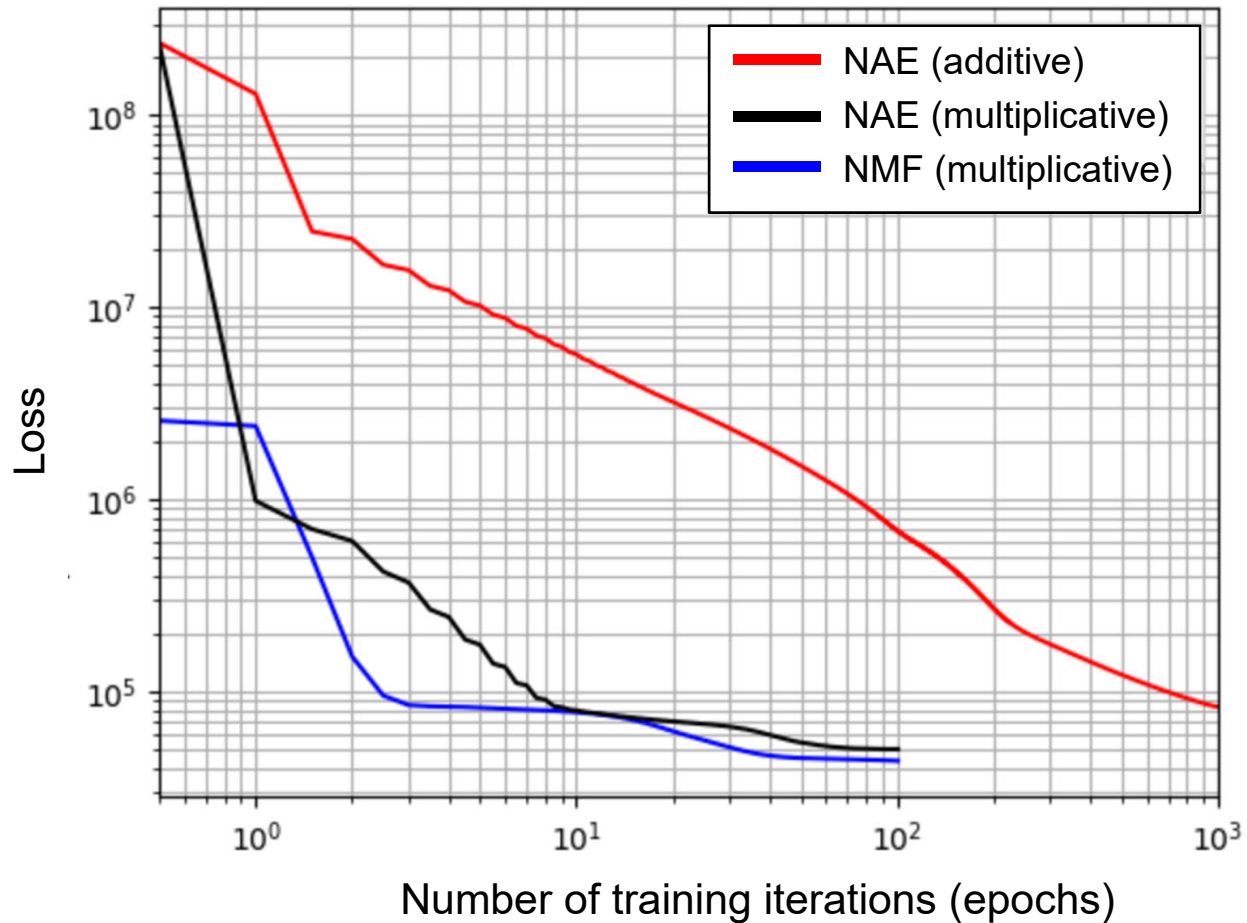
$$\hat{V} = W_{\mathcal{D}}H'$$

$$W_{\mathcal{D},kr}^{(\ell+1)} = W_{\mathcal{D},kr}^{(\ell)} \cdot \frac{(V H'^{\top})_{kr}}{(W_{\mathcal{D}}^{(\ell)} H' H'^{\top})_{kr}}$$

Similar idea and computation as for NMF.

