

Guest Lecture  
Sound and Music Computing (CS4347/CS5647)  
National University of Singapore

## Nonnegative Autoencoders with Applications to Music Audio Decomposing

**Meinard Müller**

International Audio Laboratories Erlangen  
meinard.mueller@audiolabs-erlangen.de

12.09.2022



## Meinard Müller



- Mathematics (Diplom/Master)  
Computer Science (PhD)  
Information Retrieval (Habilitation)

- Since 2012: Professor  
Semantic Audio Processing

- Former President of the International Society for  
Music Information Retrieval (MIR)

- IEEE Fellow for contributions to  
Music Signal Processing

## Meinard Müller: Research Group Semantic Audio Processing



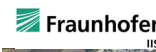
- Michael Krause
- Yigitcan Özer
- Simon Schwär
- Peter Meier (external)



- Christof Weiß
- Sebastian Rosenzweig
- Frank Zalkow
- Christian Dittmar
- Stefan Balke
- Jonathan Driedger
- Thomas Prätzlich
- ...



## International Audio Laboratories Erlangen



- Fraunhofer Institute for  
Integrated Circuits IIS
- Largest Fraunhofer  
institute with  
≈ 1000 members
- Applied research for  
sensor, audio, and  
media technology

- Friedrich-Alexander  
Universität Erlangen-  
Nürnberg (FAU)
- One of Germany's  
largest universities with  
≈ 40,000 students
- Strong Technical  
Faculty

## International Audio Laboratories Erlangen

**Audio**

## International Audio Laboratories Erlangen

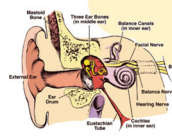
Audio Coding



3D Audio



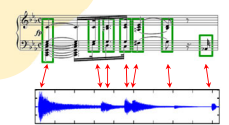
**Audio**



Psychoacoustics



Internet of Things



Music Processing

## International Audio Laboratories Erlangen

- Prof. Dr. Jürgen Herre  
Audio Coding
- Prof. Dr. Bernd Edler  
Audio Signal Analysis
- Prof. Dr. Meinard Müller  
Semantic Audio Processing
- Prof. Dr. Emanuël Habets  
Spatial Audio Signal Processing
- Prof. Dr. Nils Peters  
Audio Signal Processing
- Dr. Stefan Turowski  
Coordinator AudioLabs-FAU



## Source Separation

- Decomposition of audio stream into different sound sources
- Central task in digital signal processing
- “Cocktail party effect”

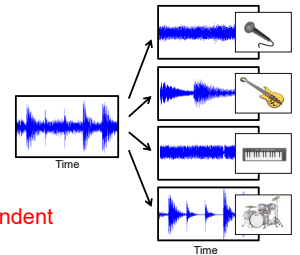


## Source Separation

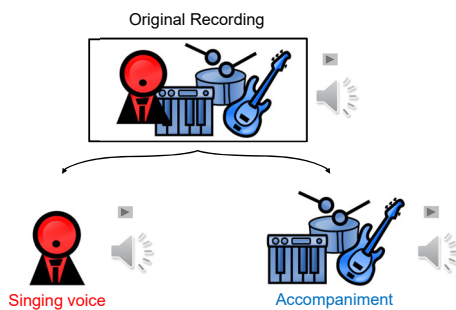
- Decomposition of audio stream into different sound sources
- Central task in digital signal processing
- “Cocktail party effect”
- Several input signals
- Sources are assumed to be statistically independent

## Source Separation (Music)

- Main melody, accompaniment, drum track
- Instrumental voices
- Individual note events
- Only mono or stereo
- Sources are often highly dependent

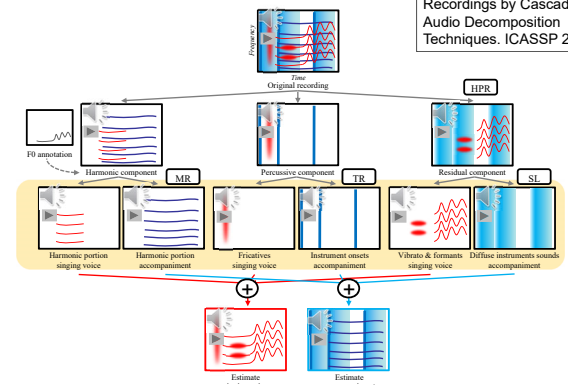


## Singing Voice Extraction



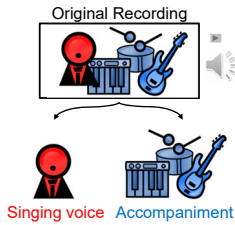
## Singing Voice Extraction

**Traditional Approach**  
Driedger, Müller: Extracting Singing Voice from Music Recordings by Cascading Audio Decomposition Techniques. ICASSP 2015.



## Singing Voice Extraction

Deep learning has led to breakthrough



**DL-Based Approach**  
Stöter, Ulich Luitkus, Mitsufuji: Open-Unmix – A Reference Implementation for Music Source Separation. JOSS 2019.



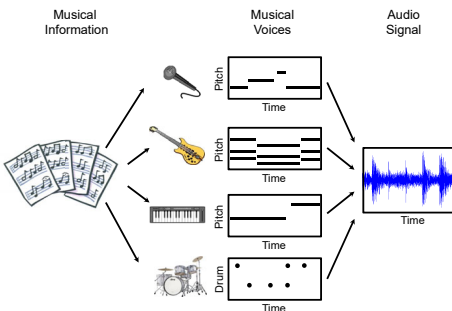
## Score-Informed Source Separation

Exploit musical score to support decomposition process



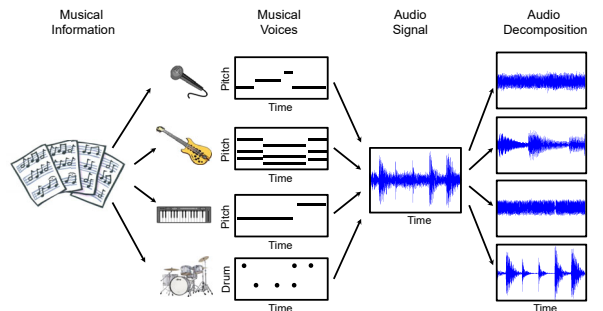
## Score-Informed Source Separation

Exploit musical score to support decomposition process



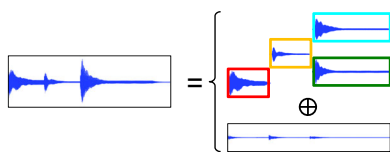
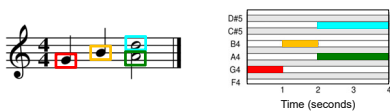
## Score-Informed Source Separation

