

INTERNATIONAL AUDIO LABORATORIES ERLANGEN  
A joint institution of Fraunhofer IIS and Universität Erlangen-Nürnberg



Tutorial 5, ISMIR  
Milan, November 5, 2023

# Learning with Music Signals: Technology Meets Education

## FMP Notebooks

**Meinard Müller**

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[meinard.mueller@audiolabs-erlangen.de](mailto:meinard.mueller@audiolabs-erlangen.de)

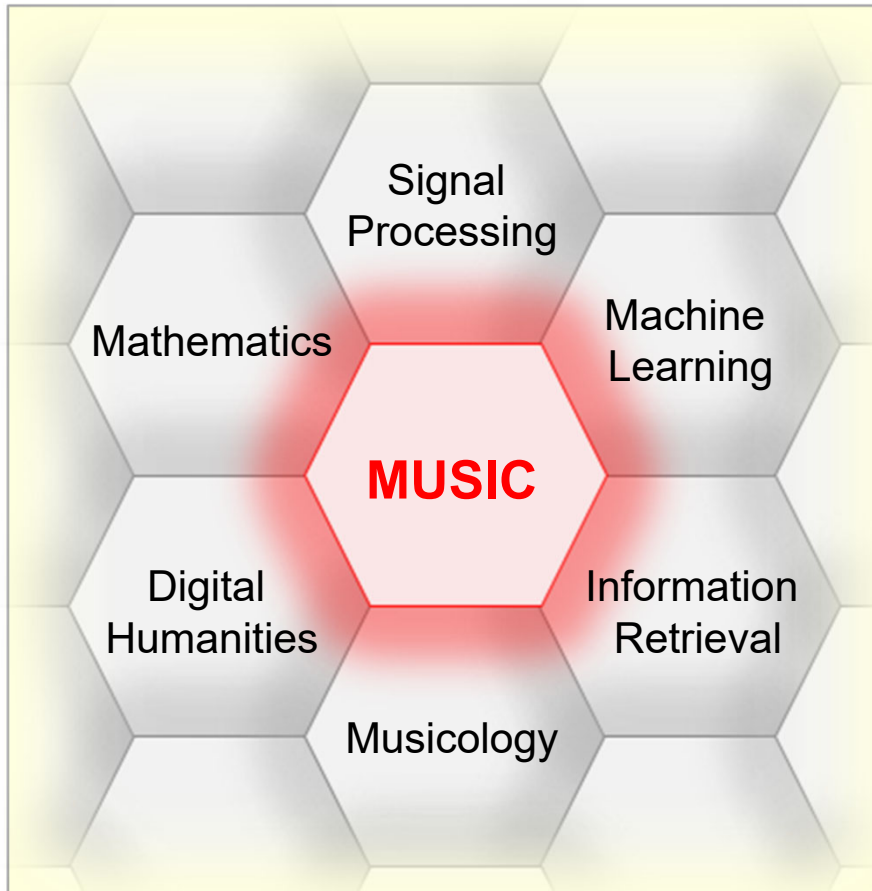


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# Music Processing



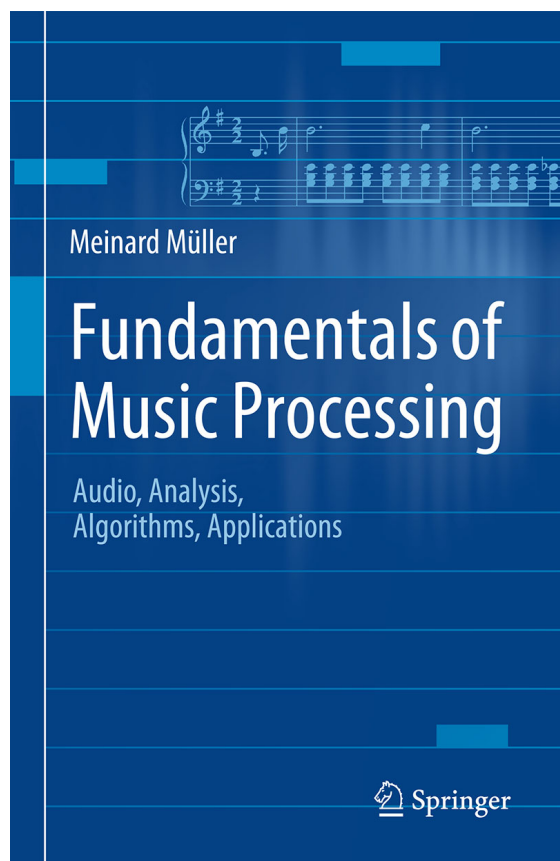
# Music Processing: A Multifaceted Research Area



## Music ...

- important part of our lives ...
- ... Spotify, Pandora, iTunes, ...
- interdisciplinary research
- intuitive entry point to education

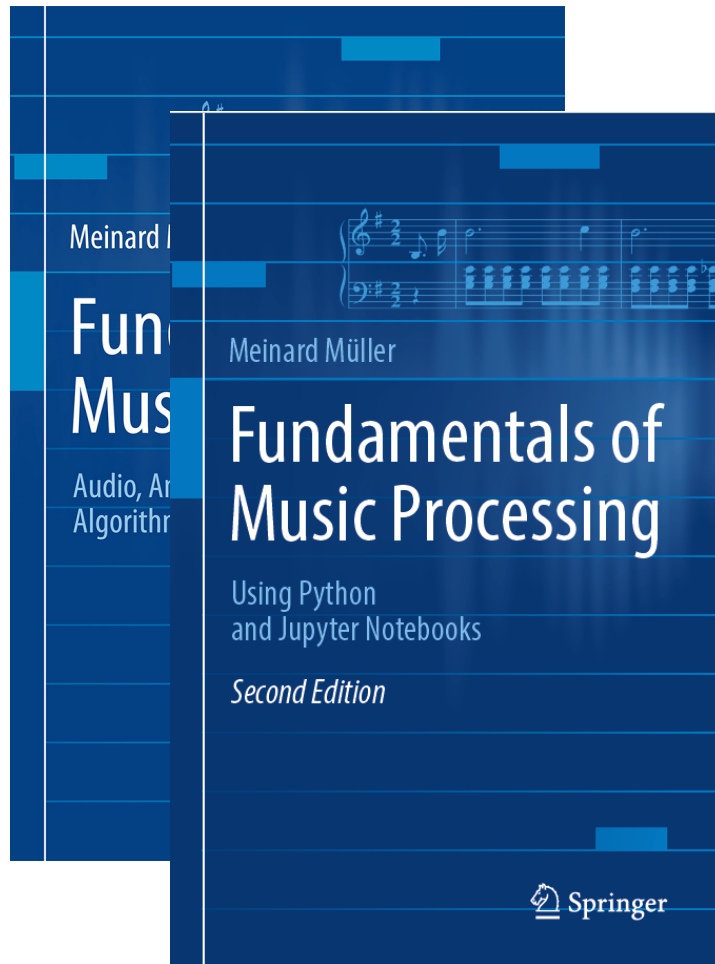
# Fundamentals of Music Processing (FMP)



Meinard Müller  
Fundamentals of Music Processing  
Audio, Analysis, Algorithms, Applications  
Springer, 2015

Accompanying website:  
[www.music-processing.de](http://www.music-processing.de)

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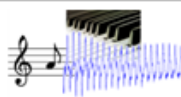

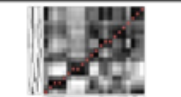
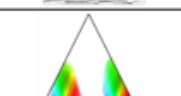

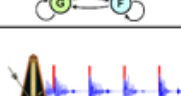




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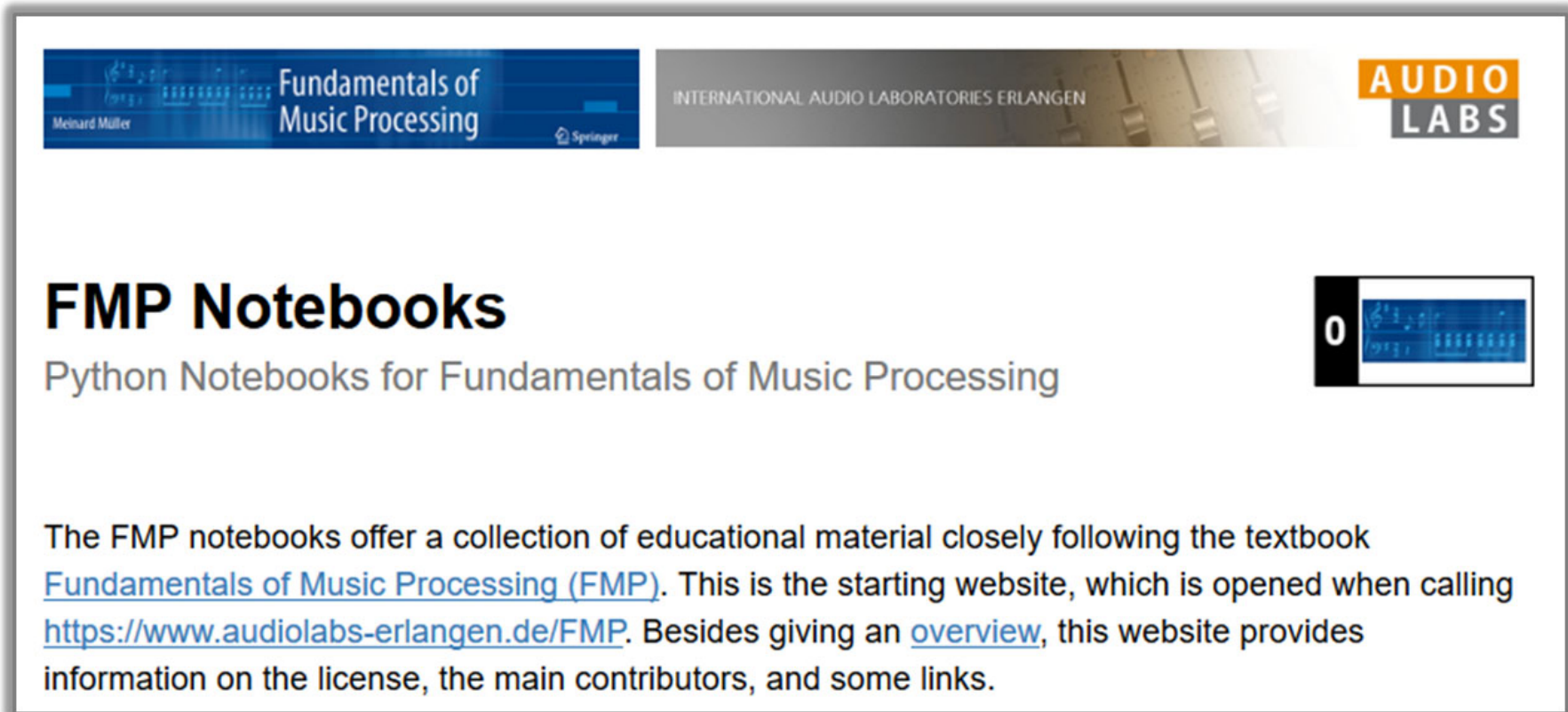
Chapter	Music Processing Scenario
1	 <b>Music Representations</b>
2	 <b>Fourier Analysis of Signals</b>
3	 <b>Music Synchronization</b>
4	 <b>Music Structure Analysis</b>
5	 <b>Chord Recognition</b>
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# FMP Notebooks: Education & Research



The screenshot shows the header of the FMP Notebooks website. On the left, there is a blue banner for the book 'Fundamentals of Music Processing' by Meinard Müller, published by Springer. To the right of this banner is the text 'INTERNATIONAL AUDIO LABORATORIES ERLANGEN' and the 'AUDIO LABS' logo. Below the banner, the main heading reads 'FMP Notebooks' in a large, bold, black font, followed by the subtitle 'Python Notebooks for Fundamentals of Music Processing' in a smaller, grey font. To the right of the subtitle is a small thumbnail image of a notebook page with a black bar on the left containing the number '0'. Below the heading and subtitle, a paragraph of text describes the notebooks: 'The FMP notebooks offer a collection of educational material closely following the textbook [Fundamentals of Music Processing \(FMP\)](#). This is the starting website, which is opened when calling <https://www.audiolabs-erlangen.de/FMP>. Besides giving an [overview](#), this website provides information on the license, the main contributors, and some links.'

<https://www.audiolabs-erlangen.de/FMP>

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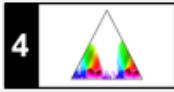

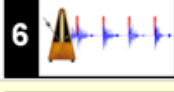

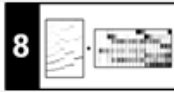
# FMP Notebooks: Education & Research

- ... provide educational material for teaching and learning fundamentals of music processing.
- ... combine textbook-like explanations, technical concepts, mathematical details, Python code examples, illustrations, and sound examples.
- ... bridge the gap between theory and practice being based on interactive Jupyter notebook framework.
- ... are freely accessible under a Creative Commons license.

<https://www.audiolabs-erlangen.de/FMP>





# FMP Notebooks

Part	Title	Notions, Techniques & Algorithms	HTML	IPYNB
	<a href="#">Basics</a>	Basic information on Python, Jupyter notebooks, Anaconda package management system, Python environments, visualizations, and other topics	<a href="#">[html]</a>	<a href="#">[ipynb]</a>
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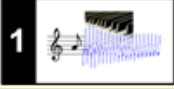
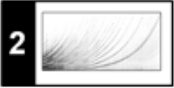
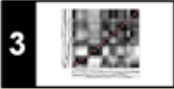
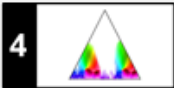

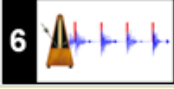

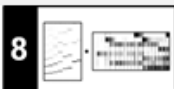
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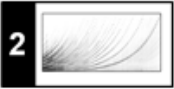
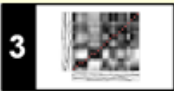
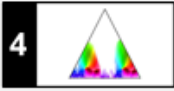


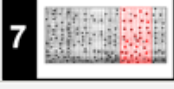
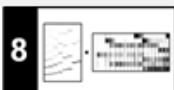
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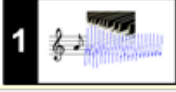

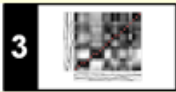
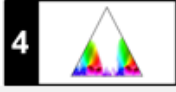



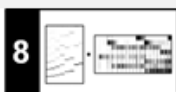
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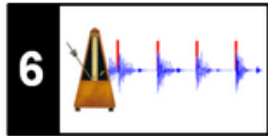
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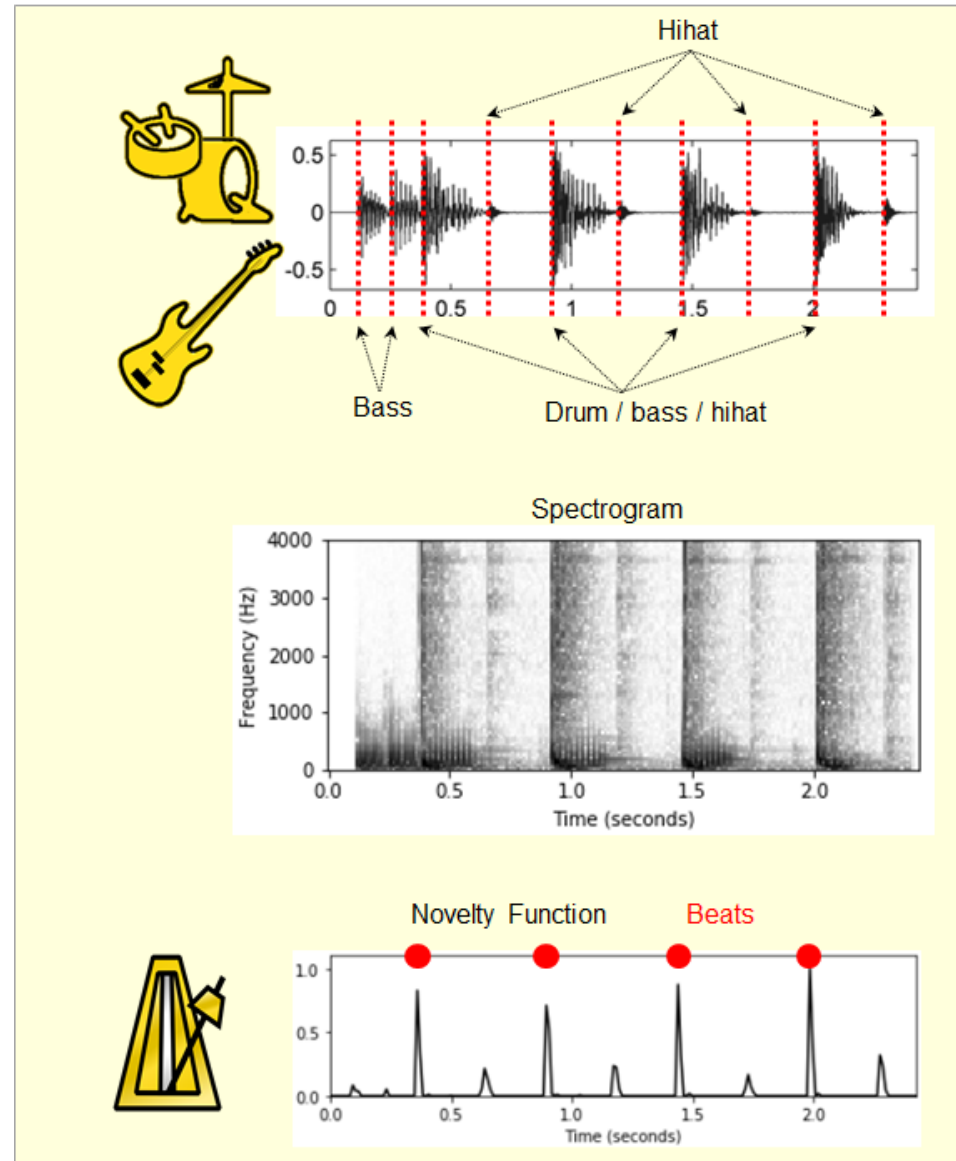
**Part 6: Tempo and Beat Tracking**



# Part 6: Tempo and Beat Tracking

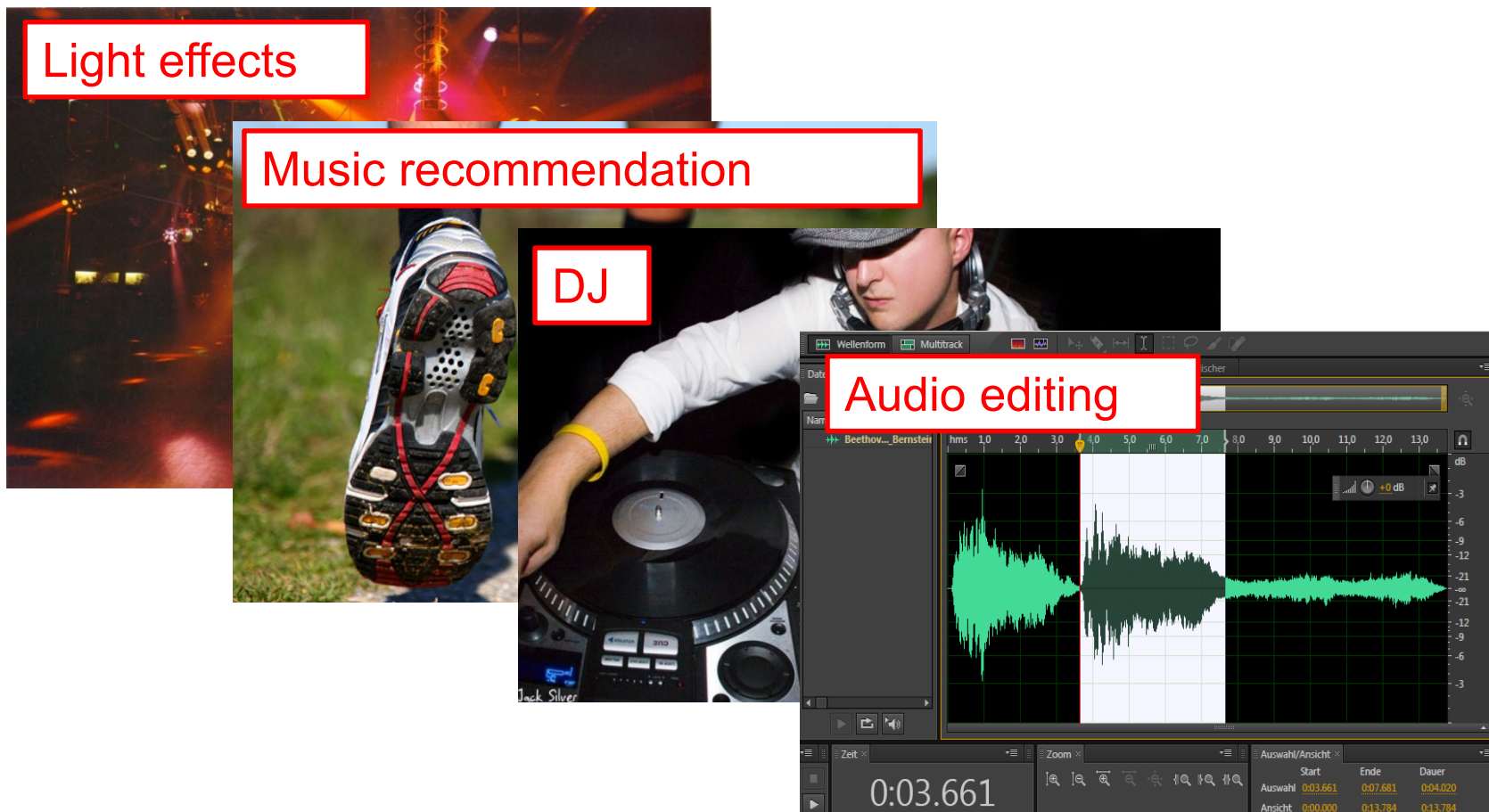


- When listening to a piece of music, we as humans are often able to tap along with the musical beat
- Automated beat tracking: Simulate this cognitive process by a computer