Zunner: Neural Networks with Nonnegativity Constraints for Decomposing Music Recordings. Master Thesis, FAU, 2021.

# Approximation Loss



Zunner: Neural Networks with Nonnegativity Constraints for Decomposing Music Recordings. Master Thesis, FAU, 2021.

# Conclusions (NAE)

- Simulation of NMF:
  - Decoder corresponds to NMF templates
  - Encoder learns a kind of pseudo-inverse
  - Code corresponds to NMF activations
- Nonnegativity can be achieved via
  - activation function (ReLU)
  - projected gradient descent
  - multiplicative update rules
- Musical knowledge can be integrated via
  - removing network weights (template constraints)
  - structured dropout (activation constraints)

# Outlook

- More complex networks
  - Deeper networks (more layers)
  - Different layer types (CNN, RNN, ...) and activation functions
  - Modification of loss function and regularization terms
- Understanding encoder – decoder relationship
  - Nonnegativity
  - Pseudo-inverse
- Update rules
  - Constraints and conversion issues
  - Adaptive learning rates and projected gradient descent

# Audio Mosaicing (Style Transfer)

Target signal: Beatles–Let it be



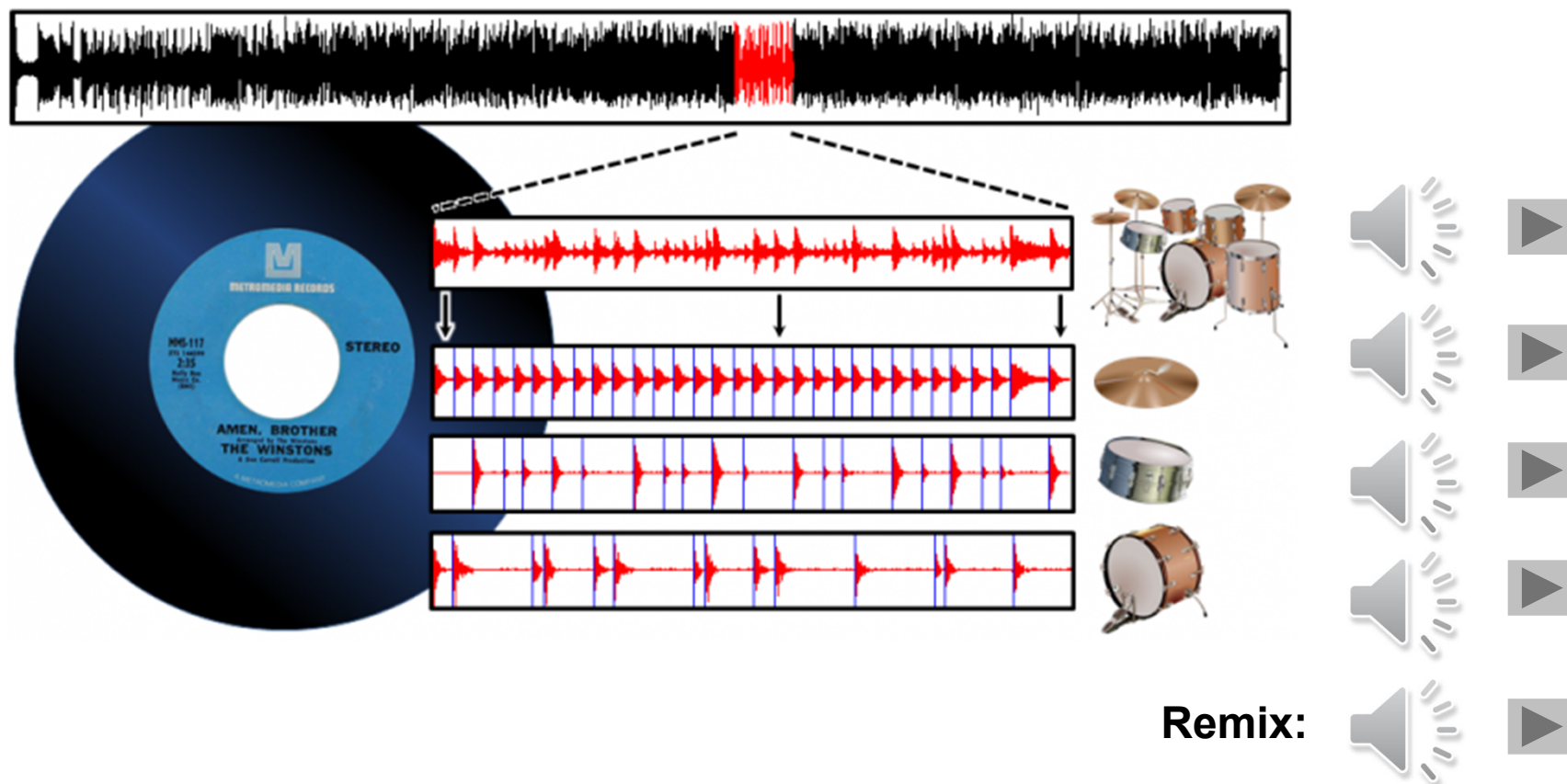
Source signal: Bees



Mosaic signal: **Let it Bee**

Driedger, Prätzlich, Müller: Let It Bee – Towards NMF-Inspired Audio Mosaicing, ISMIR 2015..

# Informed Drum-Sound Decomposition



Dittmar, Müller: Reverse Engineering the Amen Break – Score-Informed Separation and Restoration Applied to Drum Recordings, IEEE/ACM TASLP, 2016.

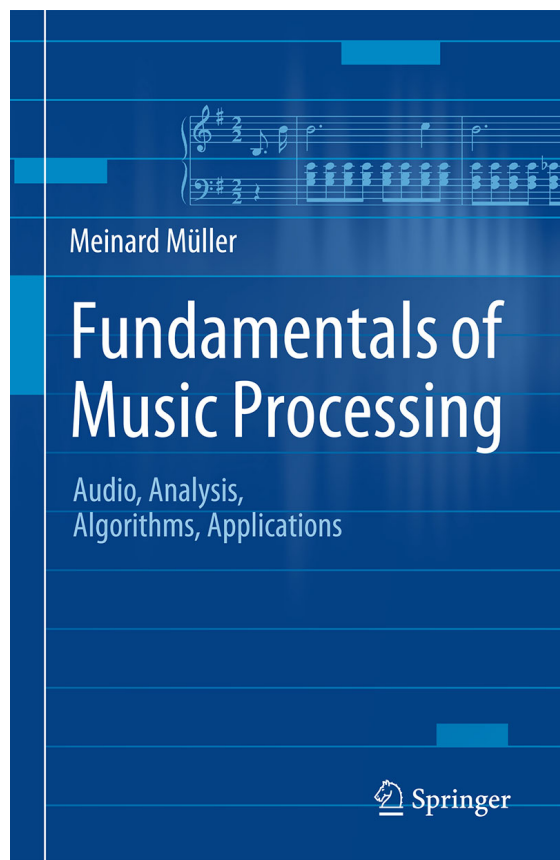
Suárez: DNN-Based Matrix Factorization with Applications to Drum Sound Decomposition. Master Thesis, FAU, 2020.

# Reconstruction of Sound Events

- Reconstruction via spectral masking (Wiener filtering)
- Alternative: Resynthesis approach
- Differentiable Digital Signal Processing (DDSP) combines classical DSP and deep learning
- Generative adversarial networks may help to reduce the artifacts