Exploit musical score to support decomposition process



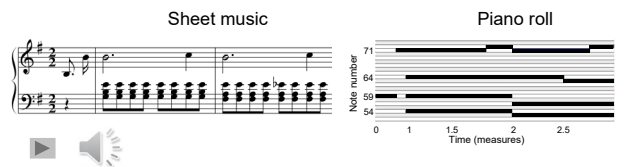
## Score-Informed Audio Decomposition

Notewise decomposition

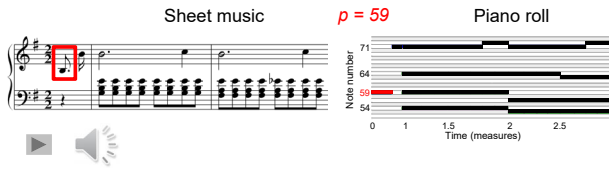


**Prior Knowledge**  
Ewert, Pardo, Müller, Plumbley: Score-Informed Source Separation for Musical Audio Recordings. IEEE SPM, 2014.

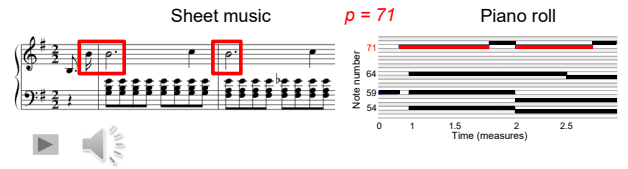
## Score-Informed Audio Decomposition



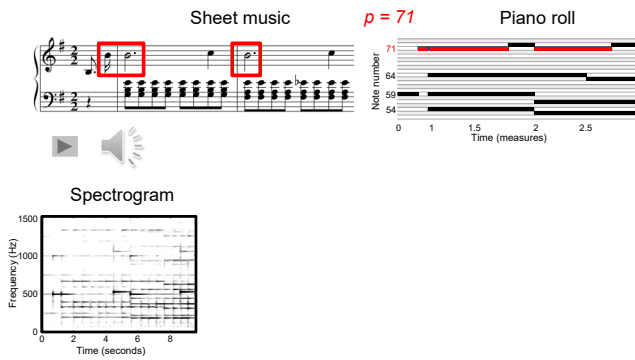
## Score-Informed Audio Decomposition



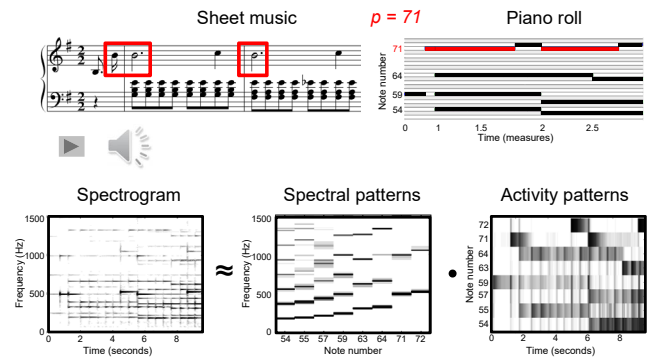
## Score-Informed Audio Decomposition



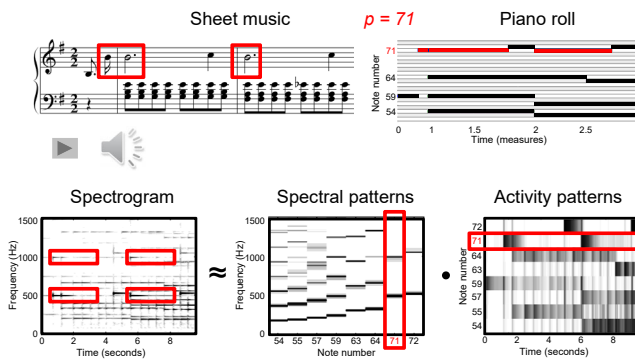
## Score-Informed Audio Decomposition



## Score-Informed Audio Decomposition

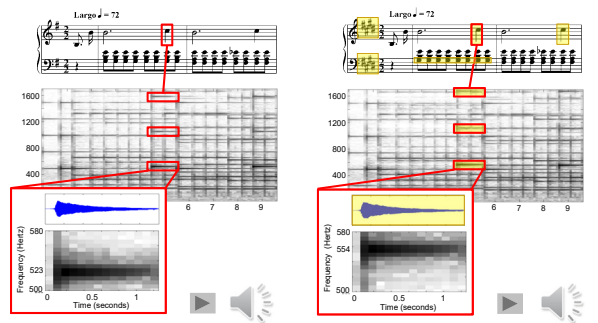


## Score-Informed Audio Decomposition

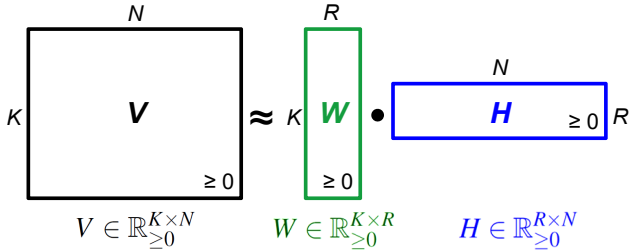


## Score-Informed Audio Decomposition

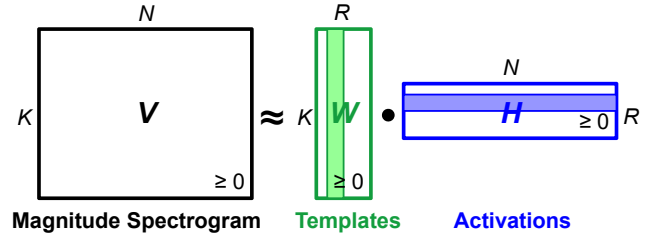
Application: Audio editing



## Nonnegative Matrix Factorization (NMF)



## Nonnegative Matrix Factorization (NMF)



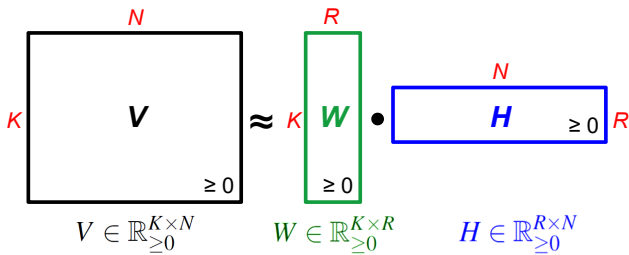
**Templates:** Pitch + Timbre

“How does it sound”