# Tempo and Beat Tracking

Basic task: “Tapping the foot when listening to music”

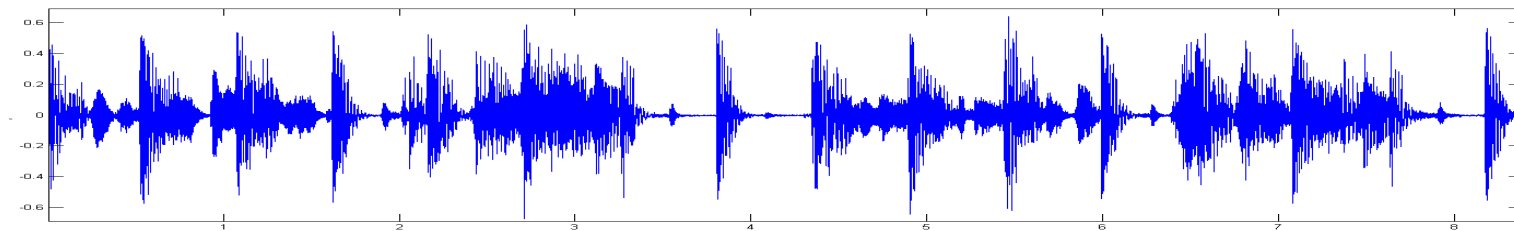




# Tempo and Beat Tracking

Basic task: “Tapping the foot when listening to music”

Example: Queen – Another One Bites The Dust

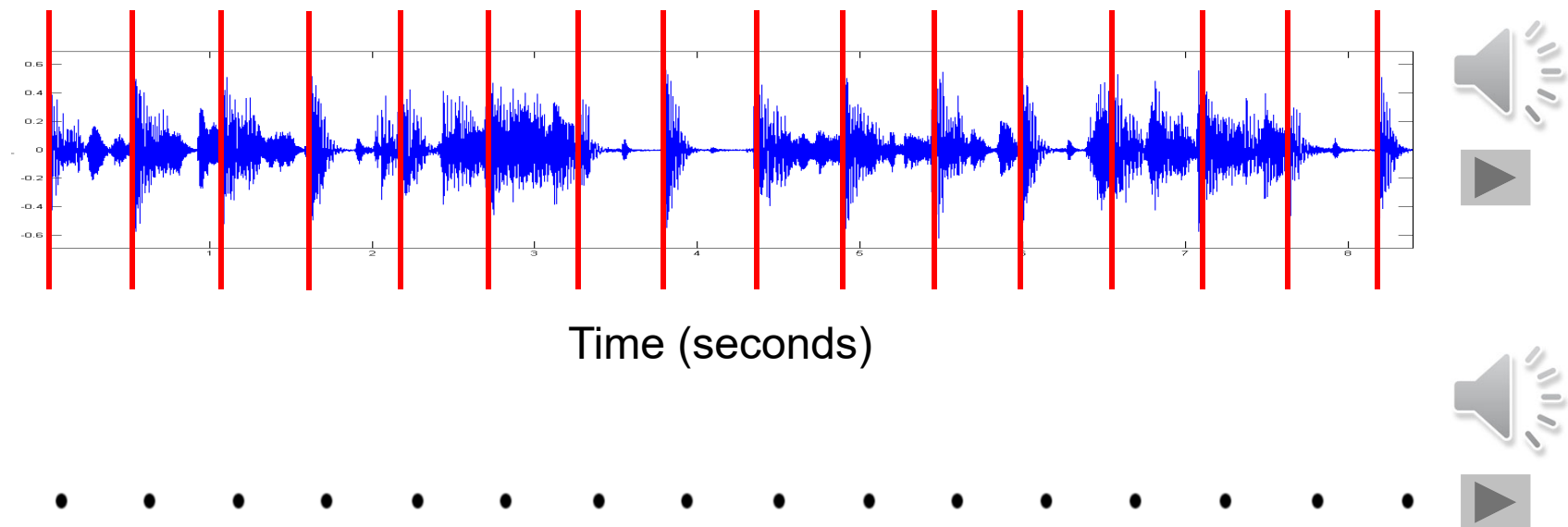


Time (seconds)

# Tempo and Beat Tracking

Basic task: “Tapping the foot when listening to music”

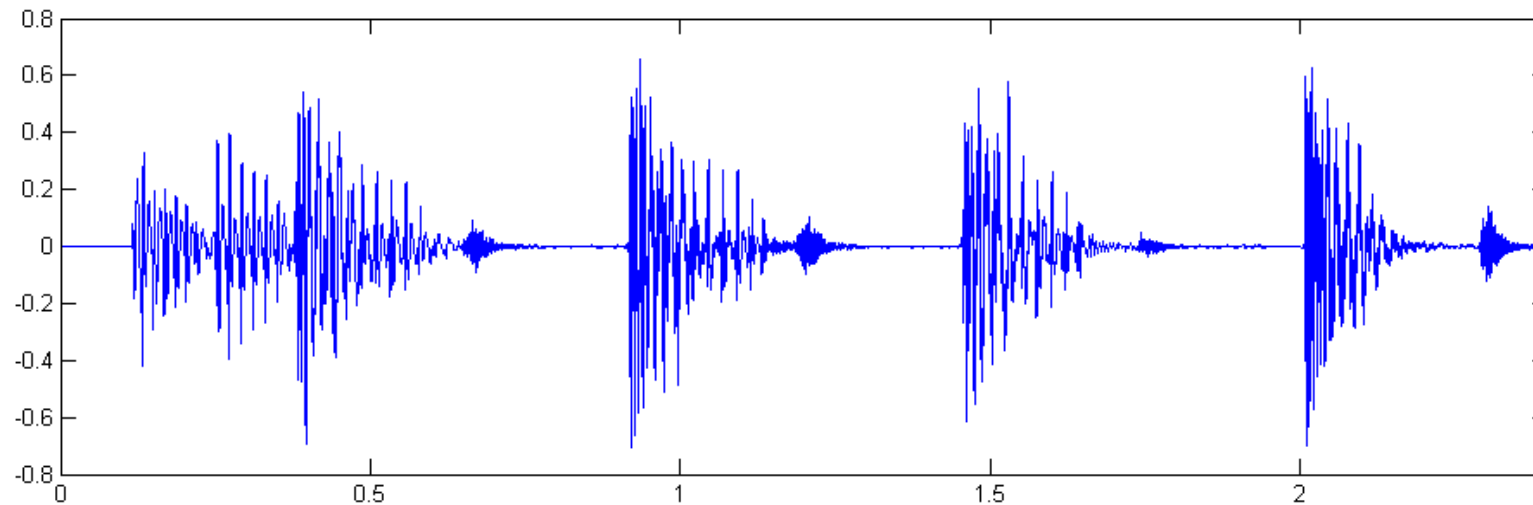
Example: Queen – Another One Bites The Dust



# Tempo and Beat Tracking

## Tasks

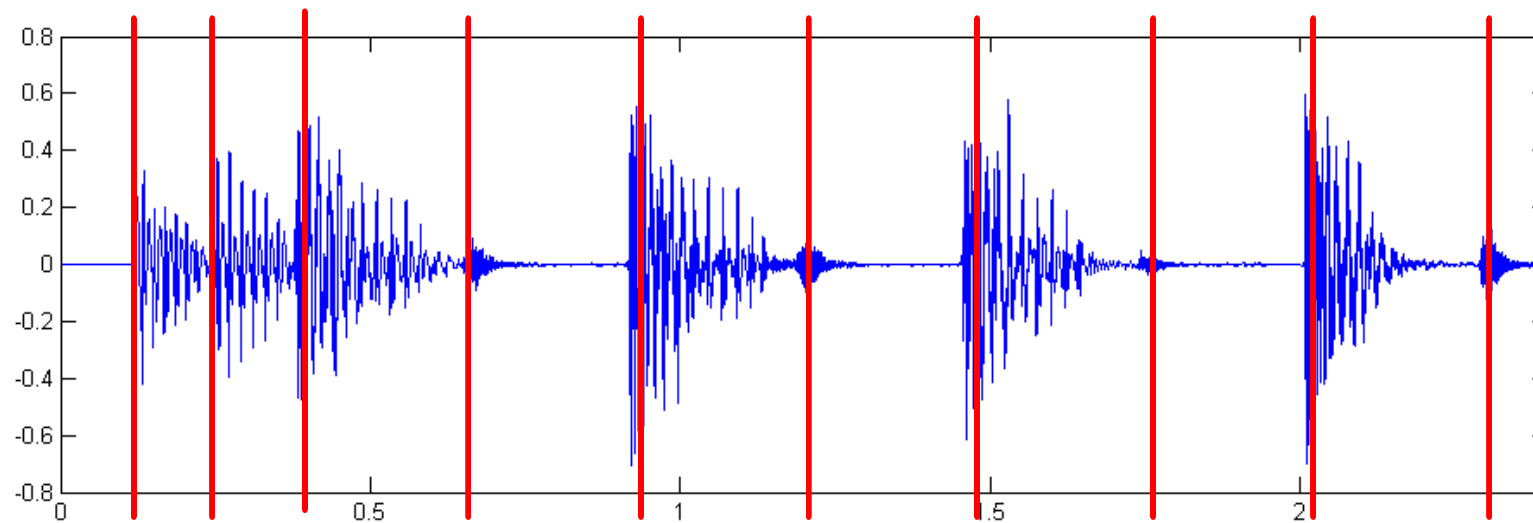
- Onset detection
- Beat tracking
- Tempo estimation



# Tempo and Beat Tracking

## Tasks

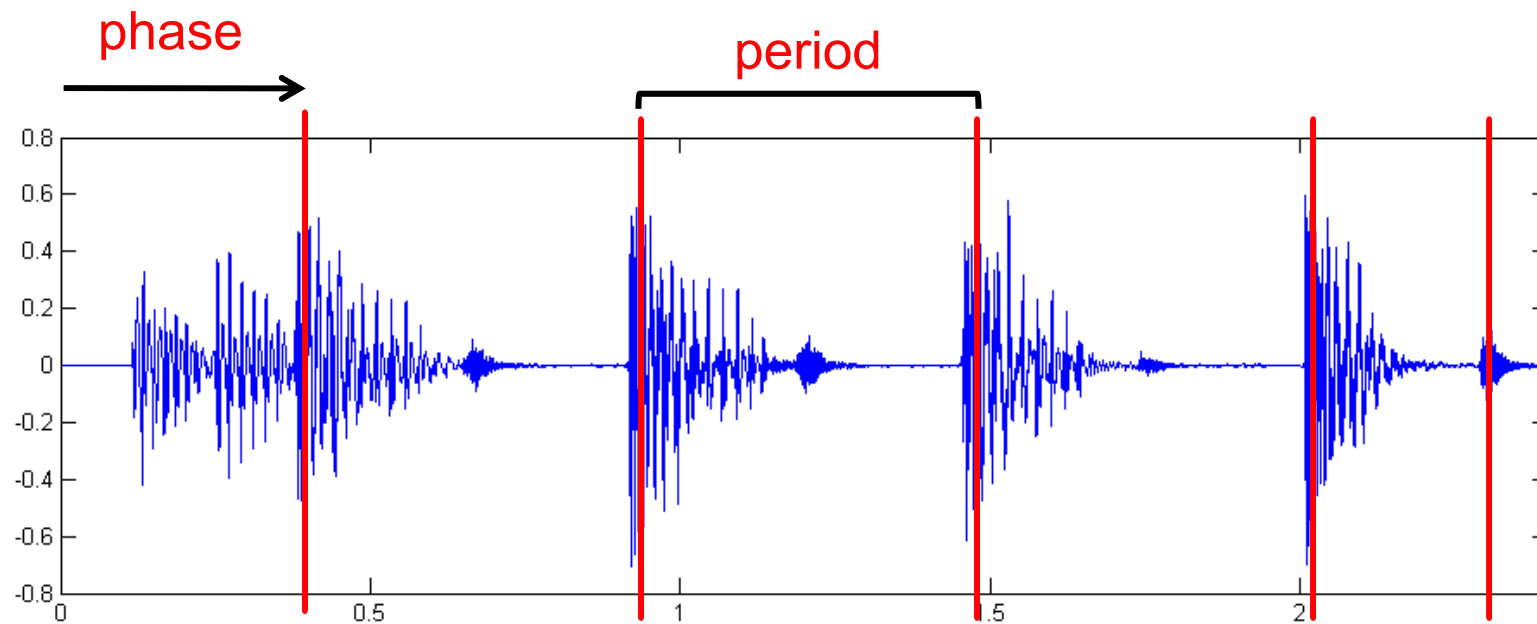
- Onset detection
- Beat tracking
- Tempo estimation



# Tempo and Beat Tracking

## Tasks

- Onset detection
- **Beat tracking**
- Tempo estimation



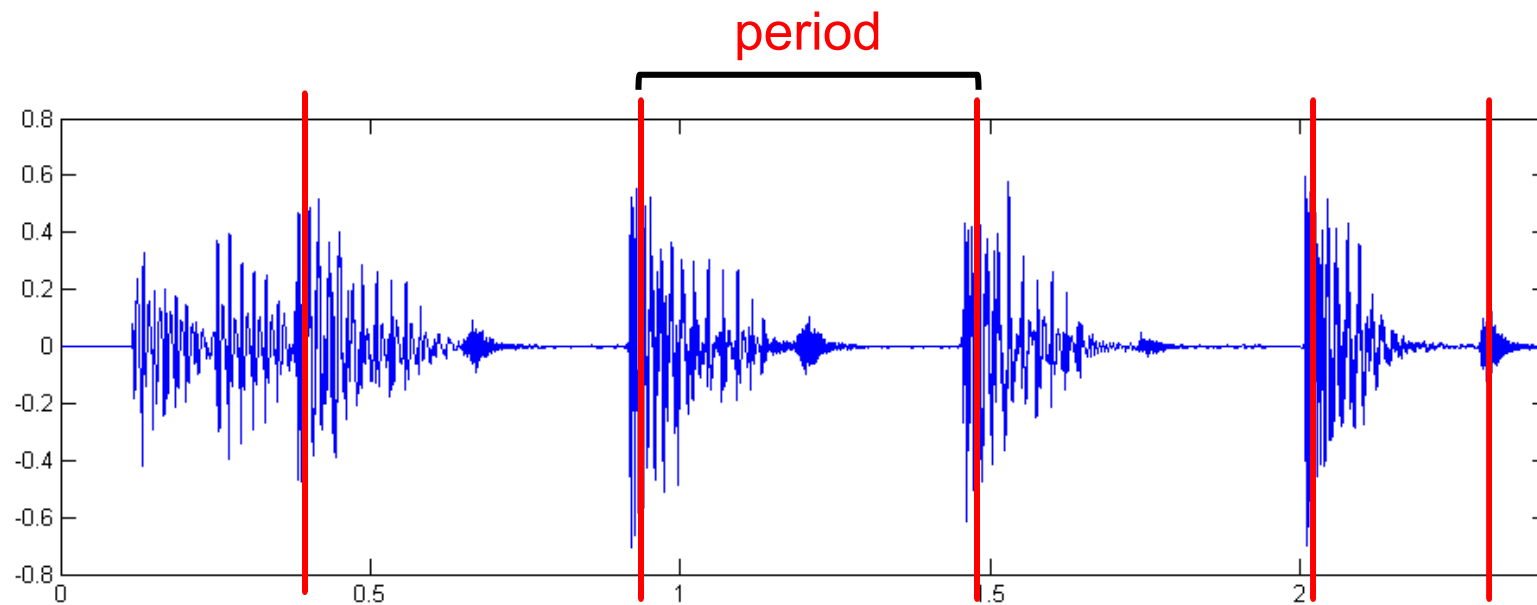
# Tempo and Beat Tracking

## Tasks

- Onset detection
- Beat tracking
- **Tempo estimation**

**Tempo := 60 / period**

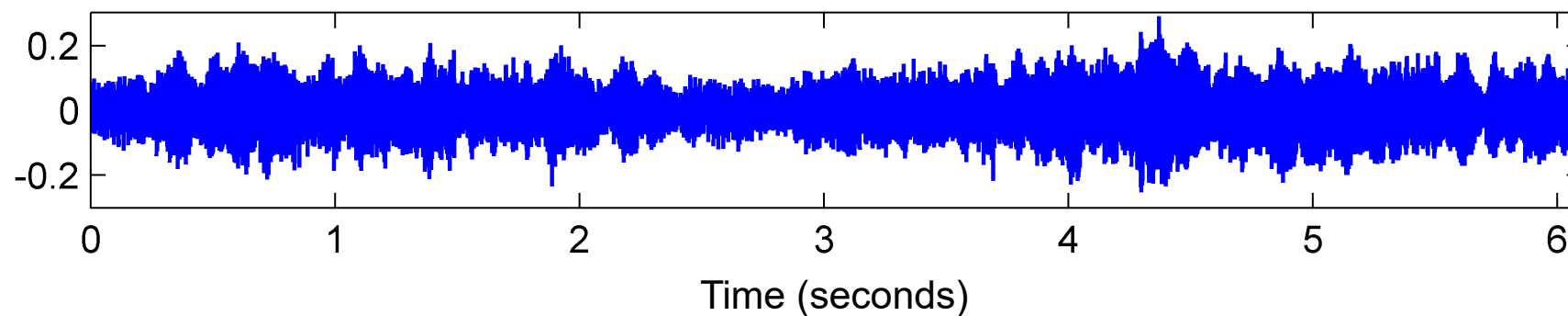
**Beats per minute (BPM)**



# Onset Detection (Spectral Flux)

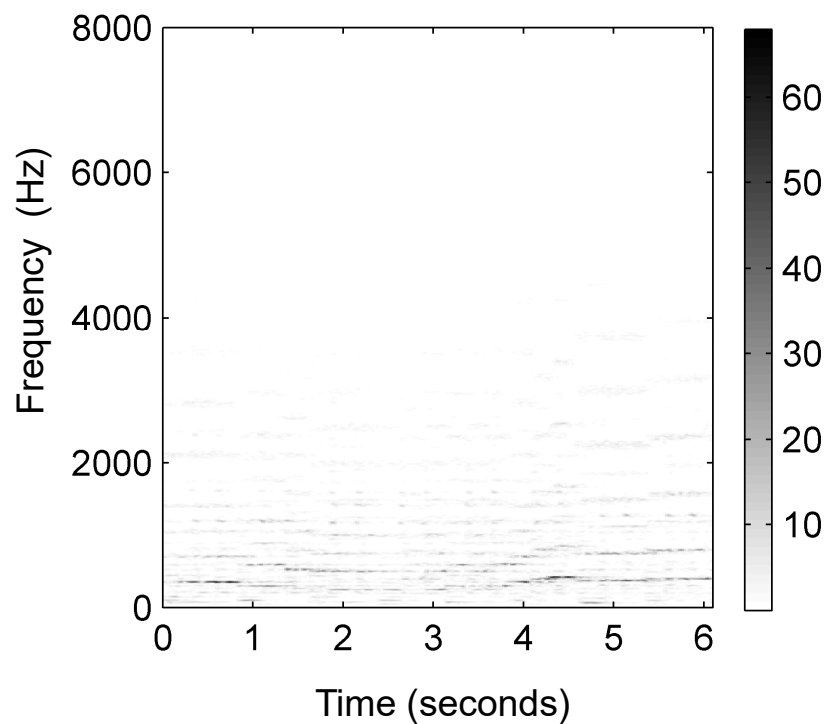


Audio recording



# Onset Detection (Spectral Flux)

Magnitude spectrogram  $|X|$



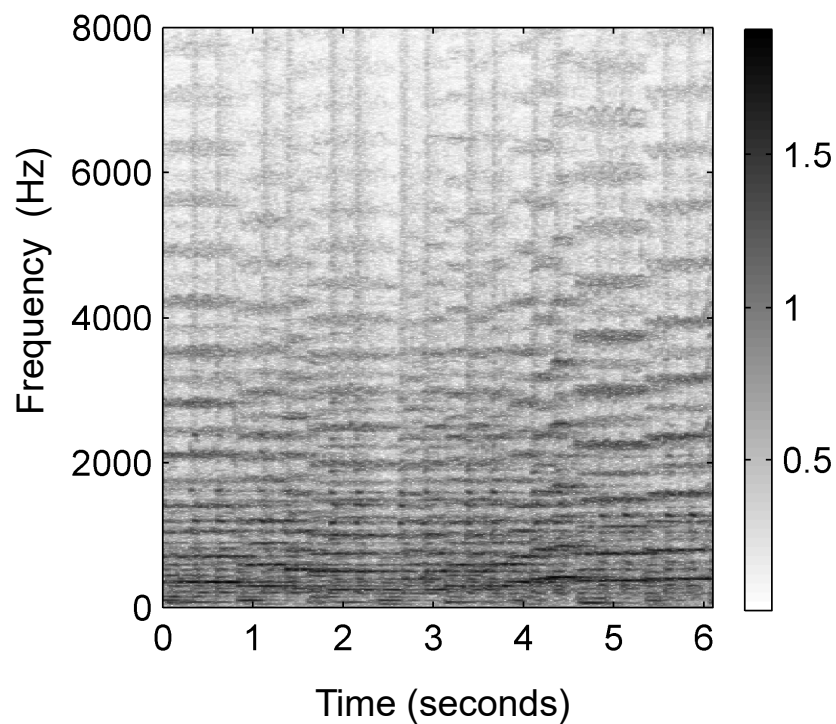
## Steps:

1. Spectrogram



# Onset Detection (Spectral Flux)

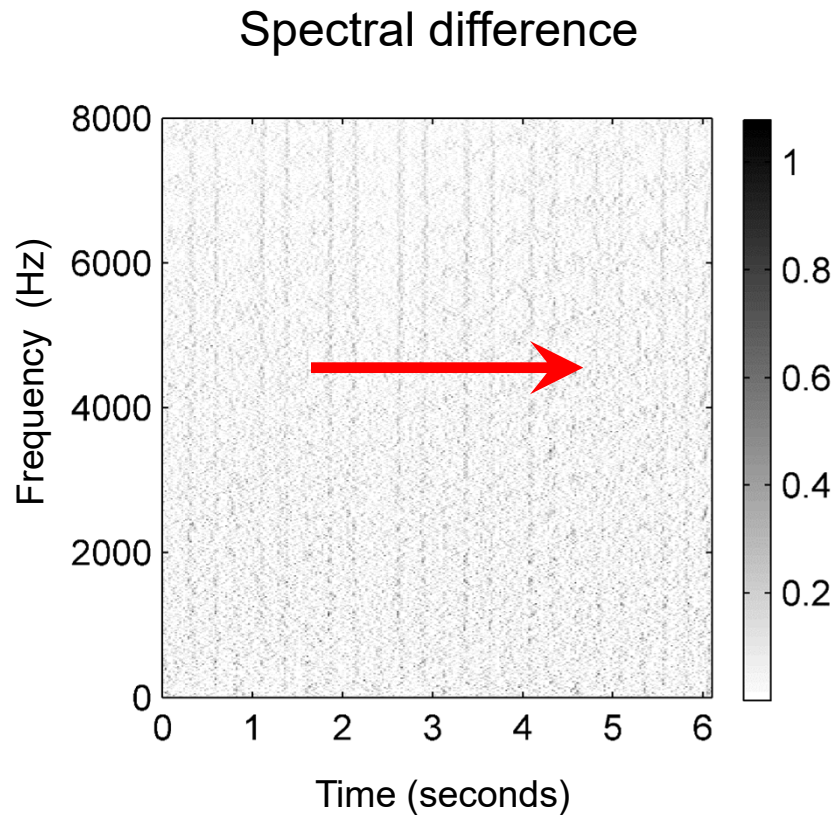
Compressed spectrogram  $Y$



## Steps:

1. Spectrogram
2. Logarithmic compression

# Onset Detection (Spectral Flux)

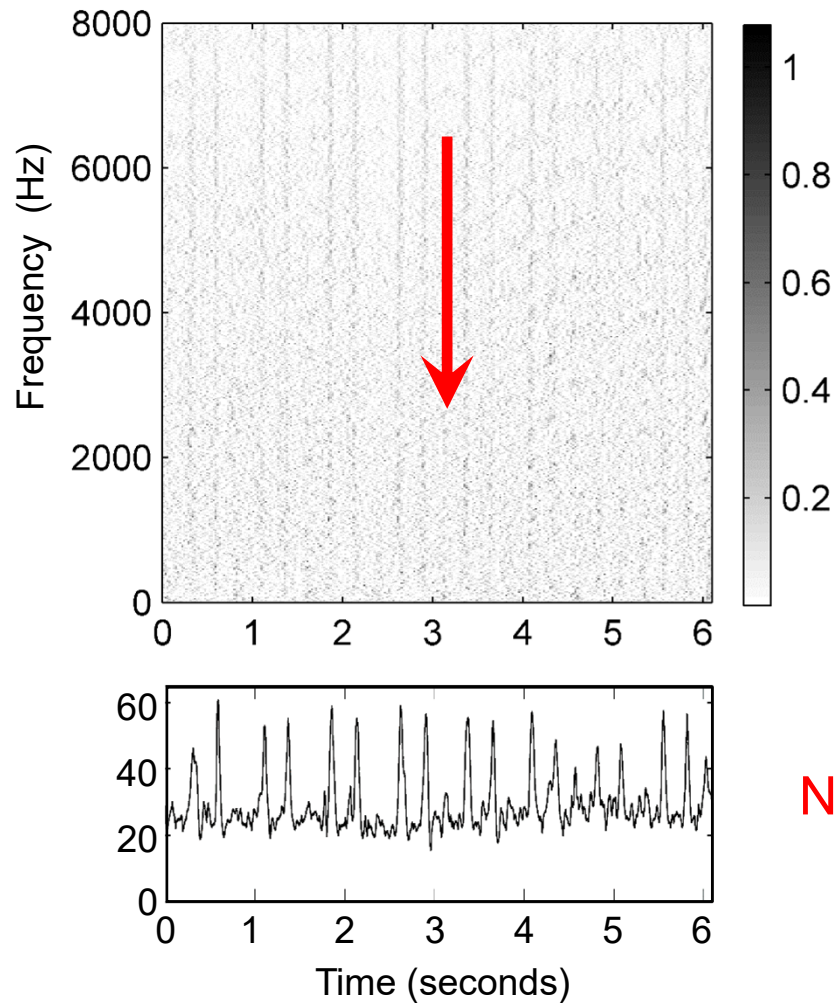


## Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification

# Onset Detection (Spectral Flux)

Spectral difference



## Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation

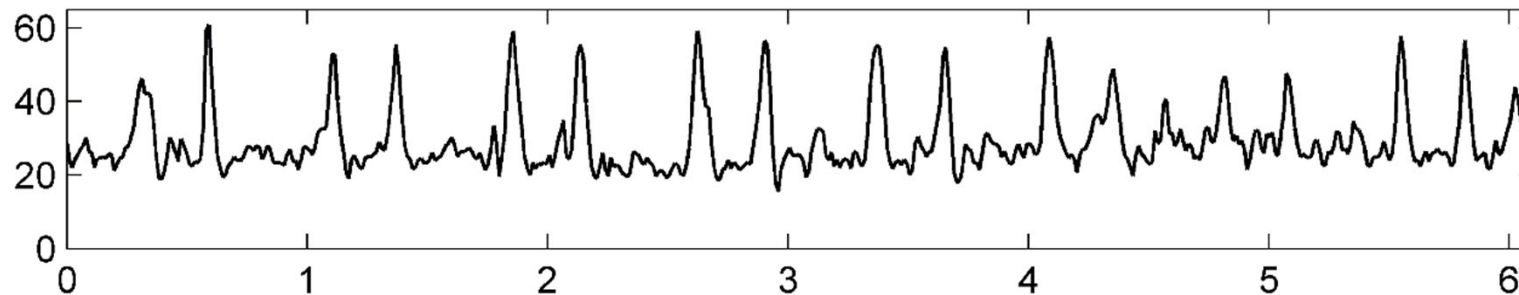
Novelty curve

# Onset Detection (Spectral Flux)

## Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation

## Novelty function



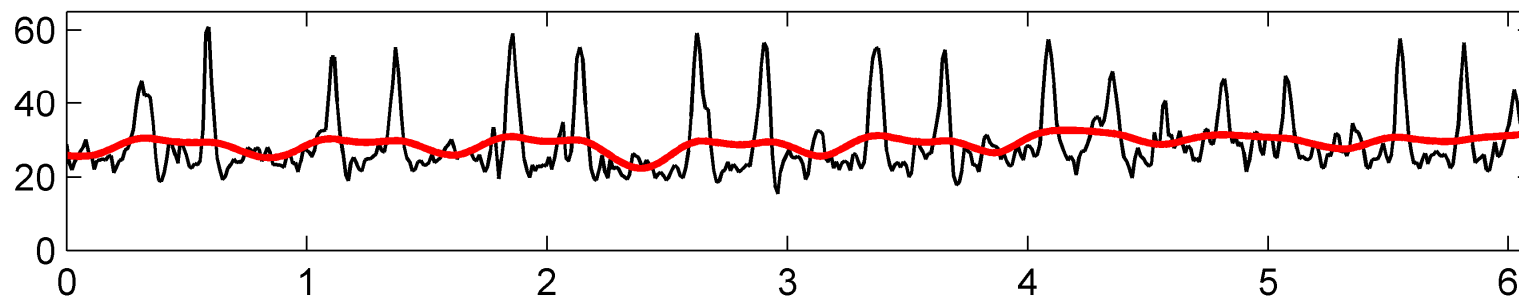
# Onset Detection (Spectral Flux)

## Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation
5. Normalization

Novelty function

Substraction of local average

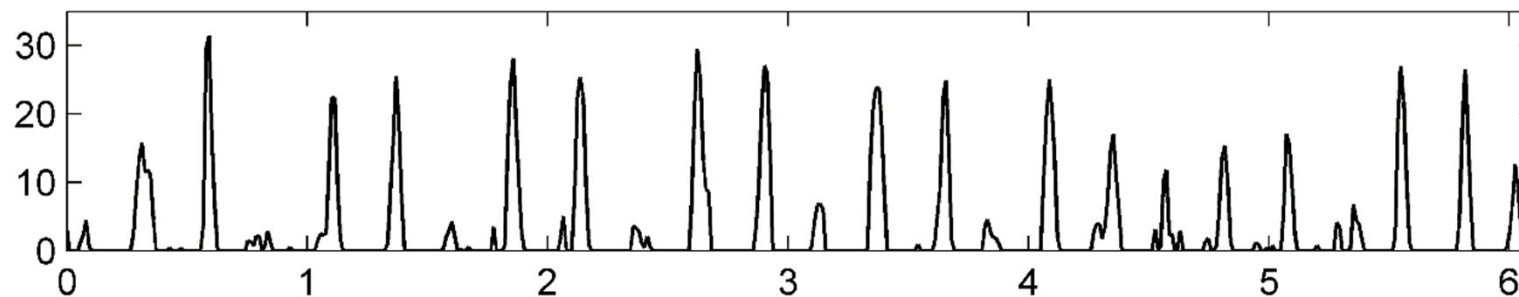


# Onset Detection (Spectral Flux)

## Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation
5. Normalization

## Normalized novelty function



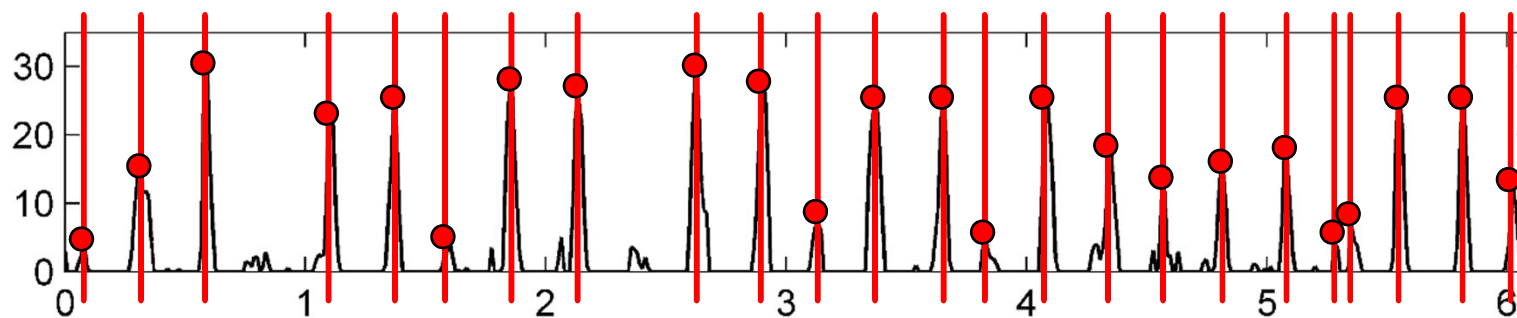
# Onset Detection (Spectral Flux)

## Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation
5. Normalization

Normalized novelty function

Peak positions indicate beat candidates



# Onset Detection (Spectral Flux)

## Deep Learning

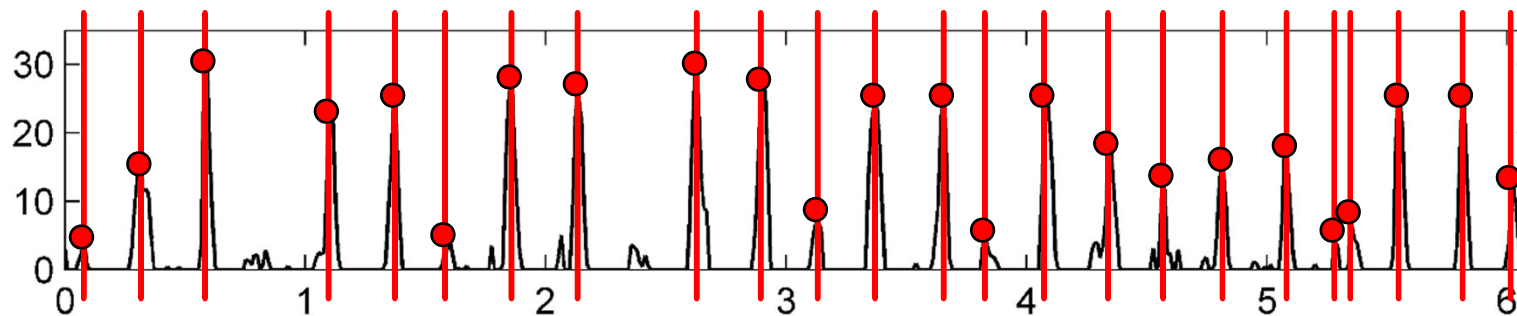
1. Input representation
2. Sigmoid activation
3. Convolution & rectified linear unit (ReLU)
4. Pooling
5. Convolution & ReLU

## Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation
5. Normalization

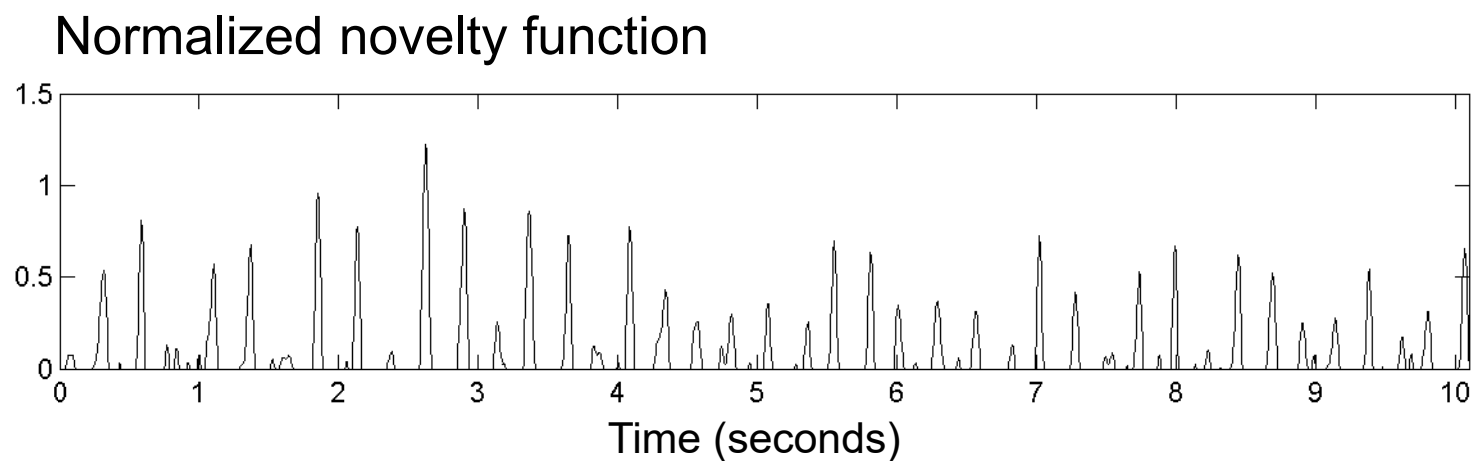
Normalized novelty function

Peak positions indicate beat candidates



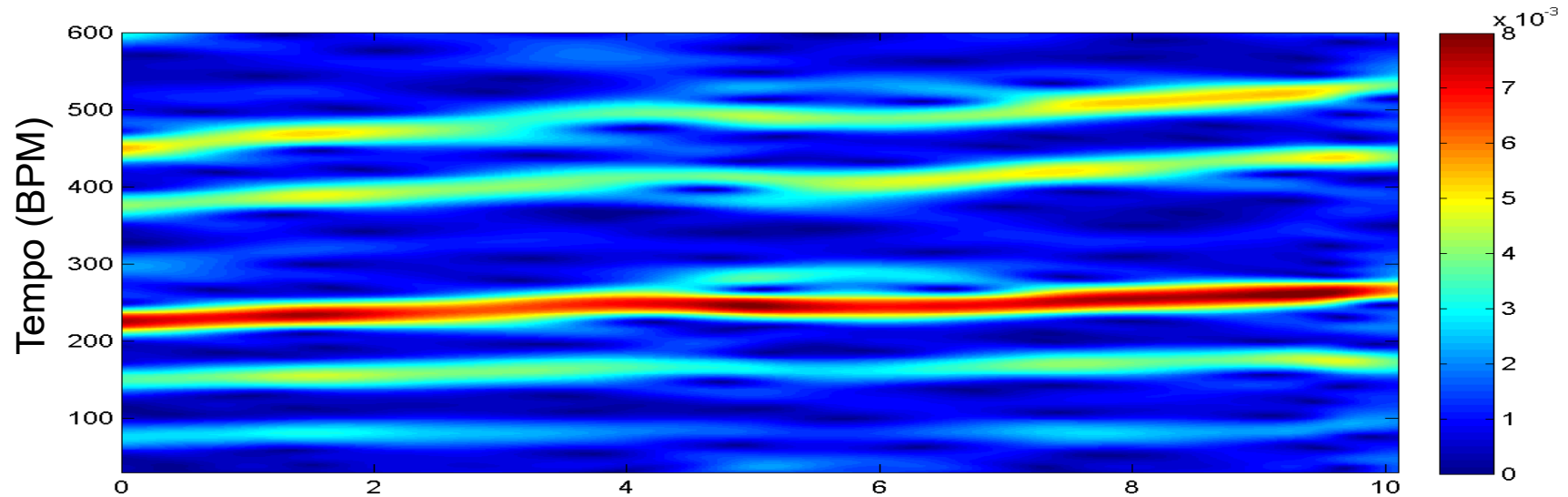


# Local Pulse and Tempo Tracking

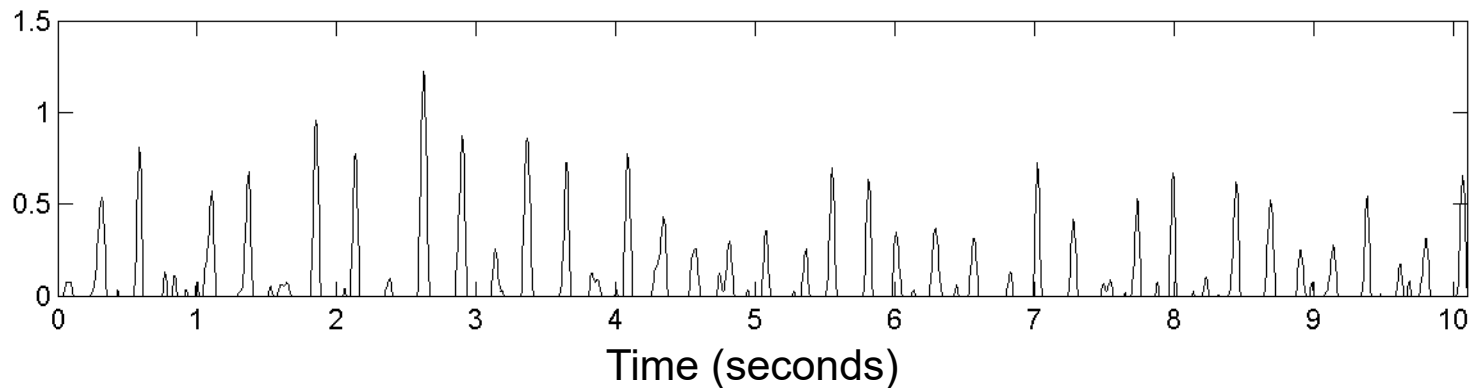


# Local Pulse and Tempo Tracking

## Fourier temogram (STFT of novelty function)

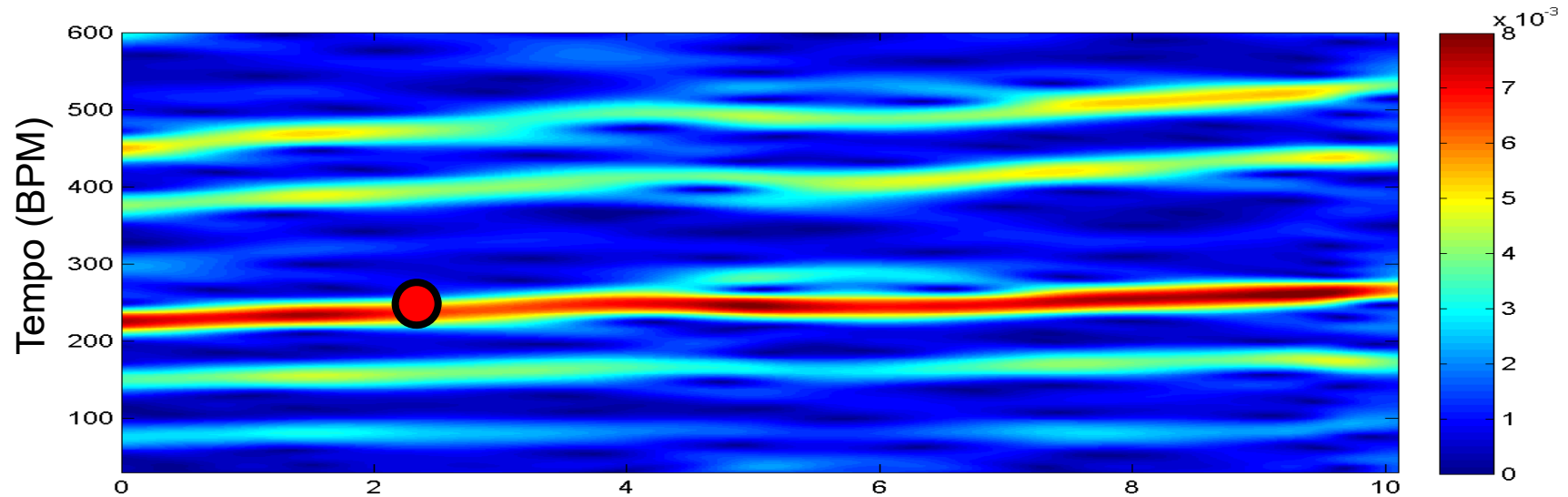


## Normalized novelty function

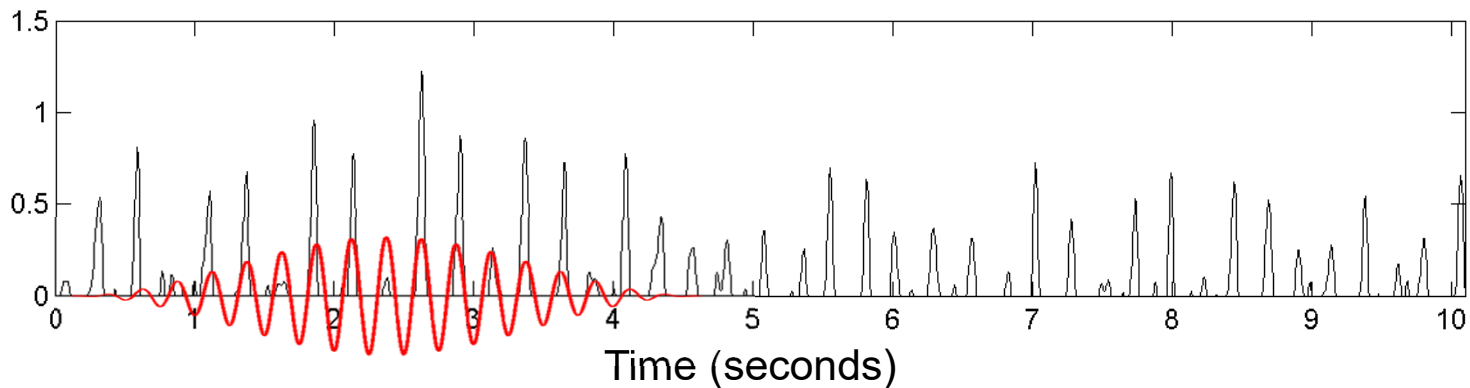


# Local Pulse and Tempo Tracking

## Fourier temogram (STFT of novelty function)

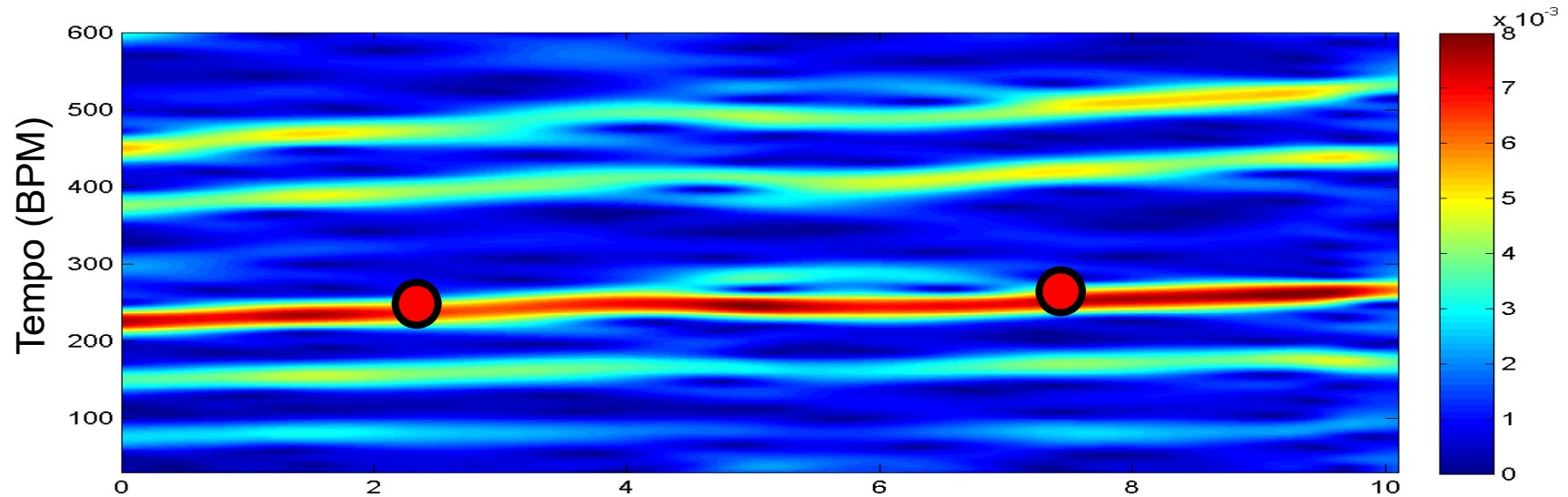


## Optimizing local periodicity kernel

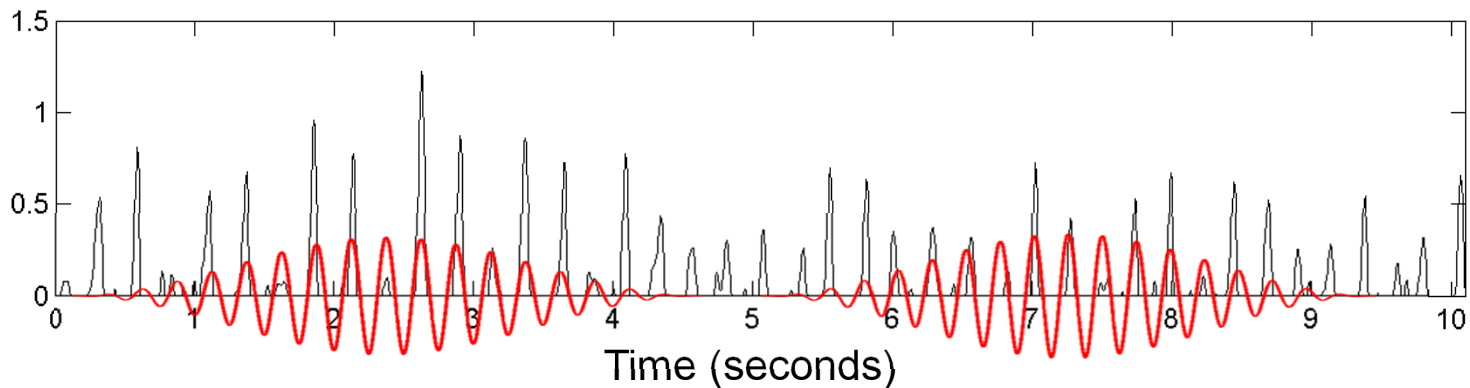


# Local Pulse and Tempo Tracking

## Fourier temogram (STFT of novelty function)

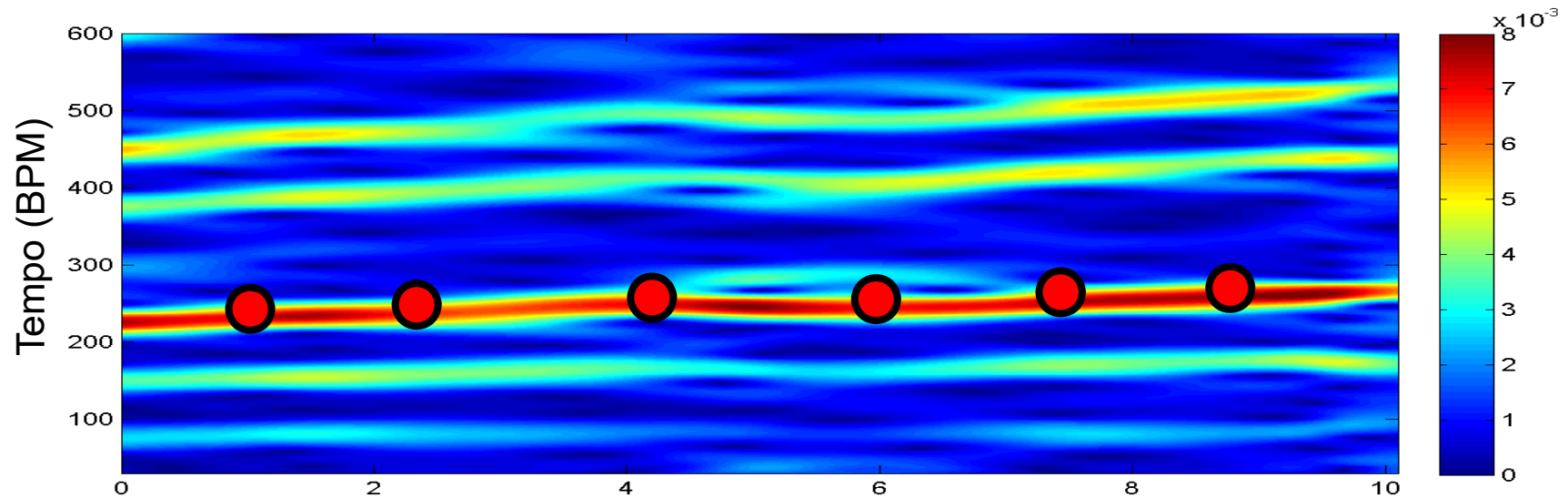


## Optimizing local periodicity kernel

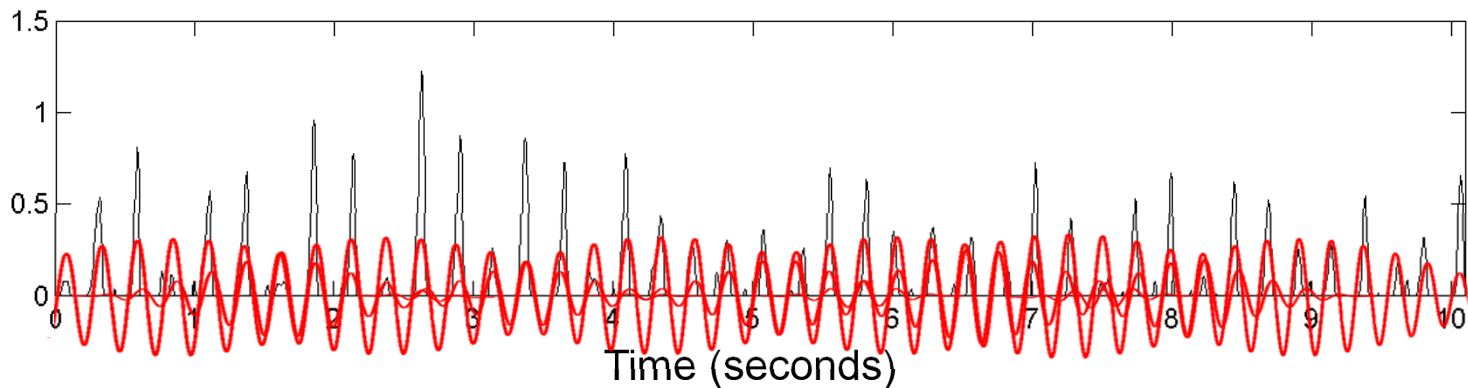


# Local Pulse and Tempo Tracking

## Fourier temogram (STFT of novelty function)

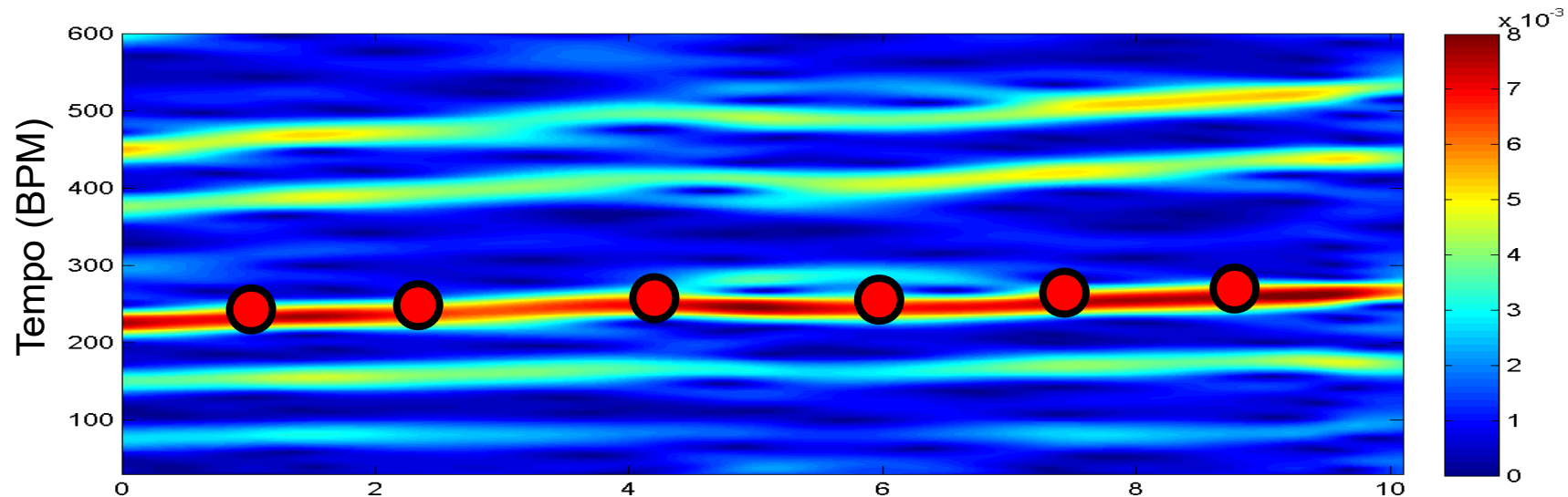


## Optimizing local periodicity kernel

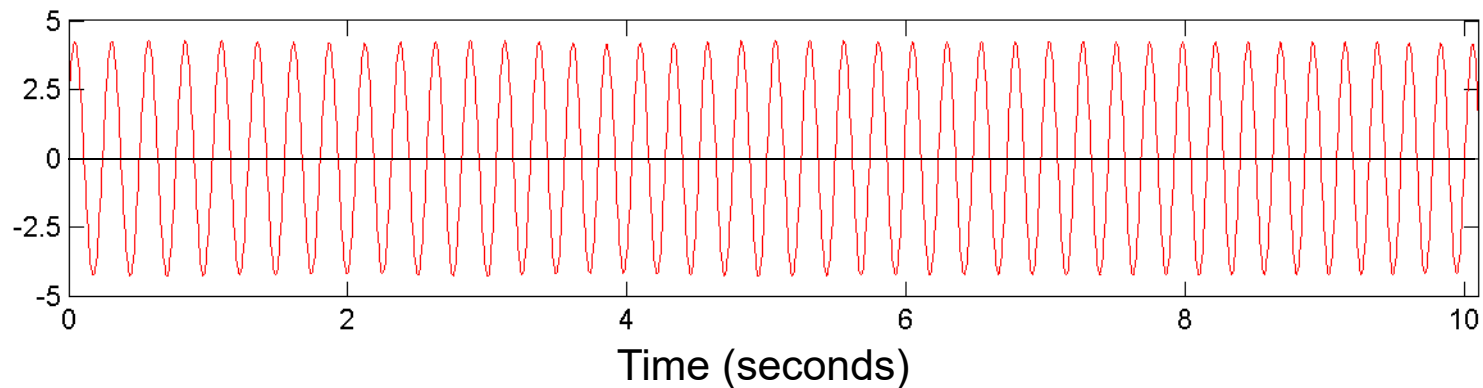


# Local Pulse and Tempo Tracking

## Fourier temogram (STFT of novelty function)

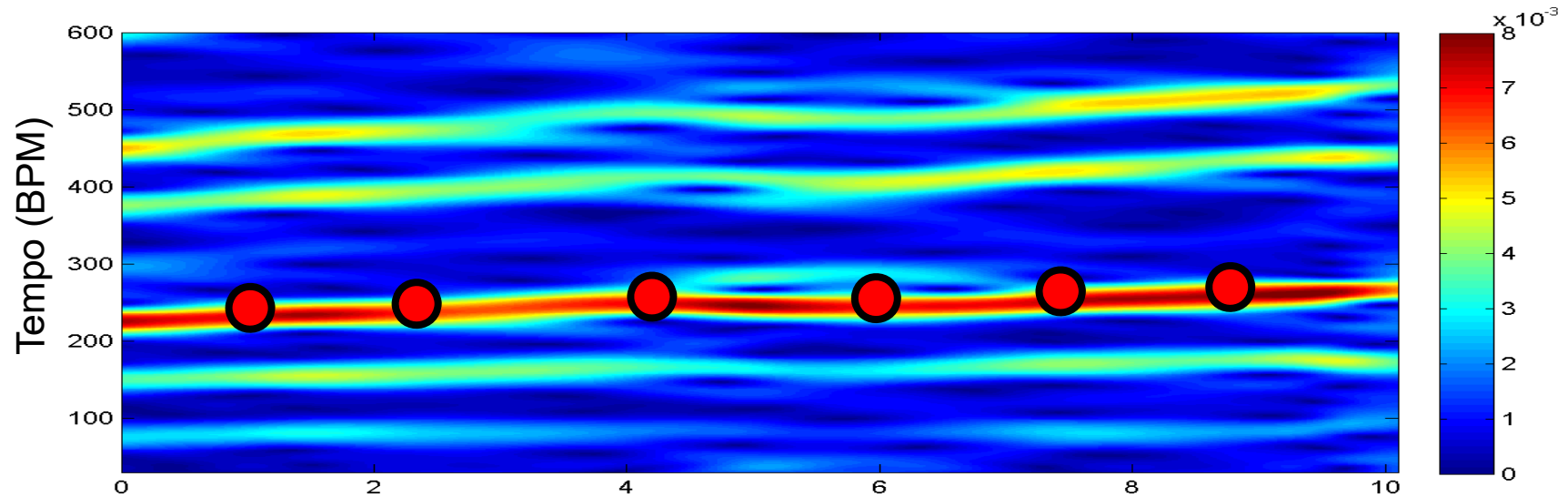


## Accumulation of kernels

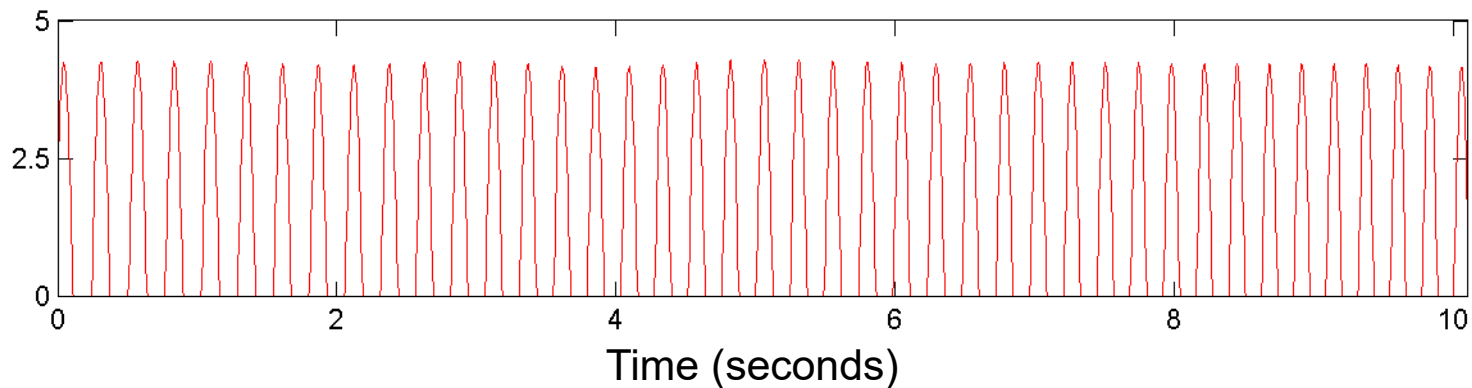


# Local Pulse and Tempo Tracking

## Fourier temogram (STFT of novelty function)

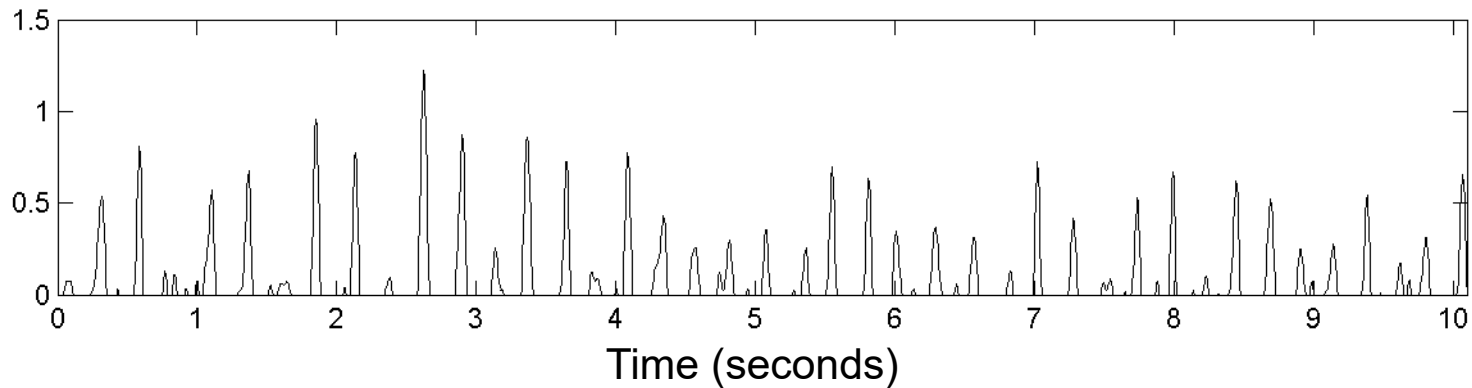


## Halfwave rectification

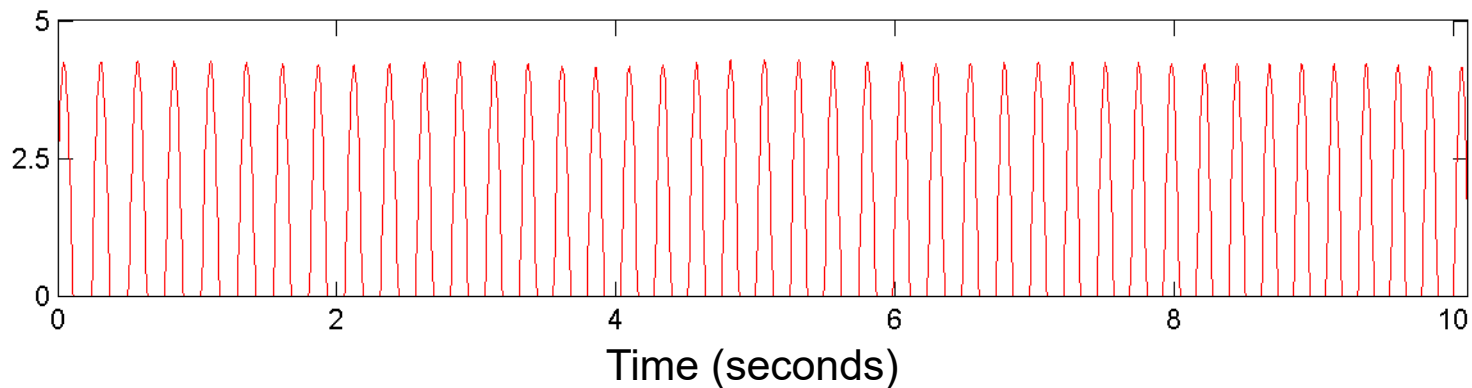


# Local Pulse and Tempo Tracking

## Novelty Curve



## Predominant Local Pulse (PLP)





# FMP Notebooks

Structured in 10 parts

- Part B: Basic introductions to
  - Jupyter notebook framework
  - Python programming
  - Other technical concepts underlying these notebooks
  
- Part 0: Starting notebook
  
- Part 1 to Part 8: Different music processing scenarios

Part	Title	Notions, Techniques & Algorithms	HTML	IPYNB
	<a href="#">Basics</a>	Basic information on Python, Jupyter notebooks, Anaconda package management system, Python environments, visualizations, and other topics	<a href="#">[html]</a>	<a href="#">[ipynb]</a>
	<a href="#">Overview</a>	Overview of the notebooks ( <a href="https://www.audiolabs-erlangen.de/FMP">https://www.audiolabs-erlangen.de/FMP</a> )	<a href="#">[html]</a>	<a href="#">[ipynb]</a>
	<a href="#">Music Representations</a>	Music notation, MIDI, audio signal, waveform, pitch, loudness, timbre	<a href="#">[html]</a>	<a href="#">[ipynb]</a>
	<a href="#">Fourier Analysis of Signals</a>	Discrete/analog signal, sinusoid, exponential, Fourier transform, Fourier representation, DFT, FFT, STFT	<a href="#">[html]</a>	<a href="#">[ipynb]</a>
	<a href="#">Music Synchronization</a>	Chroma feature, dynamic programming, dynamic time warping (DTW), alignment, user interface	<a href="#">[html]</a>	<a href="#">[ipynb]</a>
		Similarity matrix, repetition,	<a href="#">[html]</a>	<a href="#">[ipynb]</a>