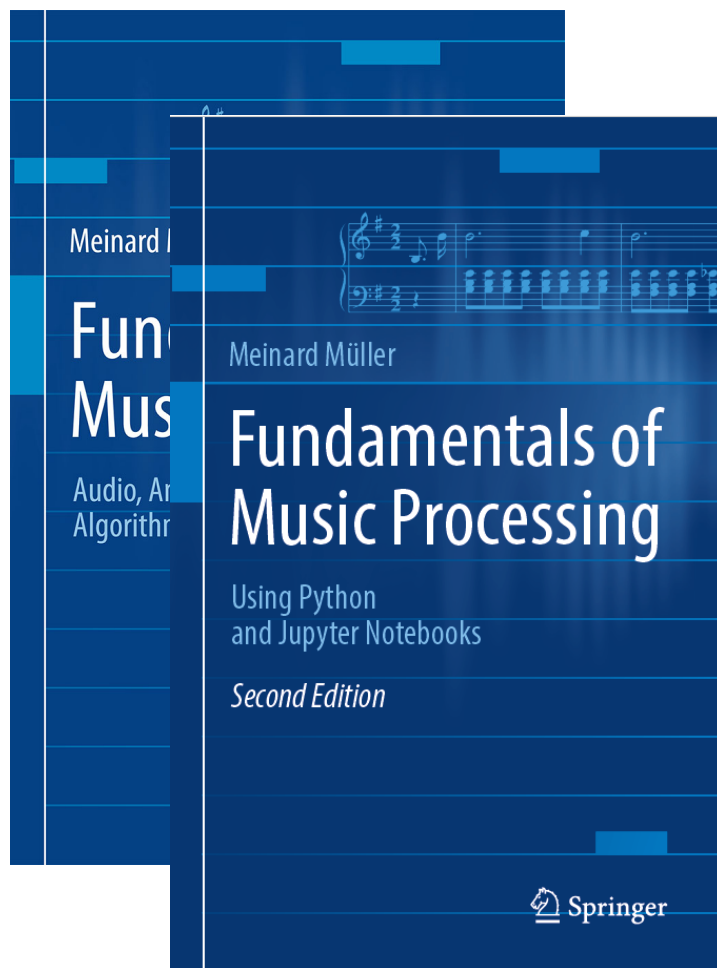
# Fundamentals of Music Processing (FMP)



Meinard Müller  
Fundamentals of Music Processing  
Audio, Analysis, Algorithms, Applications  
Springer, 2015

Accompanying website:  
[www.music-processing.de](http://www.music-processing.de)

# Fundamentals of Music Processing (FMP)

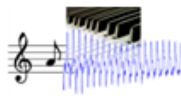

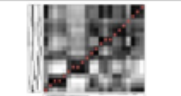


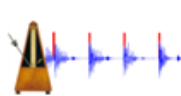
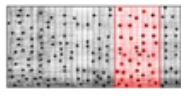
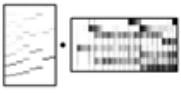


Meinard Müller  
Fundamentals of Music Processing  
Audio, Analysis, Algorithms, Applications  
Springer, 2015

Accompanying website:  
[www.music-processing.de](http://www.music-processing.de)

**2nd edition**  
**Meinard Müller**  
**Fundamentals of Music Processing**  
**Using Python and Jupyter Notebooks**  
**Springer, 2021**