**Activations:** Onset time + Duration

“When does it sound”

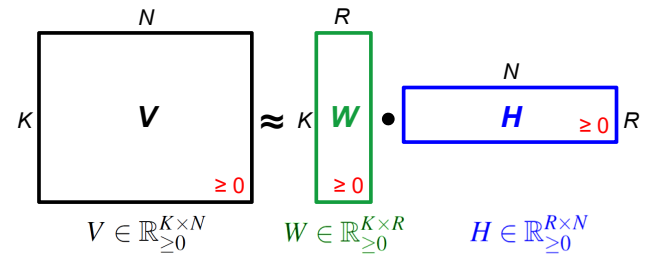
## Nonnegative Matrix Factorization (NMF)



### Dimensionality reduction

- $K, N$  typically much larger than  $R$  (maximal rank)
- Example:  $N = 1000, K = 500, R = 20$   
 $K \times N = 500,000, K \times R = 10,000, R \times N = 20,000$

## Nonnegative Matrix Factorization (NMF)



### Nonnegativity:

- Prevents mutual cancellation of template vectors
- Encourages semantically meaningful decomposition

## NMF Optimization

### Optimization problem:

Given  $V \in \mathbb{R}_{\geq 0}^{K \times N}$  and rank parameter  $R$  minimize

$$\|V - WH\|^2$$

with respect to  $W \in \mathbb{R}_{\geq 0}^{K \times R}$  and  $H \in \mathbb{R}_{\geq 0}^{R \times N}$ .

Optimization not easy:

- Nonnegativity constraints
- Nonconvexity when jointly optimizing  $W$  and  $H$

**Strategy:** Iteratively optimize  $W$  and  $H$  via gradient descent

## NMF Optimization

### Computation of gradient with respect to $H$ (fixed $W$ )

$$D := RN$$

$$\phi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

$$\phi^W(H) := \|V - WH\|^2$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

$$H_{\rho v}$$

$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

## NMF Optimization

Computation of gradient with respect to  $H$  (fixed  $W$ )

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

$$\varphi^W(H) := \|V - WH\|^2$$

$$\frac{\partial \varphi^W}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K \sum_{n=1}^N (V_{kn} - \sum_{r=1}^R W_{kr} H_{rn})^2 \right)}{\partial H_{\rho v}}$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

$$H_{\rho v}$$

$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

## NMF Optimization

Computation of gradient with respect to  $H$  (fixed  $W$ )

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

$$\varphi^W(H) := \|V - WH\|^2$$

$$\frac{\partial \varphi^W}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K \sum_{n=1}^N (V_{kn} - \sum_{r=1}^R W_{kr} H_{rn})^2 \right)}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K (V_{kv} - \sum_{r=1}^R W_{kr} H_{rv})^2 \right)}{\partial H_{\rho v}}$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

$$H_{\rho v}$$

$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

Summand that does not depend on  $H_{\rho v}$  must be zero

## NMF Optimization

Computation of gradient with respect to  $H$  (fixed  $W$ )

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

$$\varphi^W(H) := \|V - WH\|^2$$

$$\frac{\partial \varphi^W}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K \sum_{n=1}^N (V_{kn} - \sum_{r=1}^R W_{kr} H_{rn})^2 \right)}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K (V_{kv} - \sum_{r=1}^R W_{kr} H_{rv})^2 \right)}{\partial H_{\rho v}}$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

$$H_{\rho v}$$

$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

Apply chain rule from calculus

## NMF Optimization

Computation of gradient with respect to  $H$  (fixed  $W$ )

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

$$\varphi^W(H) := \|V - WH\|^2$$

$$\frac{\partial \varphi^W}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K \sum_{n=1}^N (V_{kn} - \sum_{r=1}^R W_{kr} H_{rn})^2 \right)}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K (V_{kv} - \sum_{r=1}^R W_{kr} H_{rv})^2 \right)}{\partial H_{\rho v}}$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

$$H_{\rho v}$$

$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

Rearrange summands

## NMF Optimization

Computation of gradient with respect to  $H$  (fixed  $W$ )

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

$$\varphi^W(H) := \|V - WH\|^2$$

$$\frac{\partial \varphi^W}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K \sum_{n=1}^N (V_{kn} - \sum_{r=1}^R W_{kr} H_{rn})^2 \right)}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K (V_{kv} - \sum_{r=1}^R W_{kr} H_{rv})^2 \right)}{\partial H_{\rho v}}$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

$$H_{\rho v}$$

$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

Introduce transposed  $W^T$

## NMF Optimization

Computation of gradient with respect to  $H$  (fixed  $W$ )

$$D := RN$$

$$\varphi^W : \mathbb{R}^D \rightarrow \mathbb{R}$$

$$\varphi^W(H) := \|V - WH\|^2$$

$$\frac{\partial \varphi^W}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K \sum_{n=1}^N (V_{kn} - \sum_{r=1}^R W_{kr} H_{rn})^2 \right)}{\partial H_{\rho v}} = \frac{\partial \left( \sum_{k=1}^K (V_{kv} - \sum_{r=1}^R W_{kr} H_{rv})^2 \right)}{\partial H_{\rho v}}$$

Variables

$$H \in \mathbb{R}^{R \times N}$$

$$H_{\rho v}$$

$$\rho \in [1 : R]$$

$$v \in [1 : N]$$

$= 2((W^T WH)_{\rho v} - (W^T V)_{\rho v})$

## NMF Optimization

### Gradient descent

Initialization  $H^{(0)} \in \mathbb{R}^{R \times N}$

Iteration for  $\ell = 0, 1, 2, \dots$

$$H_{rn}^{(\ell+1)} = H_{rn}^{(\ell)} - \gamma_{rn}^{(\ell)} \cdot \left( (W^T W H^{(\ell)})_{rn} - (W^T V)_{rn} \right)$$

with suitable learning rate  $\gamma_{rn}^{(\ell)} \geq 0$

## NMF Optimization

### Gradient descent

Initialization  $H^{(0)} \in \mathbb{R}^{R \times N}$

Iteration for  $\ell = 0, 1, 2, \dots$

$$H_{rn}^{(\ell+1)} = H_{rn}^{(\ell)} - \gamma_{rn}^{(\ell)} \cdot \left( (W^T W H^{(\ell)})_{rn} - (W^T V)_{rn} \right)$$

with suitable learning rate  $\gamma_{rn}^{(\ell)} \geq 0$

Issues:

- How to do the initialization?
- How to choose the learning rate?
- How to ensure nonnegativity?