			<a href="#">[html]</a>	<a href="#">[ipynb]</a>
	<a href="#">Tempo and Beat Tracking</a>	Onset, novelty, tempo, tempogram, beat, periodicity, Fourier analysis, autocorrelation	<a href="#">[html]</a>	<a href="#">[ipynb]</a>
	<a href="#">Content-Based Audio Retrieval</a>	Identification, fingerprint, indexing, inverted list, matching, version, cover song	<a href="#">[html]</a>	<a href="#">[ipynb]</a>
	<a href="#">Musically Informed Audio Decomposition</a>	Harmonic/percussive separation, signal reconstruction, instantaneous frequency, fundamental frequency (F0), trajectory, nonnegative matrix factorization (NMF)	<a href="#">[html]</a>	<a href="#">[ipynb]</a>

**Part 6: Tempo and Beat Tracking**



# Part B: Basics

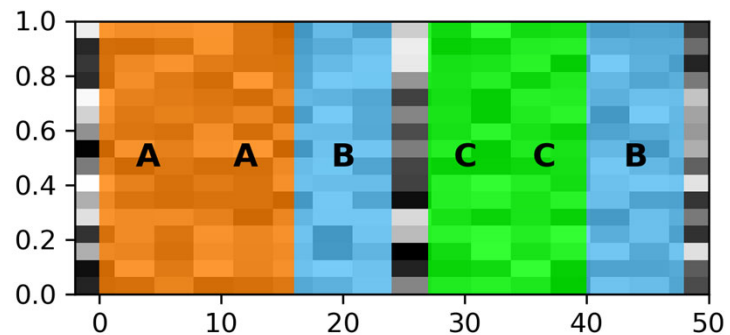
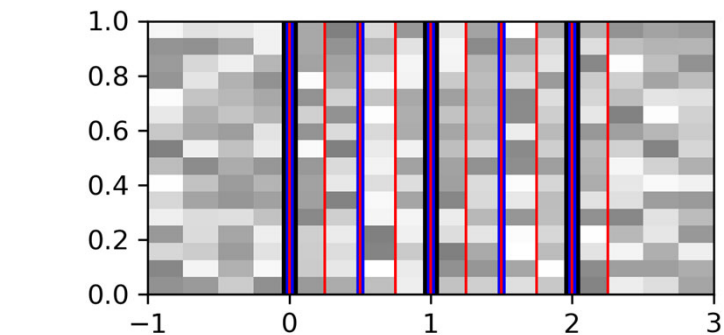
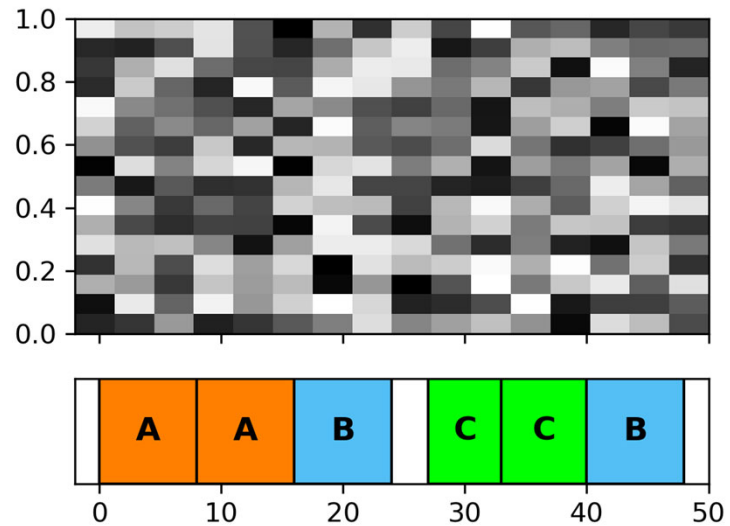
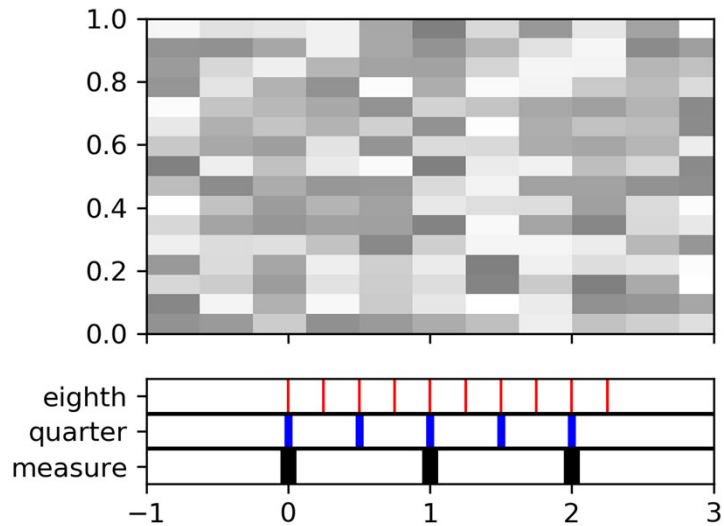


Topic	Description
<a href="#">Get Started</a>	Explanation on how to install and use the FMP notebooks
<a href="#">Installation</a>	Installation of Python using Conda
<a href="#">Jupyter Notebook</a>	Usage of Jupyter notebook framework
<a href="#">Python Basics</a>	Introduction of data types, control structures, and functions
<a href="#">Python Style Guide</a>	Recommendations for programming style
<a href="#">Multimedia</a>	Integration of multimedia objects into notebooks
<a href="#">Python Visualization</a>	Generation of figures and images
<a href="#">Python Audio</a>	Reading and writing audio files
<a href="#">Numba</a>	Acceleration of Python functions via JIT compilation
<a href="#">Annotation Visualization</a>	Visualization of annotations (single value, segments)
<a href="#">Sonification</a>	Sonification methods (onsets, F0 trajectories, pitch, chroma)
<a href="#">libfmp</a>	Library of FMP-specific Python functions
<a href="#">MIR Resources</a>	Links to resources that are useful for MIR

# Part B: Basics

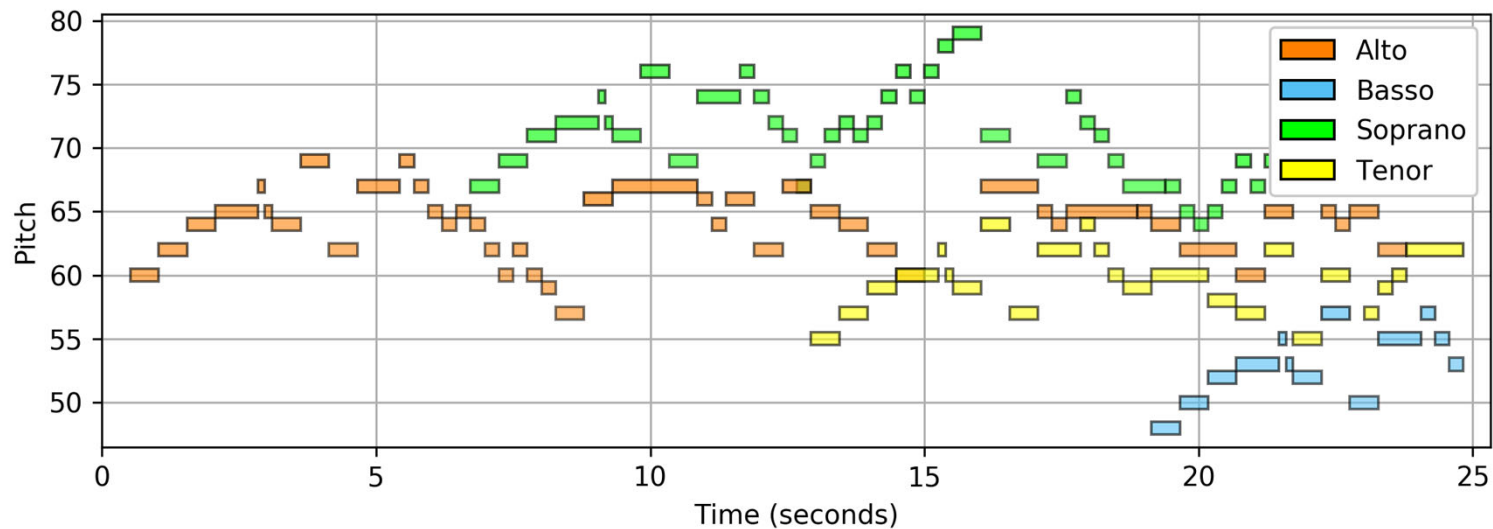
## Annotation Visualization

Examples for visualizing annotations of time positions and segments.



# Part 1: Music Representations

## Symbolic Format: CSV



Visualization of a piano-roll representation (Fugue BWV 846 by Bach).



# Part 1: Music Representations

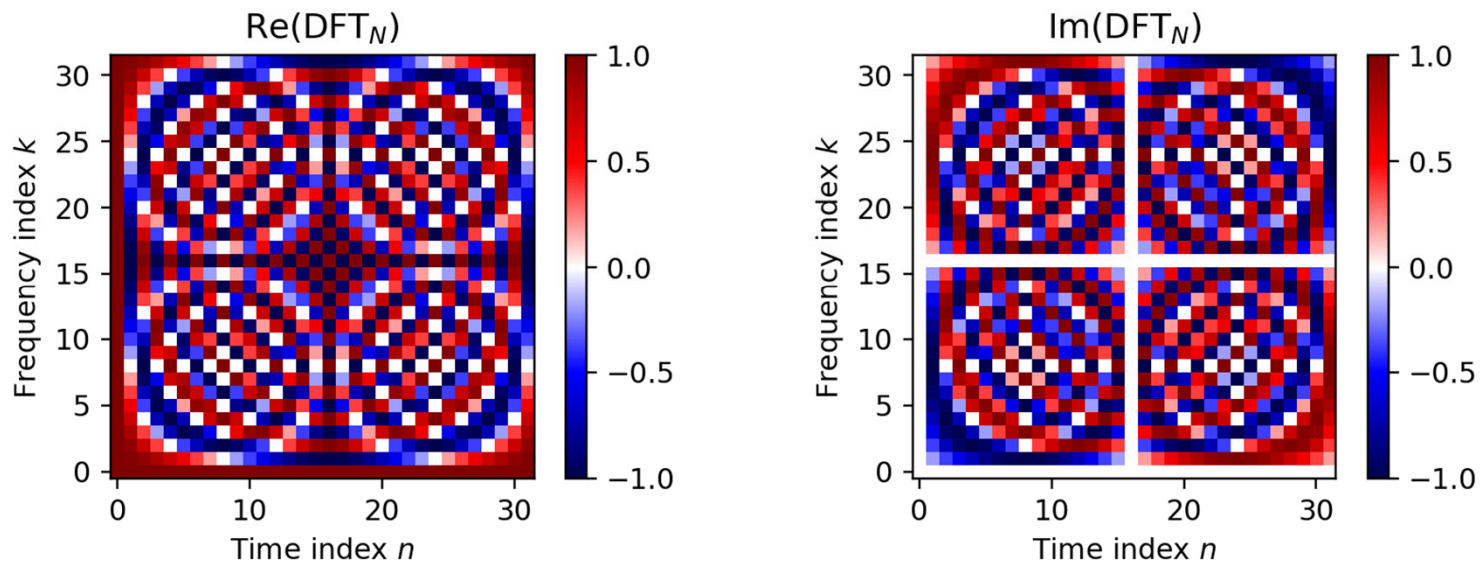
## Waves and Waveforms



Videos illustrating the concepts of transverse, longitudinal, and combined waves.

# Part 2: Fourier Analysis of Signals

## Discrete Fourier Transform (DFT)



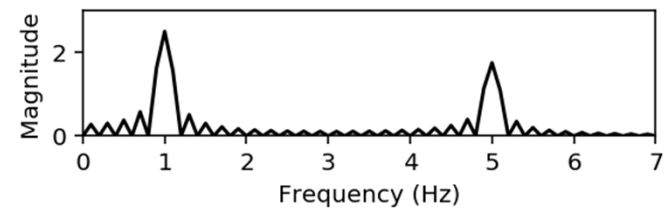
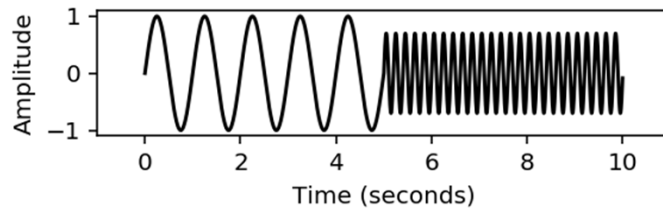
The matrix  $DFT_N$  and a visualization of its real and imaginary parts for the case  $N = 32$

# Part 2: Fourier Analysis of Signals

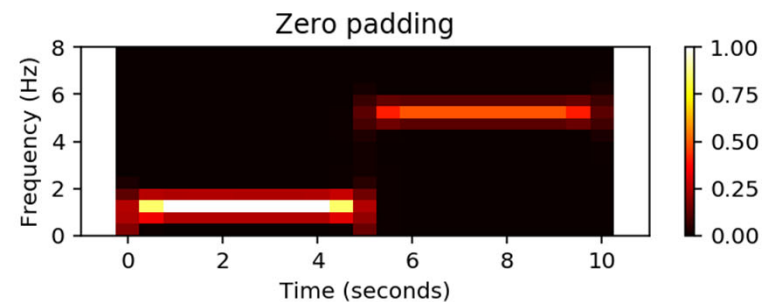
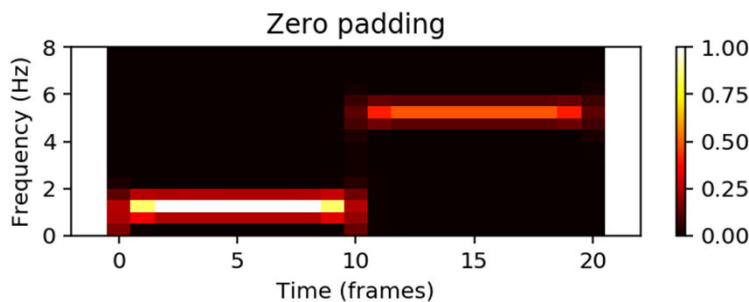
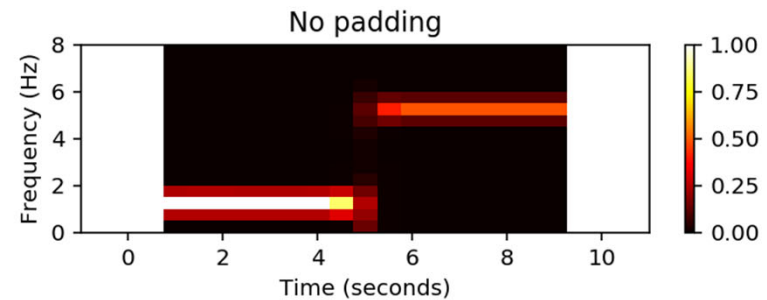
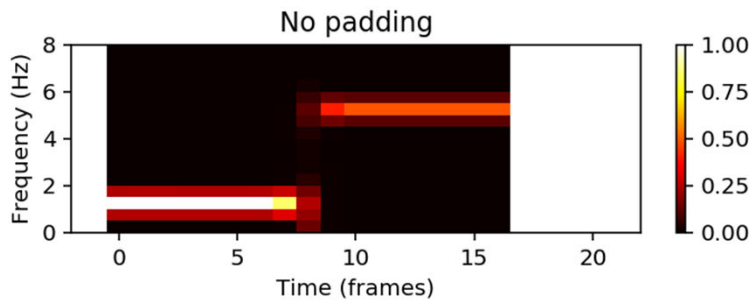


## STFT: Padding

Time-domain signal and magnitude Fourier transform.



Magnitude STFT.