# Fundamentals of Music Processing (FMP)

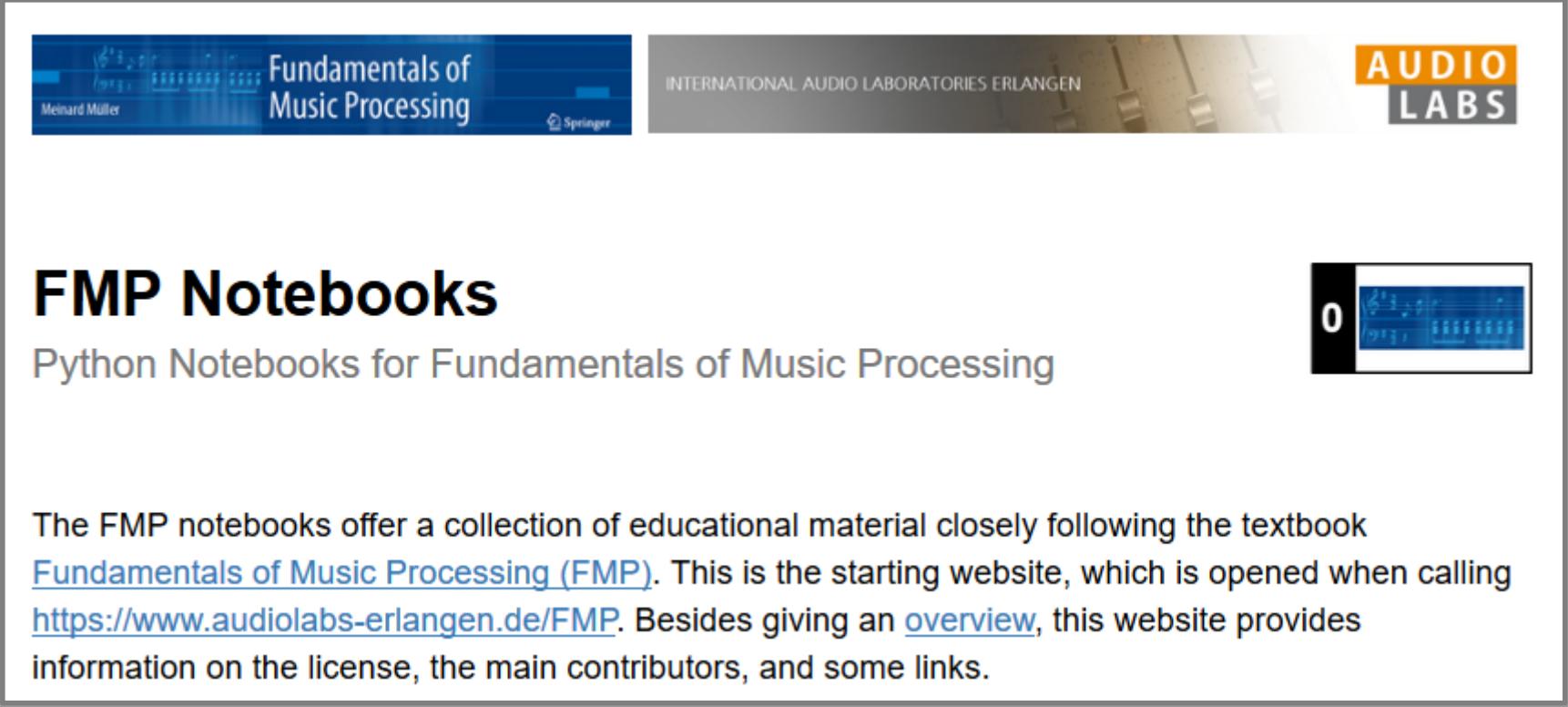
Chapter	Music Processing Scenario
1	 <b>Music Representations</b>
2	 <b>Fourier Analysis of Signals</b>
3	 <b>Music Synchronization</b>
4	 <b>Music Structure Analysis</b>
5	 <b>Chord Recognition</b>
6	 <b>Tempo and Beat Tracking</b>
7	 <b>Content-Based Audio Retrieval</b>
8	 <b>Musically Informed Audio Decomposition</b>

Meinard Müller  
Fundamentals of Music Processing  
Audio, Analysis, Algorithms, Applications  
Springer, 2015

Accompanying website:  
[www.music-processing.de](http://www.music-processing.de)

2nd edition  
Meinard Müller  
Fundamentals of Music Processing  
Using Python and Jupyter Notebooks  
Springer, 2021

# FMP Notebooks: Education & Research



The screenshot shows the header of the FMP Notebooks website. On the left, there is a blue banner for the book "Fundamentals of Music Processing" by Meinard Müller, published by Springer. In the center, it says "INTERNATIONAL AUDIO LABORATORIES ERLANGEN". On the right, there is the "AUDIO LABS" logo. Below the header, the main content area features the title "FMP Notebooks" in a large, bold, black font. To the right of the title is a small icon of a notebook with a blue cover and a white page. Below the title, the subtitle "Python Notebooks for Fundamentals of Music Processing" is displayed in a smaller, grey font. The main text paragraph describes the notebooks as educational material following the textbook "Fundamentals of Music Processing (FMP)", providing a starting website and an overview link.