## NMF Optimization

### Gradient descent

Initialization  $H^{(0)} \in \mathbb{R}^{R \times N}$

Iteration for  $\ell = 0, 1, 2, \dots$

$$H_{rn}^{(\ell+1)} = H_{rn}^{(\ell)} - \gamma_{rn}^{(\ell)} \cdot \left( (W^T W H^{(\ell)})_{rn} - (W^T V)_{rn} \right)$$

$$= H_{rn}^{(\ell)} \cdot \frac{(W^T V)_{rn}}{(W^T W H^{(\ell)})_{rn}}$$

Choose adaptive learning rate:

$$\gamma_{rn}^{(\ell)} := \frac{H_{rn}^{(\ell)}}{(W^T W H^{(\ell)})_{rn}}$$

Issues:

- How to do the initialization?
- How to choose the learning rate?
- How to ensure nonnegativity?

## NMF Optimization

### Gradient descent

Initialization  $H^{(0)} \in \mathbb{R}^{R \times N}$

Iteration for  $\ell = 0, 1, 2, \dots$

$$H_{rn}^{(\ell+1)} = H_{rn}^{(\ell)} - \gamma_{rn}^{(\ell)} \cdot \left( (W^T W H^{(\ell)})_{rn} - (W^T V)_{rn} \right)$$

$$= H_{rn}^{(\ell)} \cdot \frac{(W^T V)_{rn}}{(W^T W H^{(\ell)})_{rn}}$$

Choose adaptive learning rate:

$$\gamma_{rn}^{(\ell)} := \frac{H_{rn}^{(\ell)}}{(W^T W H^{(\ell)})_{rn}}$$

Issues:

- How to do the initialization?
- How to choose the learning rate?
- How to ensure nonnegativity?

- Update rule become multiplicative
- Nonnegative values stay nonnegative

## NMF Optimization

**Algorithm:** NMF ( $V \approx WH$ )

**Input:** Nonnegative matrix  $V$  of size  $K \times N$   
Rank parameter  $R \in \mathbb{N}$   
Threshold  $\epsilon$  used as stop criterion

**Output:** Nonnegative template matrix  $W$  of size  $K \times R$   
Nonnegative activation matrix  $H$  of size  $R \times N$

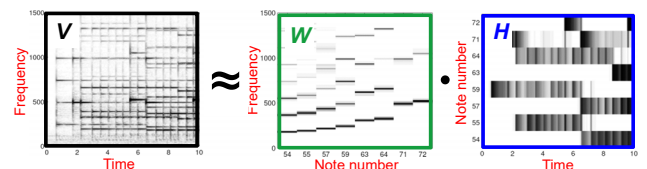
**Procedure:** Define nonnegative matrices  $W^{(0)}$  and  $H^{(0)}$  by some random or informed initialization. Furthermore set  $\ell = 0$ . Apply the following update rules (written in matrix notation):

- (1)  $H^{(\ell+1)} = H^{(\ell)} \odot \left( (W^{(\ell)})^T V \right) \oslash \left( (W^{(\ell)})^T W^{(\ell)} H^{(\ell)} \right)$
- (2)  $W^{(\ell+1)} = W^{(\ell)} \odot \left( V (H^{(\ell+1)})^T \right) \oslash \left( W^{(\ell)} H^{(\ell+1)} (H^{(\ell+1)})^T \right)$
- (3) Increase  $\ell$  by one.

Repeat the steps (1) to (3) until  $\|H^{(\ell)} - H^{(\ell-1)}\| \leq \epsilon$  and  $\|W^{(\ell)} - W^{(\ell-1)}\| \leq \epsilon$  (or until some other stop criterion is fulfilled). Finally, set  $H = H^{(\ell)}$  and  $W = W^{(\ell)}$ .

Lee, Seung: Algorithms for Non-Negative Matrix Factorization. Proc. NIPS, 2000.

## NMF-based Spectrogram Decomposition



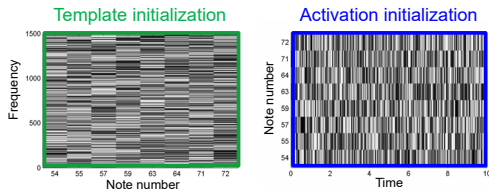
**Templates:** Pitch + Timbre

**Activations:** Onset time + Duration

"How does it sound"

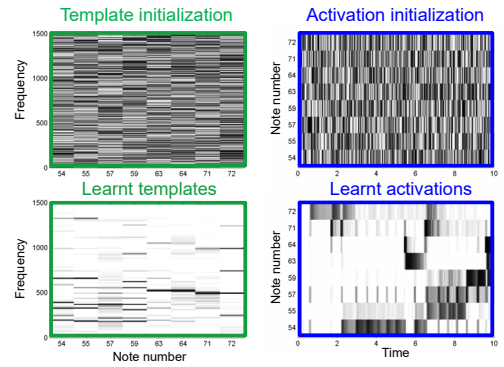
"When does it sound"

## NMF-based Spectrogram Decomposition



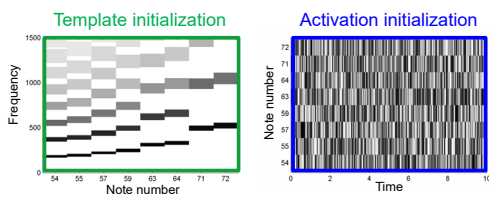
Random initialization

## NMF-based Spectrogram Decomposition



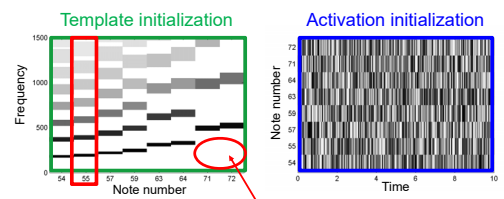
Random initialization → No semantic meaning