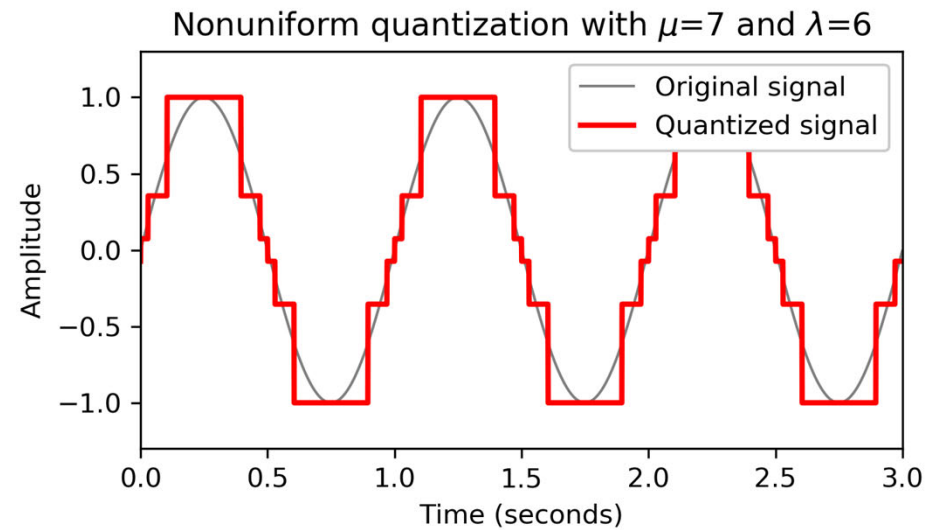
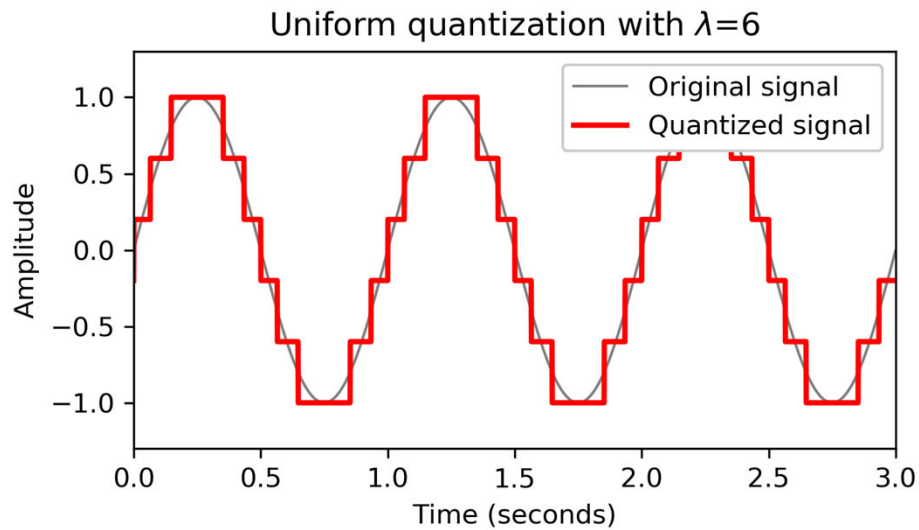


# Part 2: Fourier Analysis of Signals

## Digital Signals: Quantization



Uniform and nonuniform quantization (based on  $\mu$ -law encoding) using  $\lambda = 6$  quantization levels.



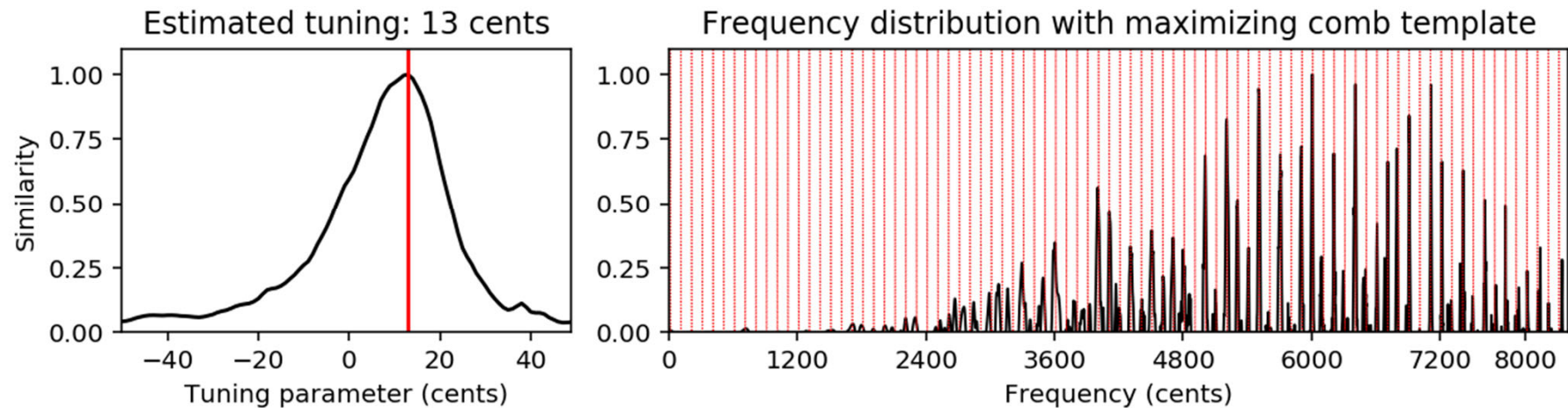


# Part 3: Music Synchronization

## Transposition and Tuning



Tuning procedure using a comb-filter approach.



*p*

*p* *leggieramente*

*cresc.*

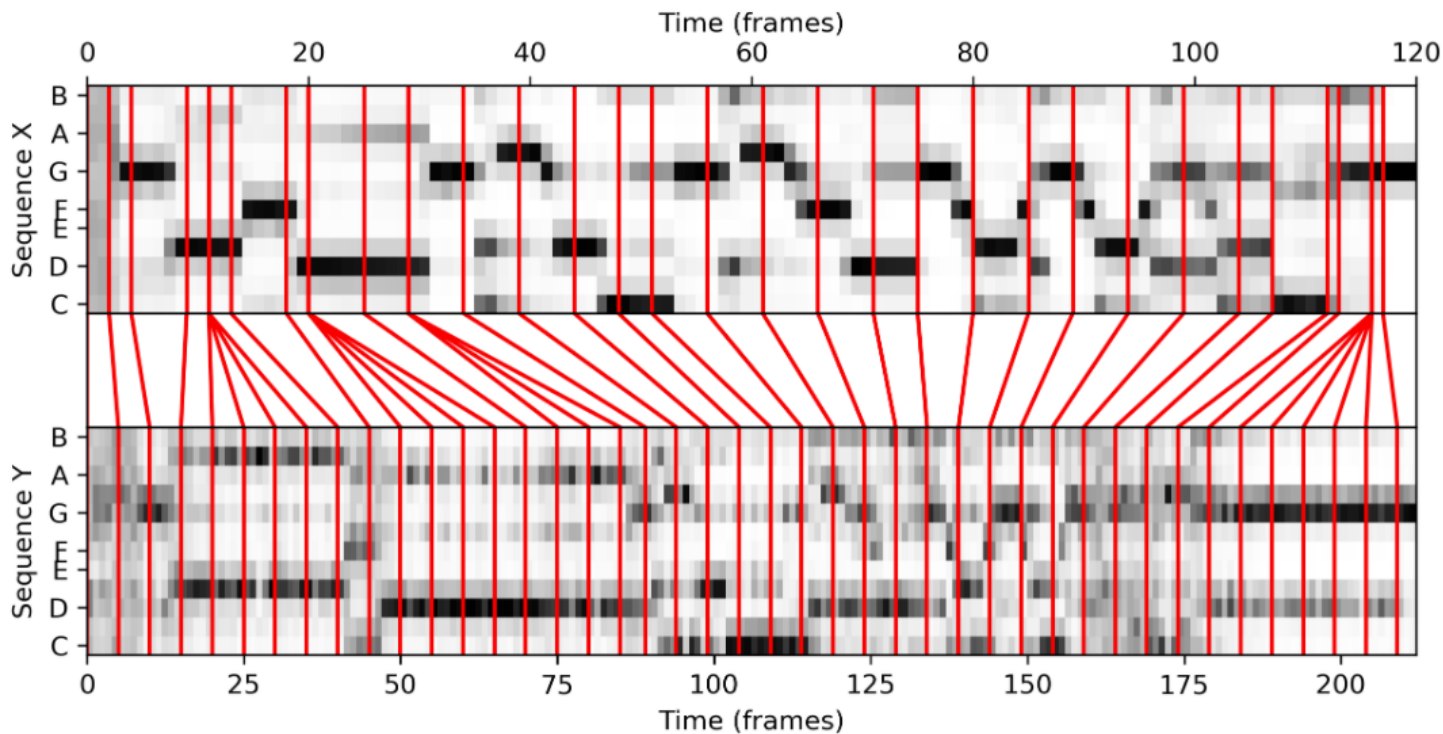
*f*



# Part 3: Music Synchronization

## Music Synchronization

Music synchronization result obtained for two input chromagrams (obtained from two recordings of the beginning of Beethoven's Fifth Symphony).



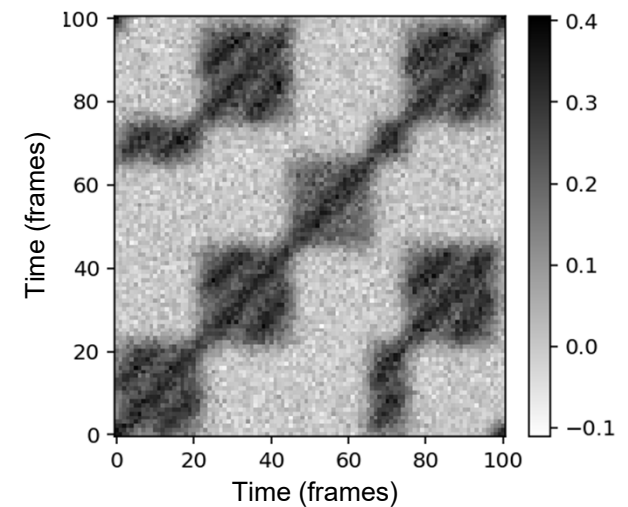
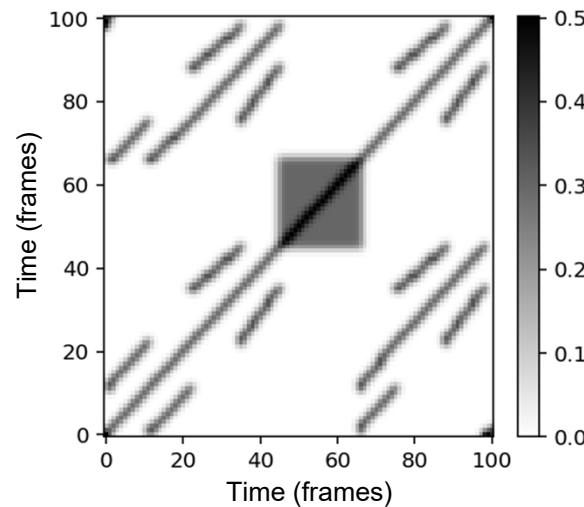
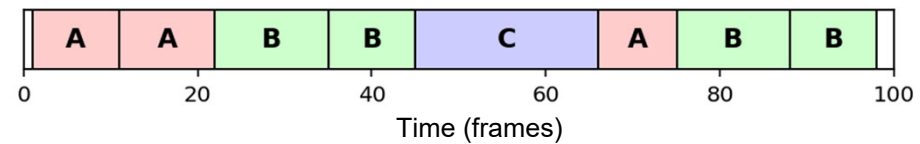
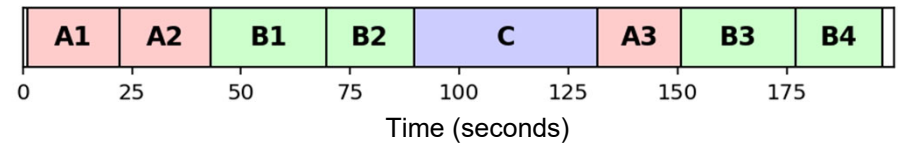
# Part 4: Music Structure Analysis

## SSM: Synthetic Generation

Structure annotation and different synthetically generated SSMs.



	start	end	label
<b>0</b>	0.00	1.01	
<b>1</b>	1.01	22.11	A1
<b>2</b>	22.11	43.06	A2
<b>3</b>	43.06	69.42	B1
<b>4</b>	69.42	89.57	B2
<b>5</b>	89.57	131.64	C
<b>6</b>	131.64	150.84	A3
<b>7</b>	150.84	176.96	B3
<b>8</b>	176.96	196.90	B4
<b>9</b>	196.90	199.64	

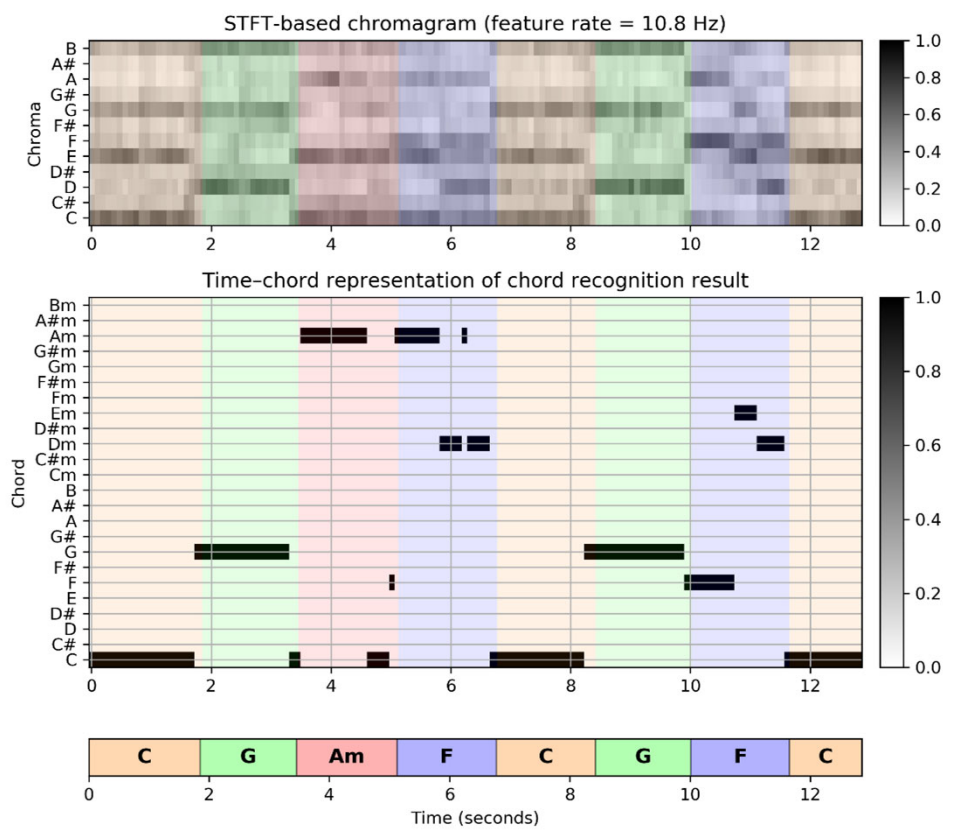


# Part 5: Chord Recognition

## Template-Based Chord Recognition



Chord recognition task illustrated by the first measures of the Beatles song "Let It Be."



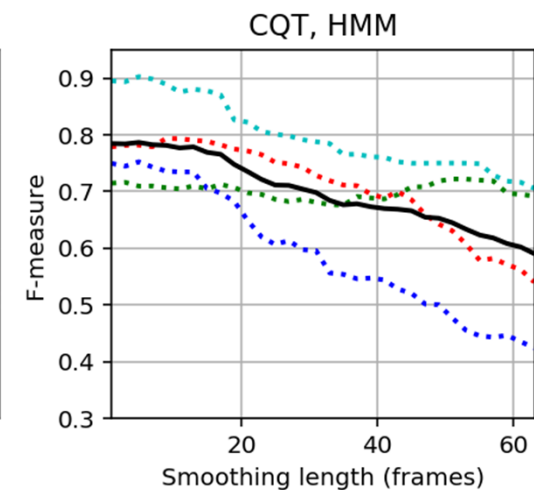
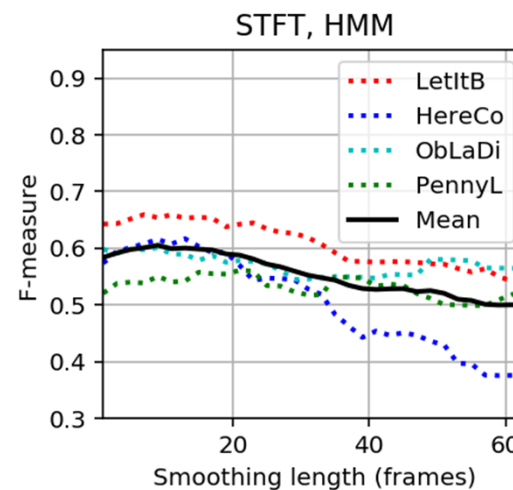
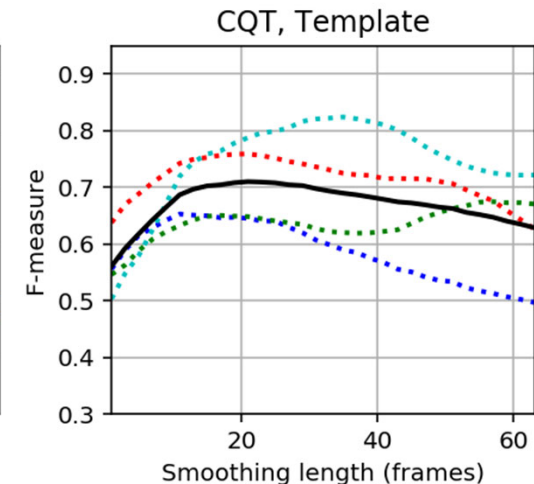
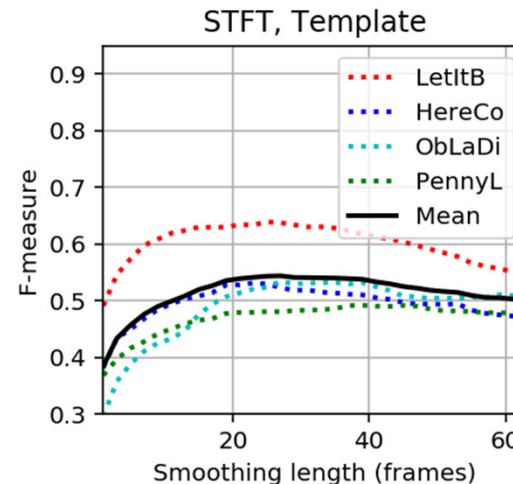
# Part 5: Chord Recognition

## Experiments: Beatles Collection



Prefiltering experiments for a template-based and an HMM-based chord recognizer applied to two different input chroma representations (STFT, CQT).

The evaluation is performed on the basis of four Beatles songs (LetItB, HereCo, ObLaDi, PennyL).