## FMP Notebooks

Python Notebooks for Fundamentals of Music Processing

The FMP notebooks offer a collection of educational material closely following the textbook [Fundamentals of Music Processing \(FMP\)](#). This is the starting website, which is opened when calling <https://www.audiolabs-erlangen.de/FMP>. Besides giving an [overview](#), this website provides information on the license, the main contributors, and some links.

<https://www.audiolabs-erlangen.de/FMP>

# References (FMP Notebooks)

- Meinard Müller: Fundamentals of Music Processing – Using Python and Jupyter Notebooks. 2nd Edition, Springer, 2021.  
<https://www.springer.com/gp/book/9783030698072>
- Meinard Müller and Frank Zalkow: libfmp: A Python Package for Fundamentals of Music Processing. Journal of Open Source Software (JOSS), 6(63): 1–5, 2021.  
<https://joss.theoj.org/papers/10.21105/joss.03326>
- Meinard Müller: An Educational Guide Through the FMP Notebooks for Teaching and Learning Fundamentals of Music Processing. Signals, 2(2): 245–285, 2021.  
<https://www.mdpi.com/2624-6120/2/2/18>
- Meinard Müller and Frank Zalkow: FMP Notebooks: Educational Material for Teaching and Learning Fundamentals of Music Processing. Proc. International Society for Music Information Retrieval Conference (ISMIR): 573–580, 2019.  
<https://zenodo.org/record/3527872#.YOhEQOgzaUk>
- Meinard Müller, Brian McFee, and Katherine Kinnaid: Interactive Learning of Signal Processing Through Music: Making Fourier Analysis Concrete for Students. IEEE Signal Processing Magazine, 38(3): 73–84, 2021.  
<https://ieeexplore.ieee.org/document/9418542>

# Resources (Group Meinard Müller)

- FMP Notebooks:

<https://www.audiolabs-erlangen.de/FMP>

- libfmp:

<https://github.com/meinardmueller/libfmp>

- synctoolbox:

<https://github.com/meinardmueller/synctoolbox>

- libtsm:

<https://github.com/meinardmueller/libtsm>

- Preparation Course Python (PCP) Notebooks:

<https://www.audiolabs-erlangen.de/resources/MIR/PCP/PCP.html>

<https://github.com/meinardmueller/PCP>

# Resources

- librosa:  
<https://librosa.org/>
- madmom:  
<https://github.com/CPJKU/madmom>
- Essentia Python tutorial:  
[https://essentia.upf.edu/essentia\\_python\\_tutorial.html](https://essentia.upf.edu/essentia_python_tutorial.html)
- mirdata:  
<https://github.com/mir-dataset-loaders/mirdata>
- open-unmix:  
<https://github.com/sigsep/open-unmix-pytorch>
- Open Source Tools & Data for Music Source Separation:  
<https://source-separation.github.io/tutorial/landing.html>



# Thanks

- Yigitcan Özer (PhD student)
- Michael Krause (PhD student)
- Tim Zunner (Master Thesis 2021)
- Edgar Suárez Guarnizo (Master Thesis 2020)
- Christian Dittmar (PhD 2018, Fraunhofer IIS)



# References (NMF, NAE)

- Daniel Lee and Sebastian Seung: **Algorithms for Non-Negative Matrix Factorization**. Proc. NIPS, 2000.
- Sebastian Ewert and Meinard Müller: **Using Score-Informed Constraints for NMF-Based Source Separation**. Proc. ICASSP, 2012.
- Paris Smaragdis and Shrikant Venkataramani: **A Neural Network Alternative to Non-Negative Audio Models**. Proc. ICASSP, 2017.
- Sebastian Ewert and Mark B. Sandler: **Structured Dropout for Weak Label and Multi-Instance Learning and Its Application to Score-Informed Source Separation**. Proc. ICASSP, 2017.
- Tim Zunner: **Neural Networks with Nonnegativity Constraints for Decomposing Music Recordings**. Master Thesis, FAU, 2021.
- Edgar Andrés Suárez Guarnizo: **DNN-Based Matrix Factorization with Applications to Drum Sound Decomposition**. Master Thesis, FAU, 2020.