## Constrained NMF: Templates



Enforce harmonic structure with zero-valued entries

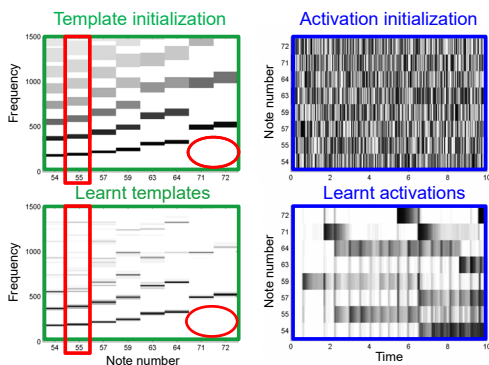
## Constrained NMF: Templates



Template constraint for  $p=55$

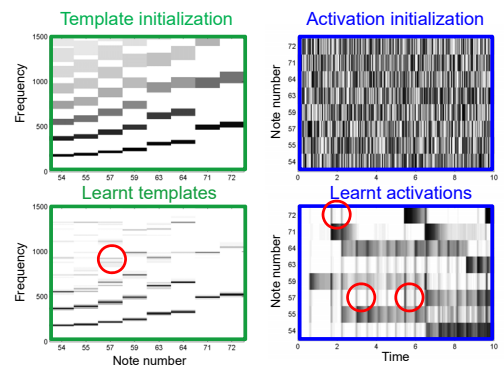
Enforce harmonic structure with zero-valued entries

## Constrained NMF: Templates



Zero-valued entries remain zero-valued entries!

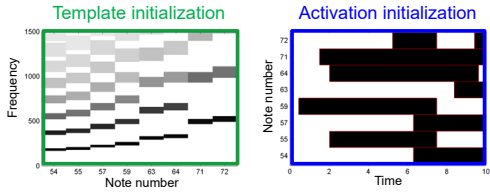
## Constrained NMF: Templates



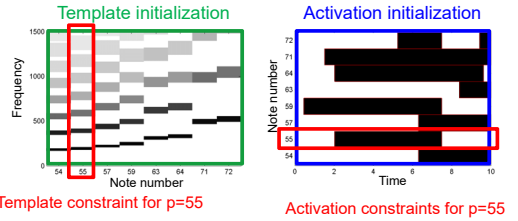
Pitch templates misused to represent onsets



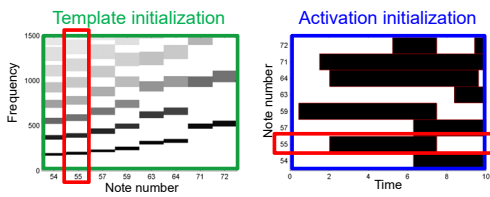
### Constrained NMF: Double Constraints



### Constrained NMF: Double Constraints



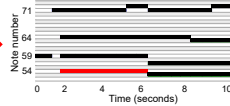
### Constrained NMF: Double Constraints



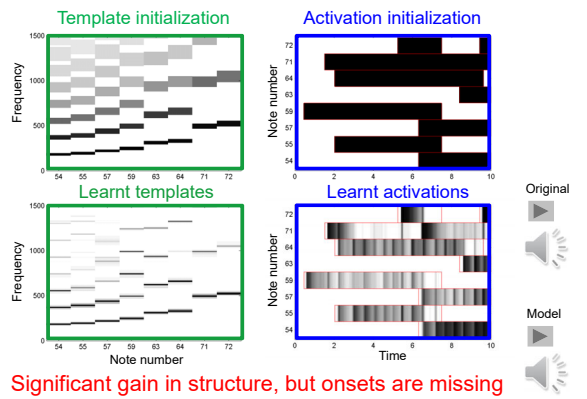
Sheet music



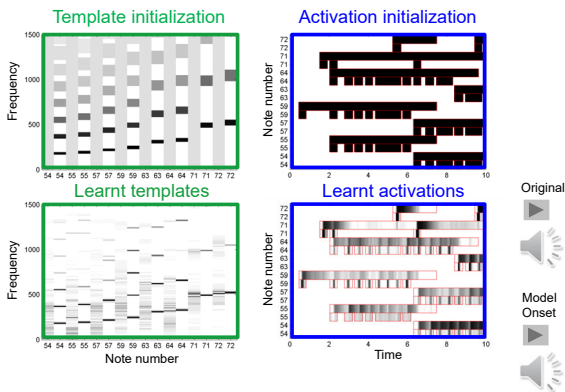
Such information may come from a synchronized score



### Constrained NMF: Double Constraints



### Constrained NMF: Onset Templates

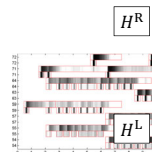


### Score-Informed Audio Decomposition

Application: Separating left and right hands for piano

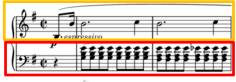


1. Split activation matrix

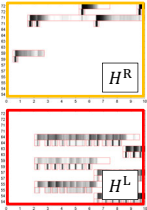


## Score-Informed Audio Decomposition

Application: Separating left and right hands for piano



1. Split activation matrix

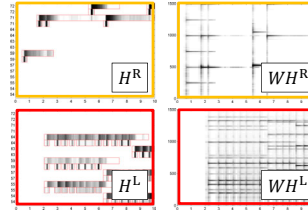


## Score-Informed Audio Decomposition

Application: Separating left and right hands for piano



1. Split activation matrix
2. Model spectrogram for left/right

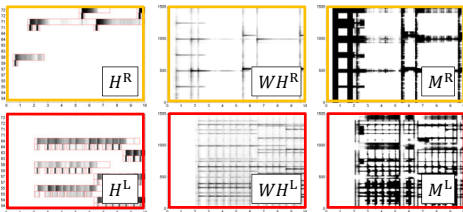


## Score-Informed Audio Decomposition

Application: Separating left and right hands for piano



1. Split activation matrix
2. Model spectrogram for left/right
3. Separation masks for left/right

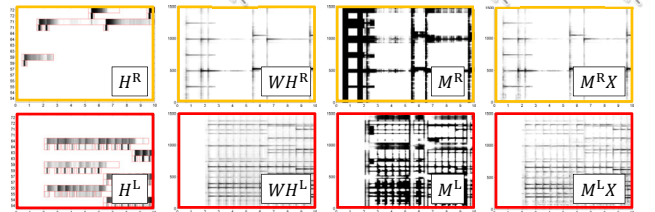


## Score-Informed Audio Decomposition

Application: Separating left and right hands for piano



1. Split activation matrix
2. Model spectrogram for left/right
3. Separation masks for left/right
4. Estimated spectrograms for left/right



## Score-Informed Audio Decomposition

Application: Separating left and right hands for piano

Chopin, Waltz Op. 64, No. 1



Original



Ewert, Müller: Using Score-Informed Constraints for NMF-based Source Separation. Proc. ICASSP, 2012.