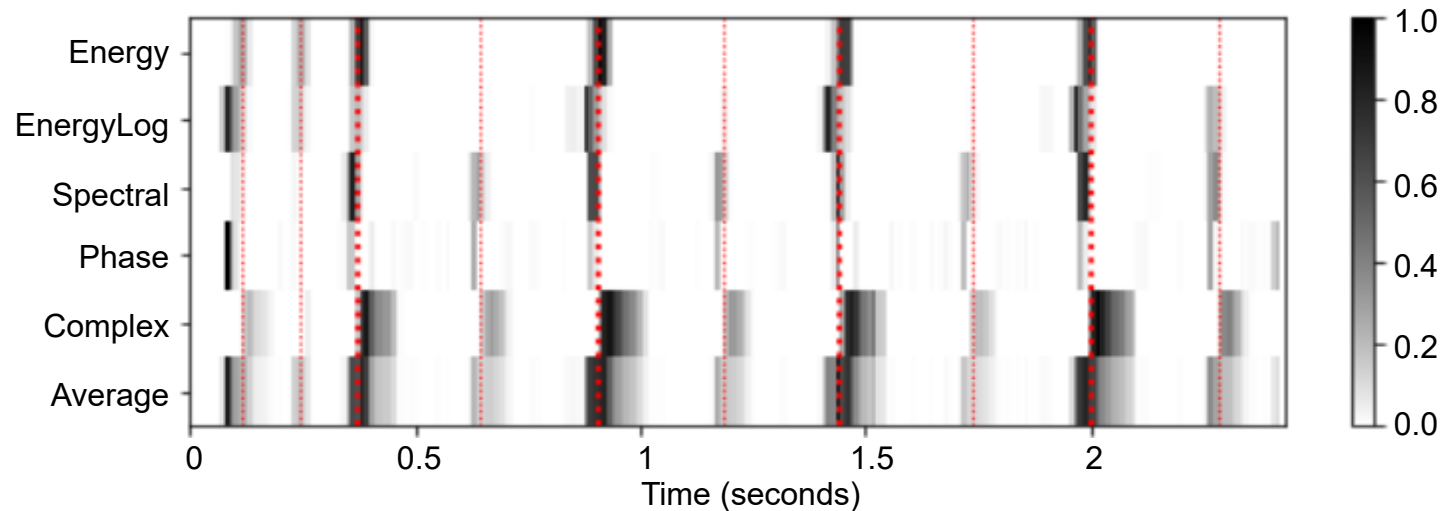
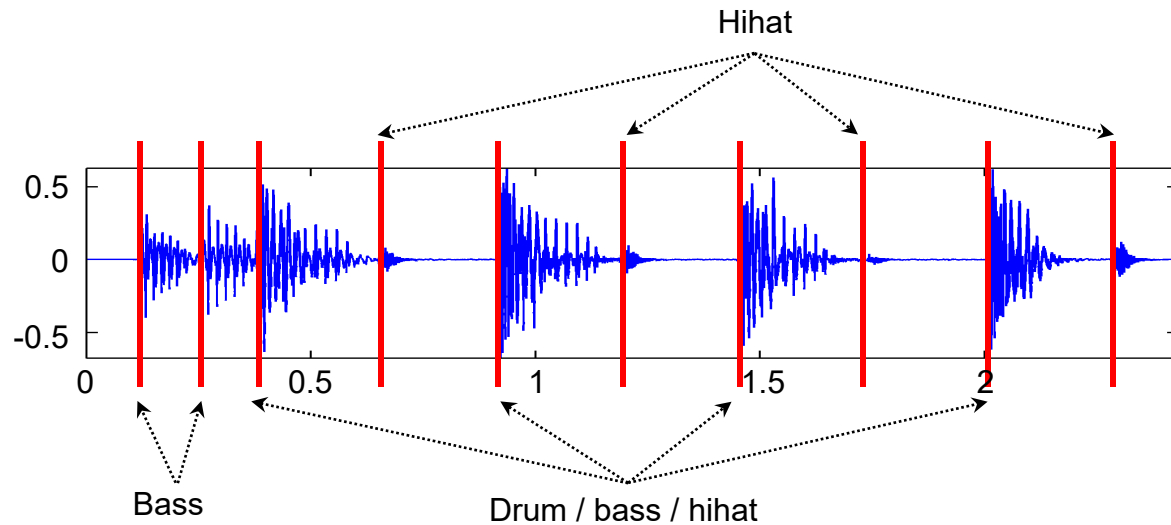


# Part 6: Tempo and Beat Tracking

## Novelty: Comparison of Approaches



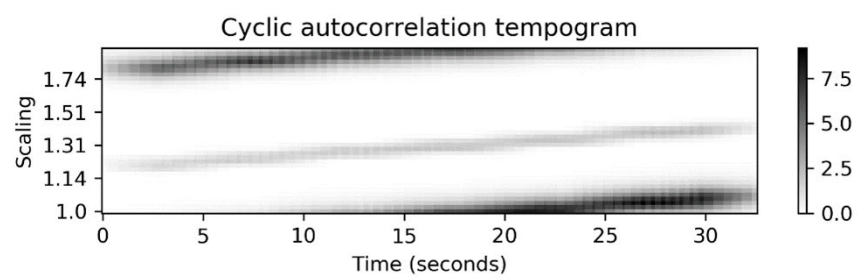
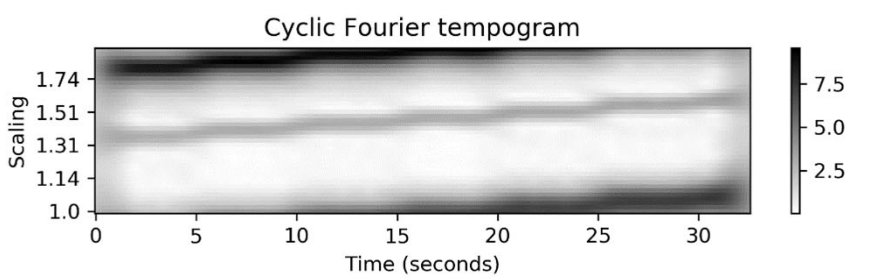
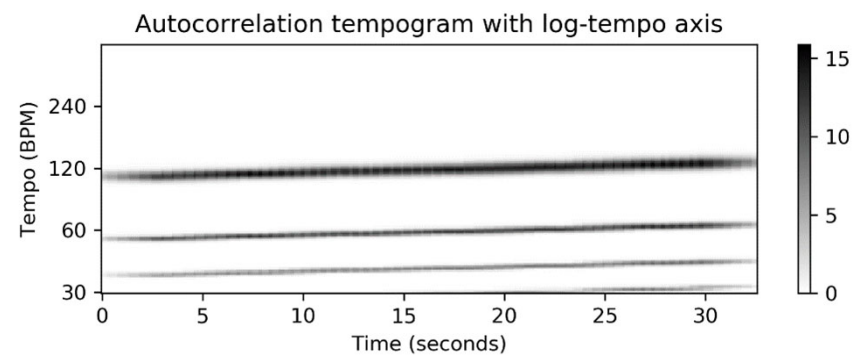
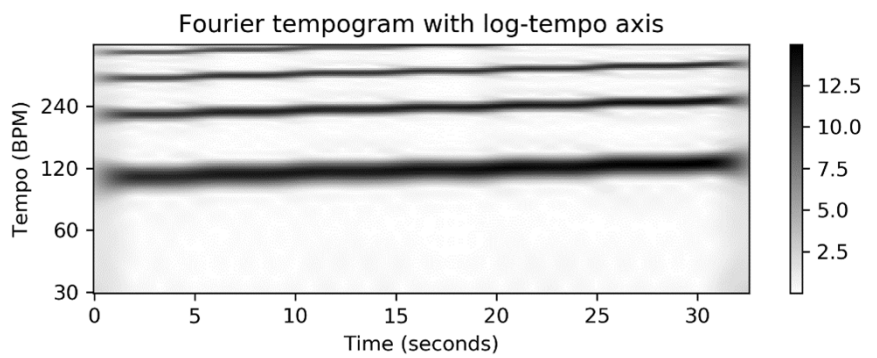
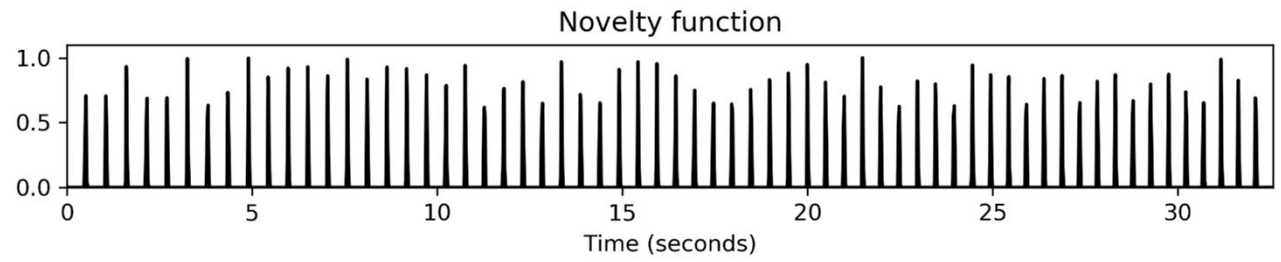
Comparison of novelty detectors using a matrix-based visualization.



# Part 6: Tempo and Beat Tracking

## Cyclic Tempogram

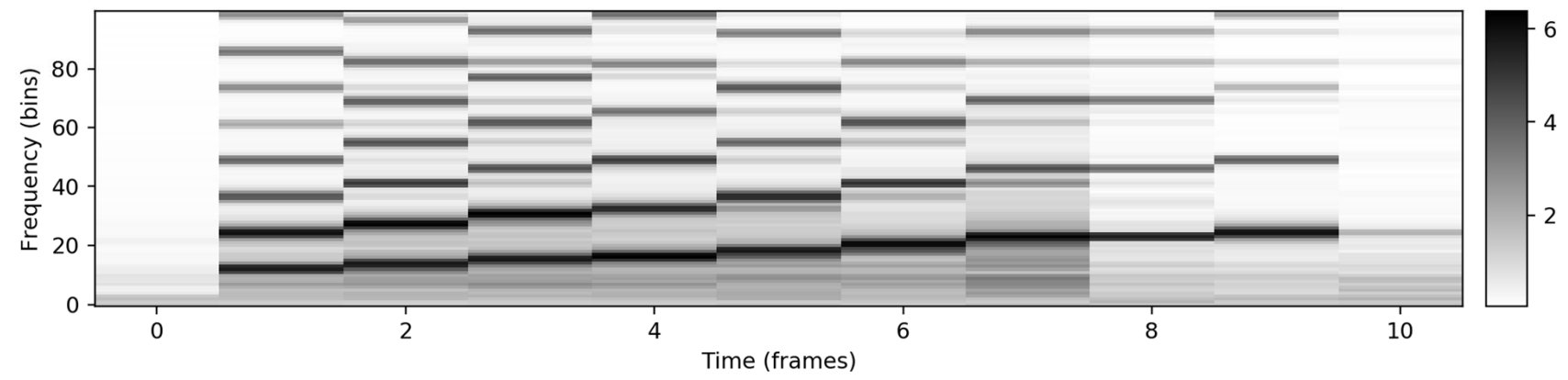
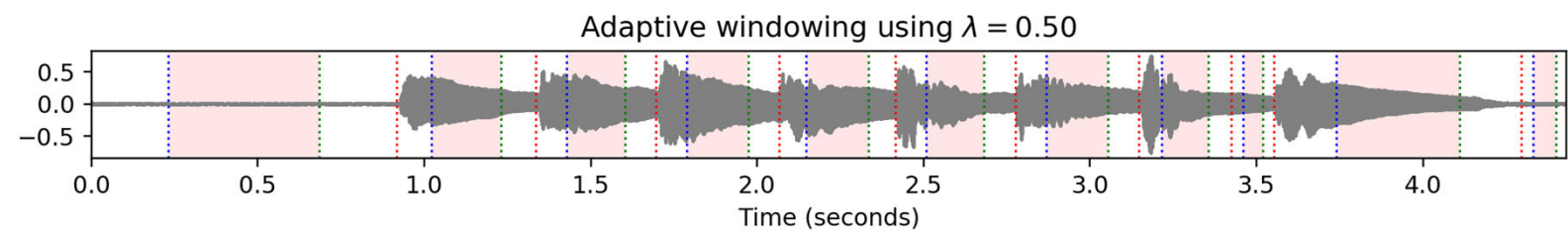
Different tempogram representations of a click track with increasing tempo.



# Part 6: Tempo and Beat Tracking

## Adaptive Windowing

Example of adaptive windowing using a parameter  $\lambda$  to control the neighborhood's relative size to be excluded.



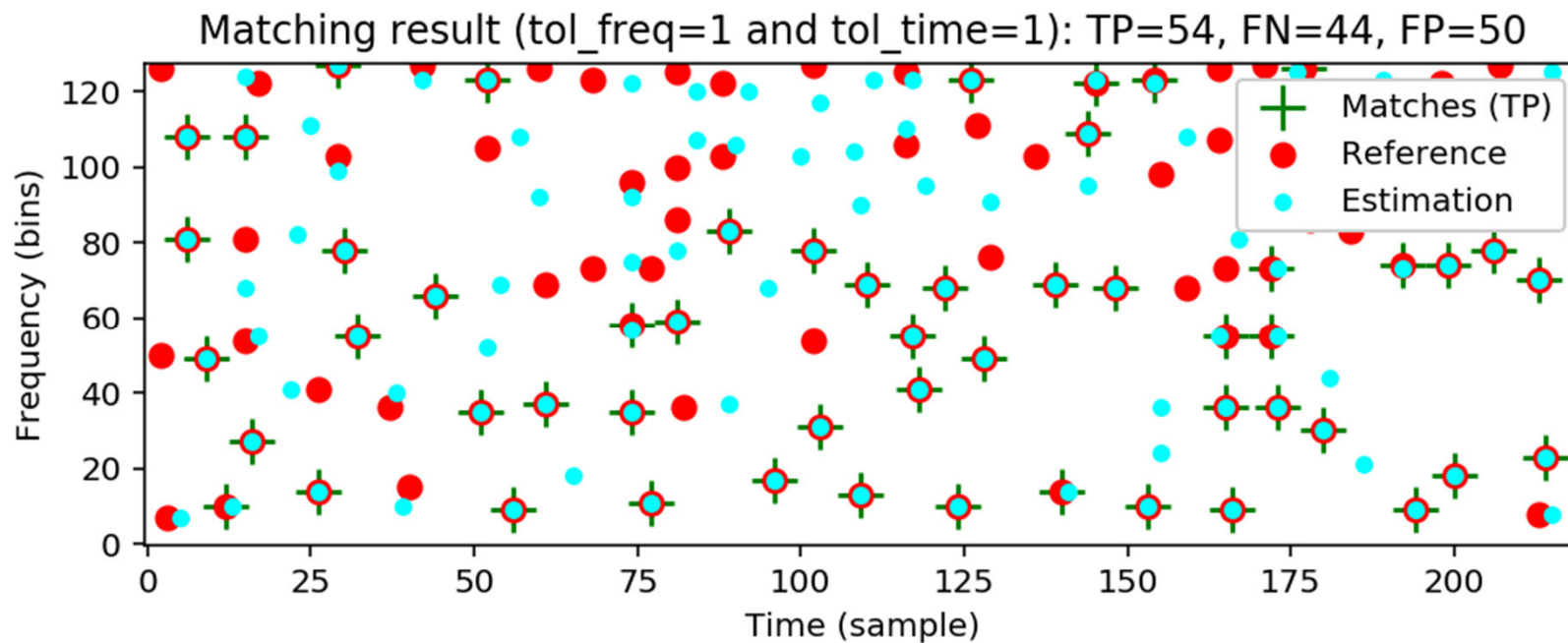


# Part 7: Content-Based Audio Retrieval

## Audio Identification



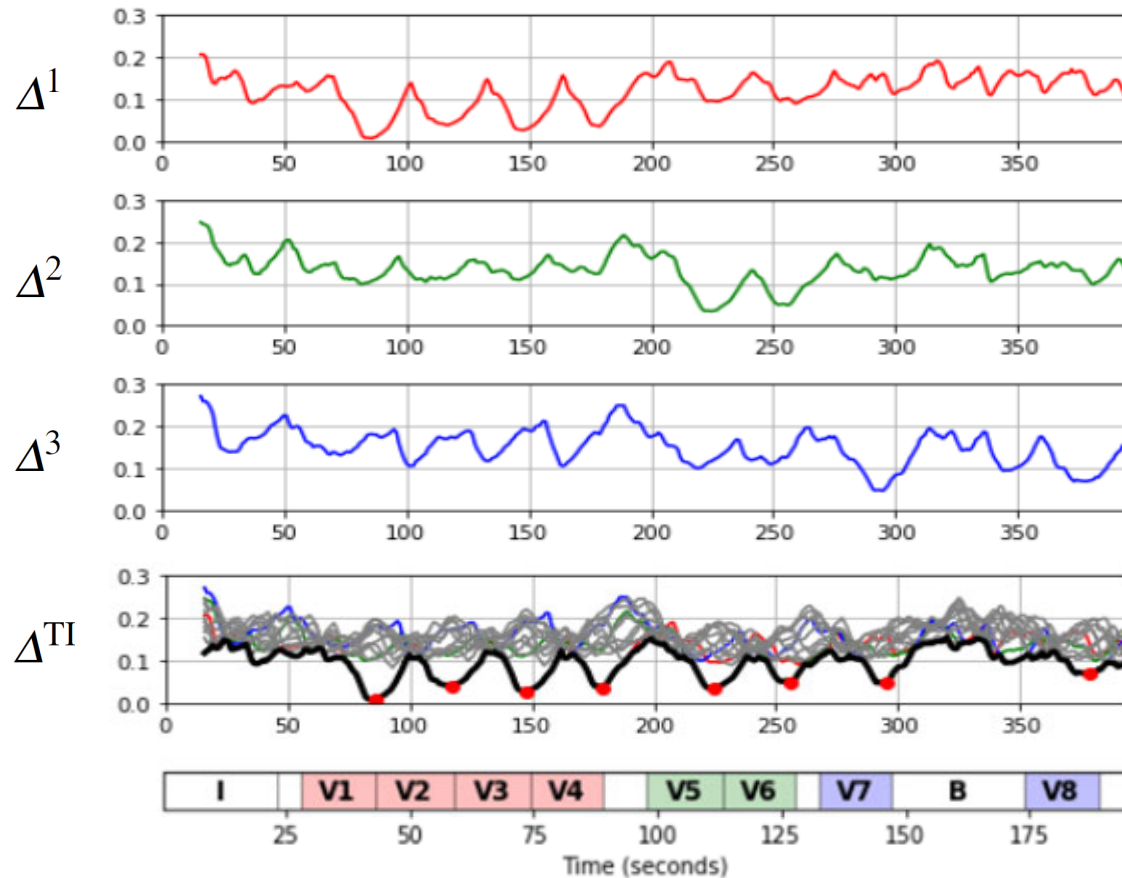
Evaluation measures that indicate the agreement between two constellation maps computed for an original version (Reference) and a noisy version (Estimation).



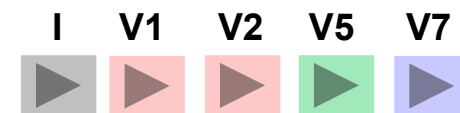
# Part 7: Content-Based Audio Retrieval



## Audio Matching



Transposition-invariant matching function illustrated by Zager and Evans' song "In the Year 2525."



# Part 7: Content-Based Audio Retrieval

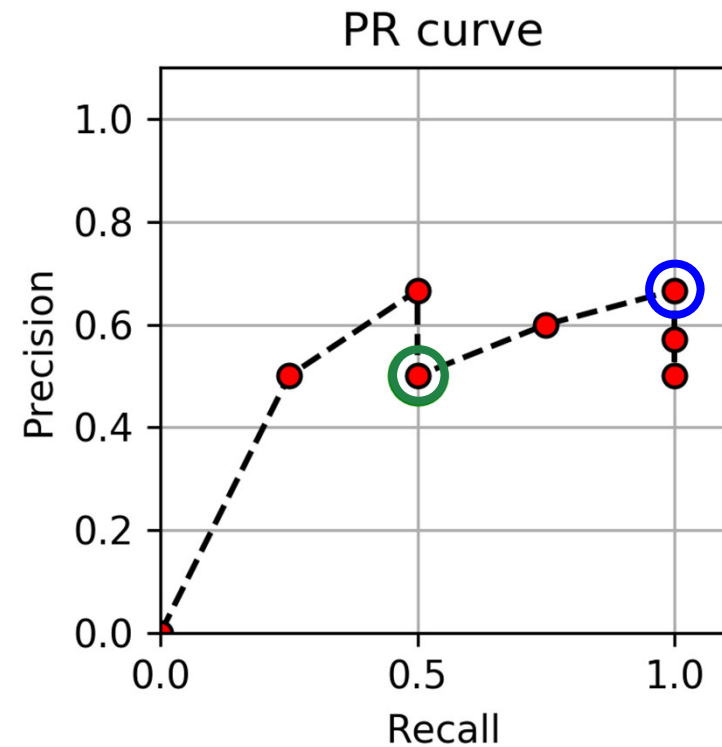
## Evaluation Measures



Various evaluation metrics applied to a toy example.

Rank	ID	Score	$\chi_Q$	P(r)	R(r)	F(r)
1	6	3.70	False	0.00	0.00	0.00
2	3	3.60	True	0.50	0.25	0.33
3	4	3.50	True	0.67	0.50	0.57
4	5	3.20	False	0.50	0.50	0.50
5	8	3.10	True	0.60	0.75	0.67
6	2	2.60	True	0.67	1.00	0.80
7	7	1.50	False	0.57	1.00	0.73
8	1	0.70	False	0.50	1.00	0.67

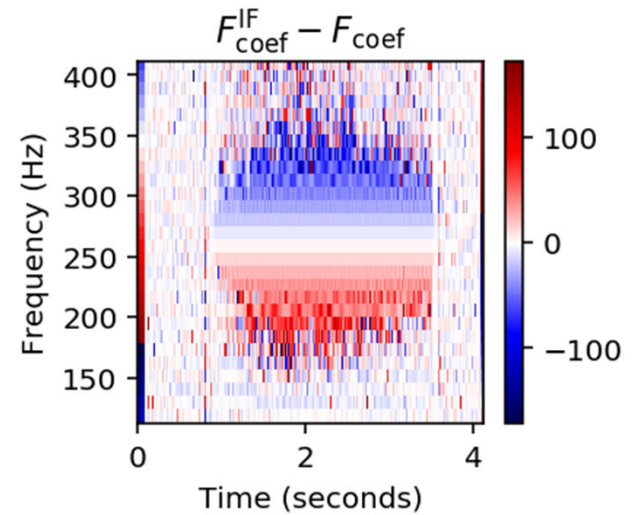
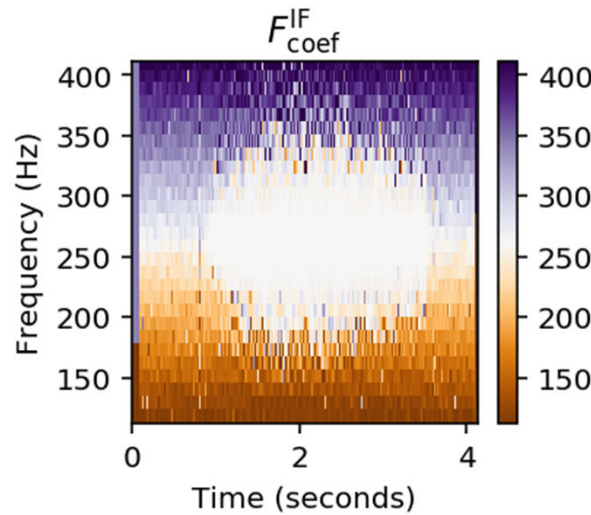
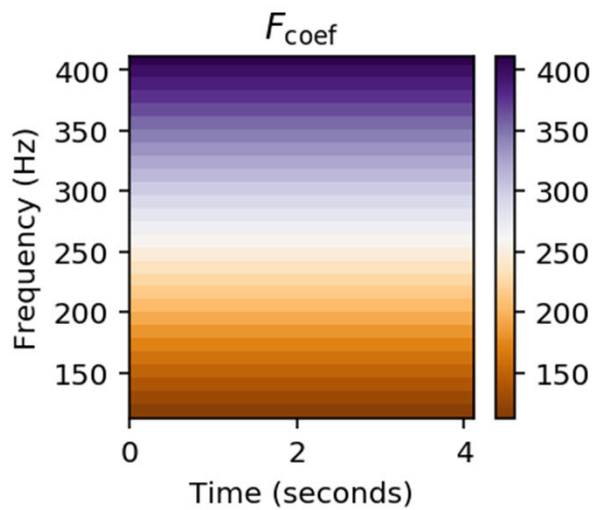
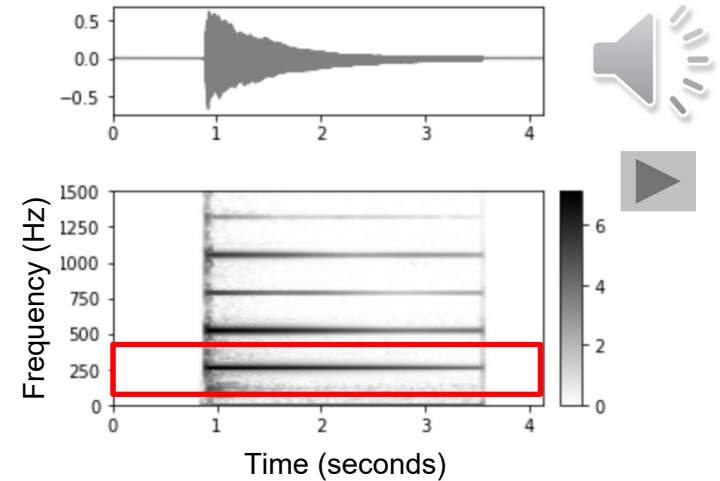
Break-even point = 0.50  
F\_max = 0.80  
Average precision = 0.60833



# Part 8: Audio Decomposition

## Instantaneous Frequency Estimation

Interpretation of time–frequency bins of an STFT using (frame-dependent) instantaneous frequency values.