Further results available at <http://www.mpi-inf.mpg.de/resources/MIR/ICASSP2012-ScoreInformedNMF/>

## Score-Informed Audio Decomposition

Application: Separating left and right hands for piano

Chopin, Waltz Op. 64, No. 1



Original



Left/right hand



Right hand



Left hand



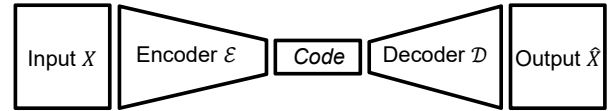
Ewert, Müller: Using Score-Informed Constraints for NMF-based Source Separation. Proc. ICASSP, 2012.

Further results available at <http://www.mpi-inf.mpg.de/resources/MIR/ICASSP2012-ScoreInformedNMF/>

## Conclusions (NMF)

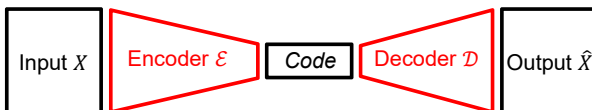
- NMF used for spectrogram decomposition
- Multiplicative update rules make it easy to constrain NMF model via zero initialization
- Exploiting score information to guide separation process (requires score–audio synchronization)
- Application: Separation of arbitrary note groups from given audio recording

## Autoencoder



- Specific type of neural network
- Encoder: Compress input  $X$  into a low-dimensional code
- Decoder: Reconstruct output  $\hat{X}$  from code

## Autoencoder



- Specific type of neural network
- Encoder: Compress input  $X$  into a low-dimensional code
- Decoder: Reconstruct output  $\hat{X}$  from code
- Goal: Learn parameters for encoder and decoder such that output is close to input with respect to some loss function:

$$\mathcal{L}(X, \hat{X}) \approx 0$$

## NMF and Autoencoder (AE)

Smaragdīs, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.

$$\text{NMF} \quad V \approx W \cdot H = \hat{V}$$

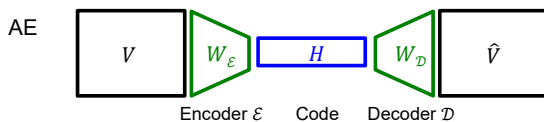
$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$

## NMF and Autoencoder (AE)

Smaragdīs, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.

$$\text{NMF} \quad V \approx W \cdot H = \hat{V}$$

$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$



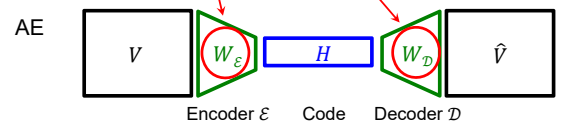
1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

## NMF and Autoencoder (AE)

Smaragdīs, Venkataramani: A Neural Network Alternative to Non-Negative Audio Models, Proc. ICASSP 2017.

$$\text{NMF} \quad V \approx W \cdot H = \hat{V}$$

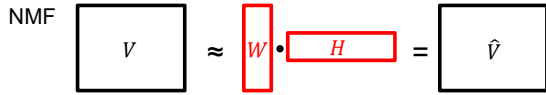
$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$



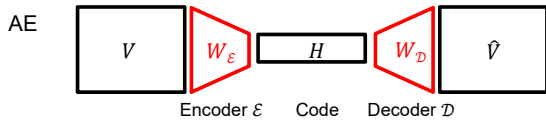
1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

Fully connected network

### NMF and Autoencoder (AE)



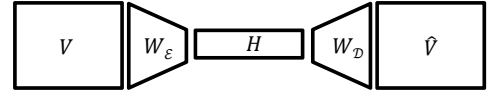
$V \approx WH$  implies  $W^+V \approx H$  with pseudoinverse  $W^+$



1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

NMF: Learn  $H$  and  $W$   
 AE: Learn  $W_\epsilon$  and  $W_D$

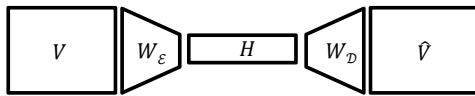
### Nonnegative Autoencoder (NAE)



1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

- How can one adjust the AE to simulate NMF?
- How can one achieve nonnegativity?
- How can one incorporate musical knowledge?
- ...

### Nonnegative Autoencoder (NAE)

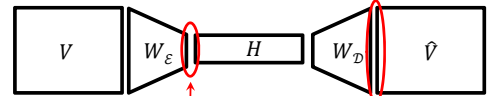


1. Layer:  $H = W_\epsilon V$
2. Layer:  $\hat{V} = W_D H$

$$\mathcal{L}(V, \hat{V}) = \|V - \hat{V}\|^2$$

- Loss function: same as in NMF

### Nonnegative Autoencoder (NAE)

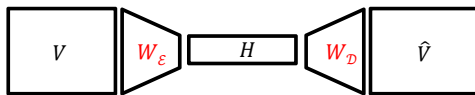


1. Layer:  $H = \max(W_\epsilon V, 0)$
2. Layer:  $\hat{V} = \max(W_D H, 0)$

$$\mathcal{L}(V, \hat{V}) = \|V - \hat{V}\|^2$$

- Loss function: same as in NMF
- Activation function (ReLU) makes  $H$  and  $\hat{V}$  nonnegative

### Nonnegative Autoencoder (NAE)



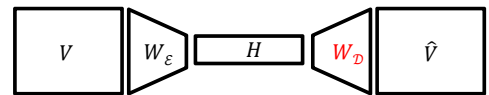
1. Layer:  $H = \max(W_\epsilon V, 0)$
2. Layer:  $\hat{V} = \max(W_D H, 0)$

$$\mathcal{L}(V, \hat{V}) = \|V - \hat{V}\|^2$$

$$W_D \leftarrow \max\left(W_D - \gamma \frac{\partial \mathcal{L}}{\partial W_D}, 0\right)$$

- Loss function: same as in NMF
- Activation function (ReLU) makes  $H$  and  $\hat{V}$  nonnegative
- Projected gradient descent can be used to keep  $W_D$  (and  $W_\epsilon$ ) nonnegative

### Musical Constraints



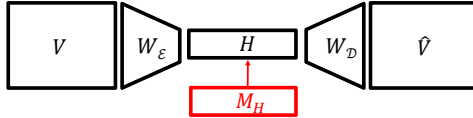
$$H = \max(W_\epsilon V, 0)$$

$$\hat{V} = \max(W_D H, 0)$$

- Template constraints: Project certain entries in  $W_D$  to zero values (using projected gradient decent)

## Musical Constraints

Ewert, Sandler: Structured Dropout for Weak Label and Multi-Instance Learning and Its Application to Score-Informed Source Separation. Proc. ICASSP, 2017.



$$H' = H \odot M_H$$

$$\hat{V} = \max(W_D H', 0)$$

- Template constraints: Project certain entries in  $W_D$  to zero values (using projected gradient decent)
- Activation constraints: Use structured dropout by applying pointwise multiplication with binary mask  $M_H$

## NAE with Multiplicative Update Rules

- Multiplicative update rules in NMF:
  - Preserve nonnegativity
  - Lead to fast convergence
- Question: Can one introduce multiplicative update rules to train network weights for NAE?
- Use in additive gradient descent

$$W^{(\ell+1)} = W^{(\ell)} - \gamma \cdot \frac{\partial \mathcal{L}}{\partial W}$$

a suitable (adaptive) learning rate  $\gamma$ .

## NAE with Multiplicative Update Rules

- Encoder:
 
$$H = W_\epsilon V$$
- Structured Dropout:
 
$$H' = H \odot M_H$$
- Decoder:
 
$$\hat{V} = W_D H'$$

Özer, Hansen, Zünner, Müller: Investigating Nonnegative Autoencoders for Efficient Audio Decomposition. Proc. EUSIPCO, 2022.

## NAE with Multiplicative Update Rules

- Encoder:
 
$$H = W_\epsilon V$$
- Structured Dropout:
 
$$H' = H \odot M_H$$
- Decoder:
 
$$\hat{V} = W_D H'$$

$$W_{\epsilon, rk}^{(\ell+1)} = W_{\epsilon, rk}^{(\ell)} \cdot \frac{\left( (W_D^\top V) \odot M_H \right) V^\top}{\left( (W_D^\top W_D H'^{(\ell)}) \odot M_H \right) V^\top}_{rk}$$

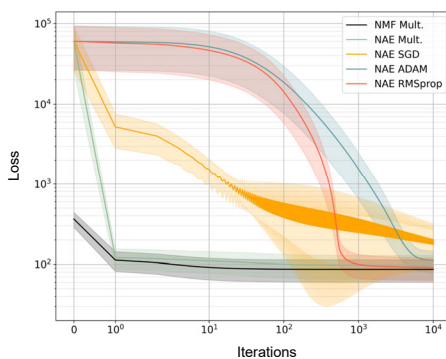
$$W_{D, kr}^{(\ell+1)} = W_{D, kr}^{(\ell)} \cdot \frac{(V H'^\top)_{kr}}{(W_D^\top H' H'^\top)_{kr}}$$

Similar idea and computation as for NMF.

Özer, Hansen, Zünner, Müller: Investigating Nonnegative Autoencoders for Efficient Audio Decomposition. Proc. EUSIPCO, 2022.

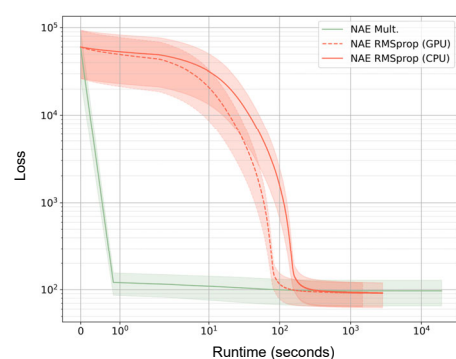
## Approximation Loss

Özer, Hansen, Zünner, Müller: Investigating Nonnegative Autoencoders for Efficient Audio Decomposition. Proc. EUSIPCO, 2022.



## Approximation Loss

Özer, Hansen, Zünner, Müller: Investigating Nonnegative Autoencoders for Efficient Audio Decomposition. Proc. EUSIPCO, 2022.



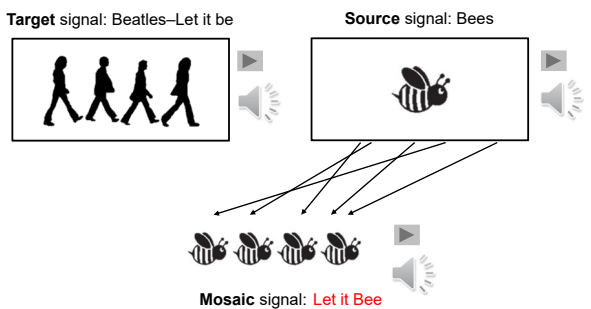
## Conclusions (NAE)

- Simulation of NMF:
  - Decoder corresponds to NMF templates
  - Encoder learns a kind of pseudo-inverse
  - Code corresponds to NMF activations
- Nonnegativity can be achieved via
  - activation function (ReLU)
  - projected gradient descent
  - multiplicative update rules
- Musical knowledge can be integrated via
  - removing network weights (template constraints)
  - structured dropout (activation constraints)

## Outlook

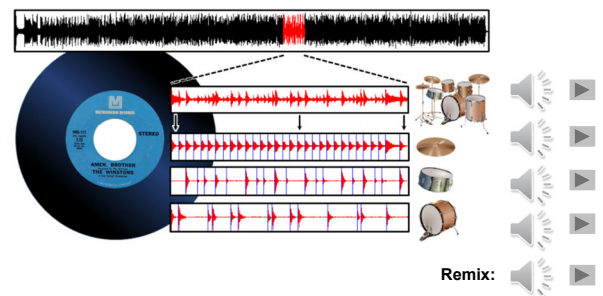
- More complex networks
  - Deeper networks (more layers)
  - Different layer types (CNN, RNN, ...) and activation functions
  - Modification of loss function and regularization terms
- Understanding encoder – decoder relationship
  - Nonnegativity
  - Pseudo-inverse
- Update rules
  - Constraints and convergence issues
  - Adaptive learning rates and projected gradient descent

## Audio Mosaicing (Style Transfer)



Driedger, Prätzlich, Müller: Let It Bee – Towards NMF-Inspired Audio Mosaicing, ISMIR 2015..

## Informed Drum-Sound Decomposition

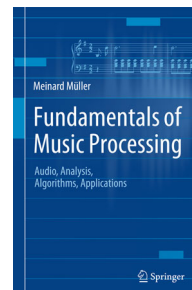


Dittmar, Müller: Reverse Engineering the Amen Break – Score-Informed Separation and Restoration Applied to Drum Recordings, IEEE/ACM TASLP, 2016.

## Reconstruction of Sound Events

- Reconstruction via spectral masking (Wiener filtering)
- Alternative: Resynthesis approach
- Differentiable Digital Signal Processing (DDSP) combines classical DSP and deep learning
- Generative adversarial networks may help to reduce the artifacts

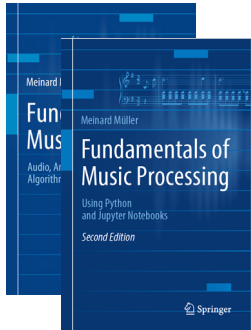
## Fundamentals of Music Processing (FMP)



Meinard Müller  
Fundamentals of Music Processing  
Audio, Analysis, Algorithms, Applications  
Springer, 2015

Accompanying website:  
[www.music-processing.de](http://www.music-processing.de)

## Fundamentals of Music Processing (FMP)



Meinard Müller  
Fundamentals of Music Processing  
Audio, Analysis, Algorithms, Applications  
Springer, 2015

Accompanying website:  
[www.music-processing.de](http://www.music-processing.de)

2nd edition  
Meinard Müller  
Fundamentals of Music Processing  
Using Python and Jupyter Notebooks  
Springer, 2021

## Fundamentals of Music Processing (FMP)

Chapter	Music Processing Scenario
1	Music Representations
2	Fourier Analysis of Signals
3	Music Synchronization
4	Music Structure Analysis
5	Chord Recognition
6	Tempo and Beat Tracking
7	Content-Based Audio Retrieval
8	Musically Informed Audio Decomposition

Meinard Müller  
Fundamentals of Music Processing  
Audio, Analysis, Algorithms, Applications  
Springer, 2015

Accompanying website:  
[www.music-processing.de](http://www.music-processing.de)

2nd edition  
Meinard Müller  
Fundamentals of Music Processing  
Using Python and Jupyter Notebooks  
Springer, 2021

## FMP Notebooks: Education & Research

**FMP Notebooks**  
Python Notebooks for Fundamentals of Music Processing

The FMP notebooks offer a collection of educational material closely following the textbook [Fundamentals of Music Processing \(FMP\)](https://www.audiolabs-erlangen.de/FMP). This is the starting website, which is opened when calling <https://www.audiolabs-erlangen.de/FMP>. Besides giving an [overview](#), this website provides information on the license, the main contributors, and some links.

<https://www.audiolabs-erlangen.de/FMP>

## References (FMP Notebooks)

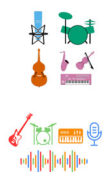
- Meinard Müller: Fundamentals of Music Processing – Using Python and Jupyter Notebooks. 2nd Edition, Springer, 2021.  
<https://www.springer.com/gp/book/9783030698072>
- Meinard Müller and Frank Zalkow: libfmp: A Python Package for Fundamentals of Music Processing. Journal of Open Source Software (JOSS), 6(63): 1–5, 2021.  
<https://joss.theoj.org/papers/10.21105/joss.03326>
- Meinard Müller: An Educational Guide Through the FMP Notebooks for Teaching and Learning Fundamentals of Music Processing. Proc. International Society for Music Information Retrieval Conference (ISMIR): 573–580, 2019.  
<https://www.mdpi.com/2624-6120/2/2/18>
- Meinard Müller and Frank Zalkow: FMP Notebooks: Educational Material for Teaching and Learning Fundamentals of Music Processing. Proc. International Society for Music Information Retrieval Conference (ISMIR): 573–580, 2019.  
[https://zenodo.org/record/3527872#\\_YOhEQOqzaUk](https://zenodo.org/record/3527872#_YOhEQOqzaUk)
- Meinard Müller, Brian McFee, and Katherine Kinnaird: Interactive Learning of Signal Processing Through Music: Making Fourier Analysis Concrete for Students. IEEE Signal Processing Magazine, 38(3): 73–84, 2021.  
<https://ieeexplore.ieee.org/document/9418542>

## Resources (Group Meinard Müller)

- FMP Notebooks:  
<https://www.audiolabs-erlangen.de/FMP>
- libfmp:  
<https://github.com/meinardmueller/libfmp>
- synctoolbox:  
<https://github.com/meinardmueller/synctoolbox>
- libtsm:  
<https://github.com/meinardmueller/libtsm>
- Preparation Course Python (PCP) Notebooks:  
<https://www.audiolabs-erlangen.de/resources/MIR/PCP/PCP.html>  
<https://github.com/meinardmueller/PCP>

## Resources

- librosa:  
<https://librosa.org/>
- madmom:  
<https://github.com/CPJKU/madmom>
- Essentia Python tutorial:  
[https://essentia.upf.edu/essentia\\_python\\_tutorial.html](https://essentia.upf.edu/essentia_python_tutorial.html)
- mirdata:  
<https://github.com/mir-dataset-loaders/mirdata>
- open-unmix:  
<https://github.com/sigsep/open-unmix-pytorch>
- Open Source Tools & Data for Music Source Separation:  
<https://source-separation.github.io/tutorial/landing.html>



---

## Thanks

- Yigitcan Özer (PhD student)
- Michael Krause (PhD student)
- Tim Zunner (Master Thesis 2021)
- Edgar Suárez Guarnizo (Master Thesis 2020)
- Christian Dittmar (PhD 2018, Fraunhofer IIS)

---

## References (NMF, NAE)

- Daniel Lee and Sebastian Seung: **Algorithms for Non-Negative Matrix Factorization**. Proc. NIPS, 2000.
- Sebastian Ewert and Meinard Müller: **Using Score-Informed Constraints for NMF-Based Source Separation**. Proc. ICASSP, 2012.
- Paris Smaragdis and Shrikant Venkataramani: **A Neural Network Alternative to Non-Negative Audio Models**. Proc. ICASSP, 2017.
- Sebastian Ewert and Mark B. Sandler: **Structured Dropout for Weak Label and Multi-Instance Learning and Its Application to Score-Informed Source Separation**. Proc. ICASSP, 2017.
- Yigitcan Özer, Jonathan Hansen, Tim Zunner, and Meinard Müller: **Investigating Nonnegative Autoencoders for Efficient Audio Decomposition**. Proc. EUSIPCO, 2022.