# Part 8: Audio Decomposition

## Fundamental Frequency Tracking

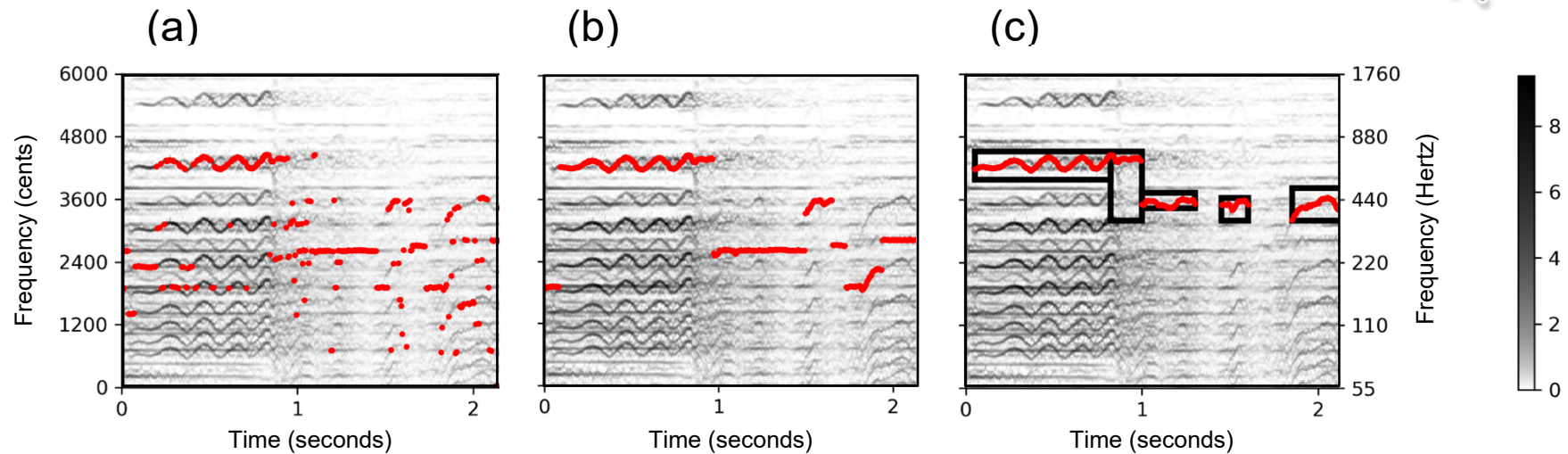


Saliency representation with trajectories computed by

- (a) a frame-wise approach,
- (b) an approach using continuity constraints, and
- (c) a score-informed approach.



Figure 8.10a from [Müller, FMP, Springer 2015]



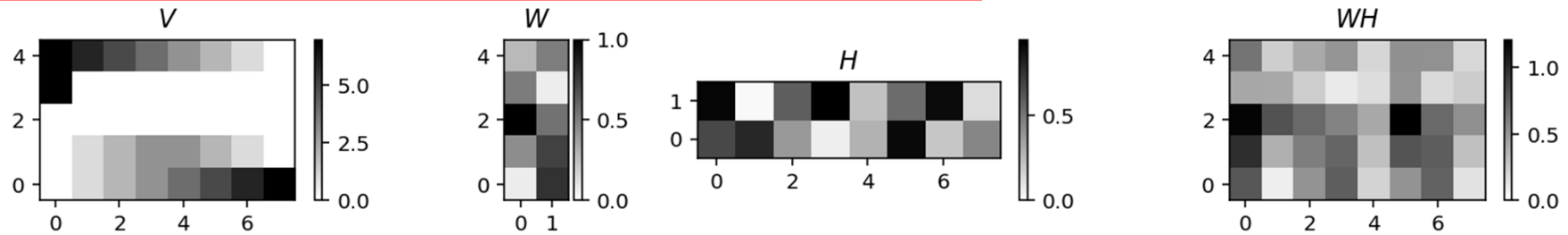
# Part 8: Audio Decomposition

## Nonnegative Matrix Factorization (NMF)

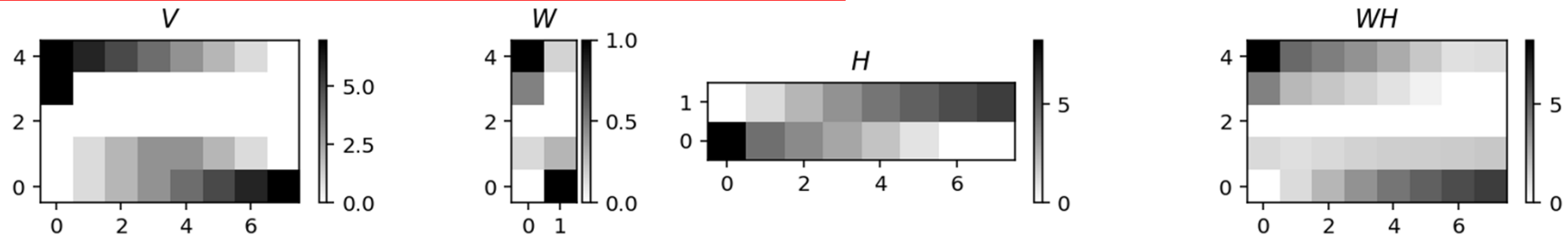
NMF procedure applied to a toy example.



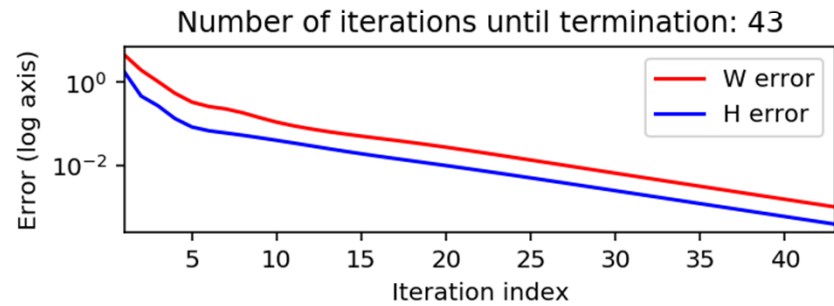
Matrix  $V$  and randomly initialized matrices  $W$  and  $H$ .

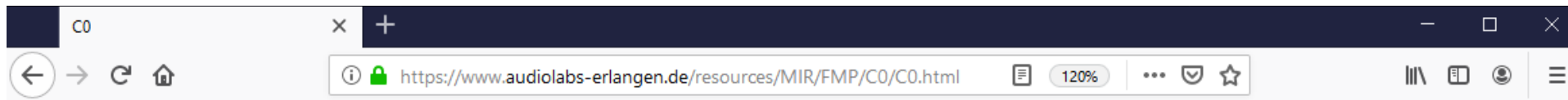


Matrix  $V$  and matrices  $W$  and  $H$  after training.



Error terms over iteration.



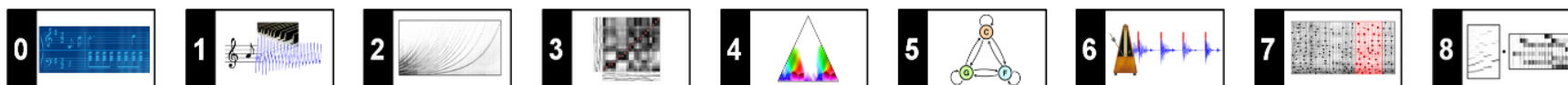


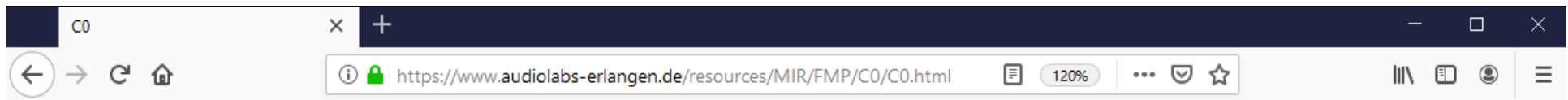
# FMP Notebooks

Python Notebooks for Fundamentals of Music Processing



<https://www.audiolabs-erlangen.de/FMP>



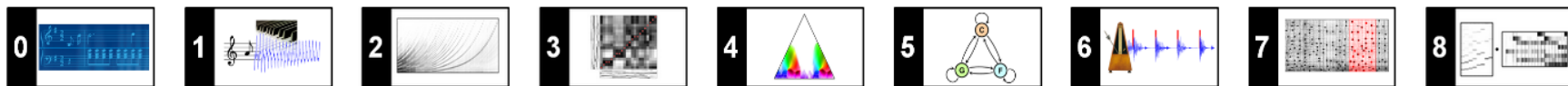


# FMP Notebooks

Python Notebooks for Fundamentals of Music Processing

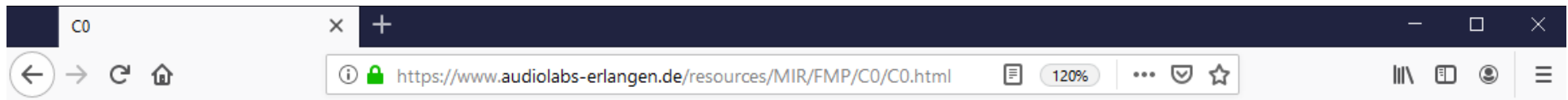


<https://www.audiolabs-erlangen.de/FMP>



**Basics + 8 Chapters**



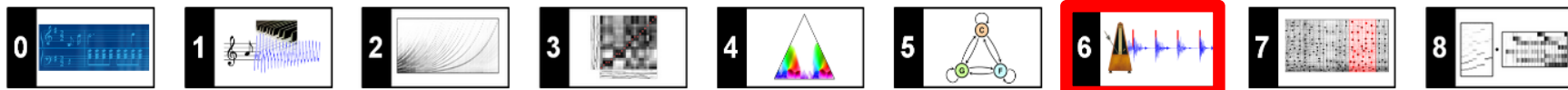


# FMP Notebooks

Python Notebooks for Fundamentals of Music Processing

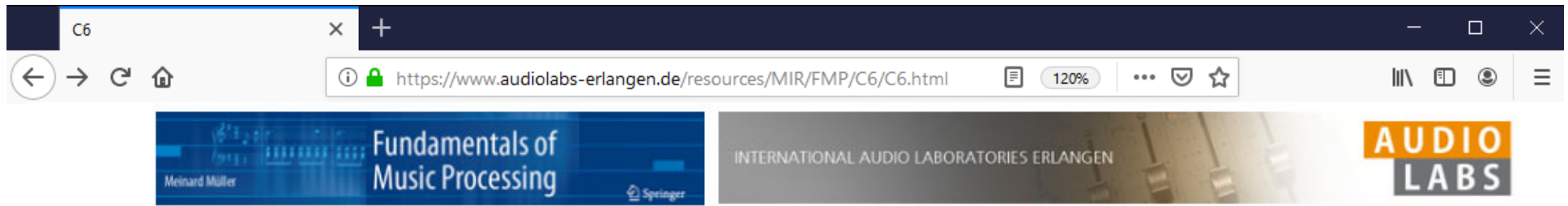


<https://www.audiolabs-erlangen.de/FMP>

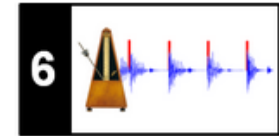


**Basics + 8 Chapters**

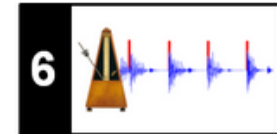
**Tempo and Beat Tracking**



# Tempo and Beat Tracking



# Tempo and Beat Tracking

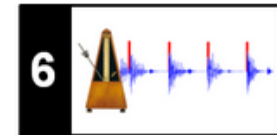


## Definition

We assume that we are given a discrete-time novelty function  $\Delta : \mathbb{Z} \rightarrow \mathbb{I}$  indicate note onset candidates. The idea of Fourier analysis is to detect local periodicity in novelty curve by comparing it with windowed sinusoids. A high correlation of  $\Delta$  with a windowed sinusoid indicates a periodicity of the novelty curve (given a suitable phase). This correlation (along with the phase) can be computed via the short-time Fourier transform. To this end, we fix a window function  $w : \mathbb{Z} \rightarrow \mathbb{R}$  of length centered at  $n = 0$  (e.g., a sampled Hann window). Then, for a frequency parameter  $\omega \in \mathbb{R}_{\geq 0}$  and time parameter  $n \in \mathbb{Z}$ , the complex Fourier coefficient is defined by

$$\mathcal{F}(n, \omega) := \sum_{m \in \mathbb{Z}} \Delta(m) \bar{w}(m - n) \exp(-2\pi i \omega m).$$

# Tempo and Beat Tracking



## Definition

We assume that we are given a discrete-time novelty function  $\Delta : \mathbb{Z} \rightarrow \mathbb{I}$  indicate note onset and in novelty curve by computing the correlation of  $\Delta$  with a window function  $w$  (given a suitable phase). This correlation (along with the phase) is analyzed using the short-time Fourier transform. To this end, we fix a window function  $w$  (e.g., a sampled Hann window). Then, for a time parameter  $n \in \mathbb{Z}$ , the complex Fourier coefficient is defined by

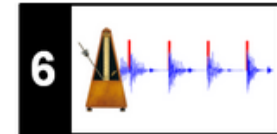
Explanations

Theory

Mathematics

$$\mathcal{F}(n, \omega) := \sum_{m \in \mathbb{Z}} \Delta(m) \bar{w}(m - n) \exp(-2\pi i \omega m).$$

# Tempo and Beat Tracking



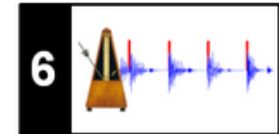
## Example: Shostakovich

In the following example, we consider an excerpt of a recording of Dimitri Shostakovich's Suite for Variety Orchestra No. 1. The score version of the excerpt.



We start with a [spectral-based novelty function](#) resampled to  $F_s^\Delta$  :  
Furthermore, we use a window size corresponding to 5 seconds ( $l$ )

# Tempo and Beat Tracking

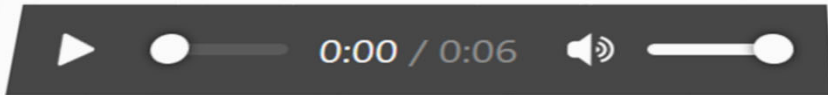


## Example: Shostakovich

In the following example, we consider an excerpt from the Suite for Variety Orchestra.

Music Examples

Annotations

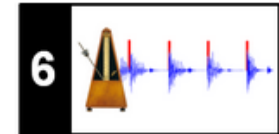


Audio

We start with a [spectral-based novelty function](#) resampled to  $F_s^\Delta$ . Furthermore, we use a window size corresponding to 5 seconds ( $\Delta$ ).

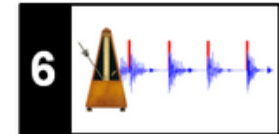
Links

# Tempo and Beat Tracking



```
In [2]: def compute_sinusoid_optimal(c, tempo, n, Fs, N
        """Compute windowed sinusoid with optimal p
        Notebook: C6/C6S2_TempogramFourier.ipynb
        Args:
            c: Coefficient of tempogram (c=X(k,n))
            tempo: Tempo parameter corresponding to
            _coef_BPM[k])
            n: Frame parameter of c
            Fs: Sampling rate
            N: Window length
            H: Hop size
```

# Tempo and Beat Tracking



```
In [2]: def compute_sinusoid_optimal_p(n, Fs, N, H):  
        """Compute windowed sinusoid coefficients for a given tempo parameter p  
        (in BPM) and frame parameter n. The function returns the coefficient c and the  
        tempo parameter corresponding to the coefficient c. The function also returns  
        the frame parameter n and the sampling rate Fs. The function also returns  
        the window length N and the hop size H. The function also returns the  
        coefficient c and the tempo parameter corresponding to the coefficient c.  
        """  
        c: Coefficient of tempogram (c=X(k,n))  
        tempo: Tempo parameter corresponding to c  
        _coef_BPM[k])  
        n: Frame parameter of c  
        Fs: Sampling rate  
        N: Window length  
        H: Hop size
```

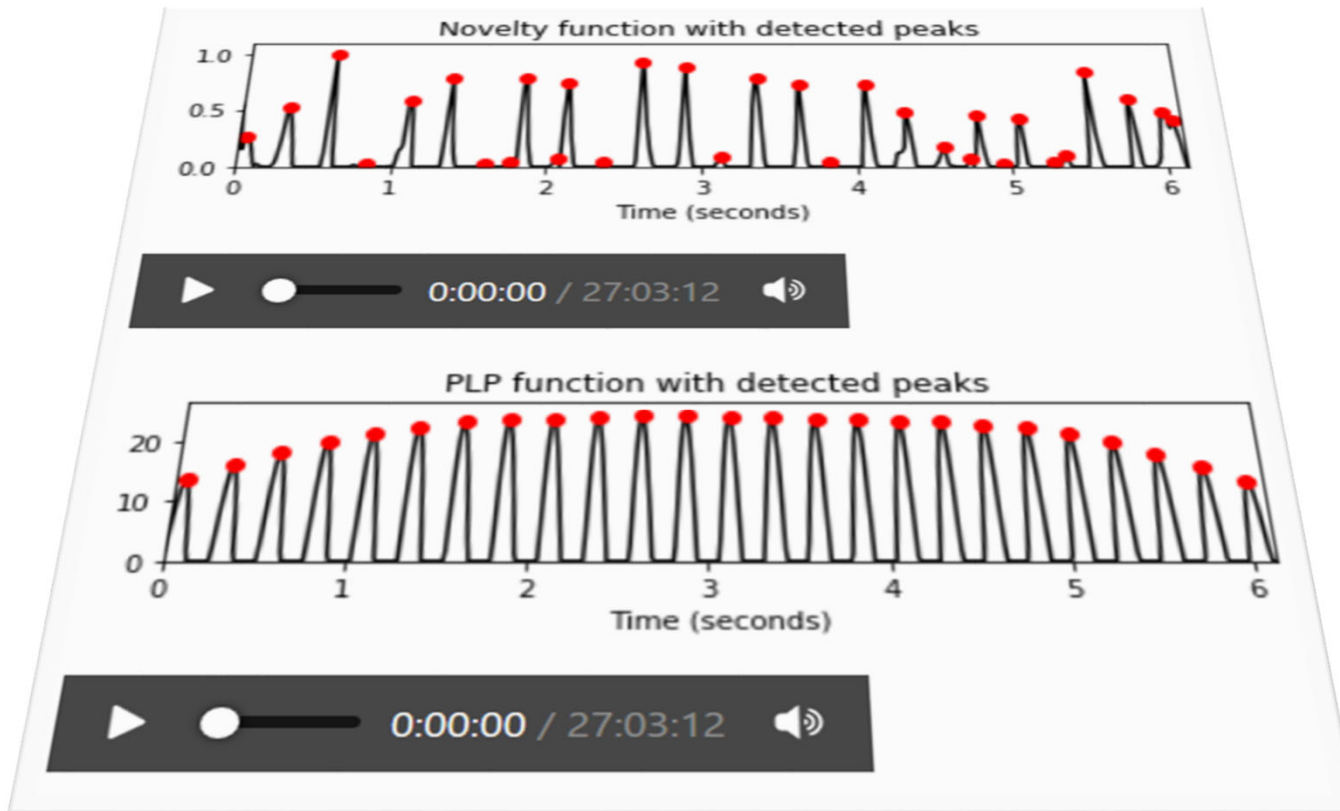
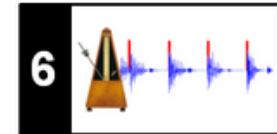
Python Code

Algorithms

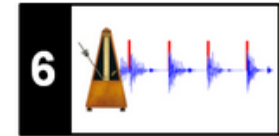
Functions



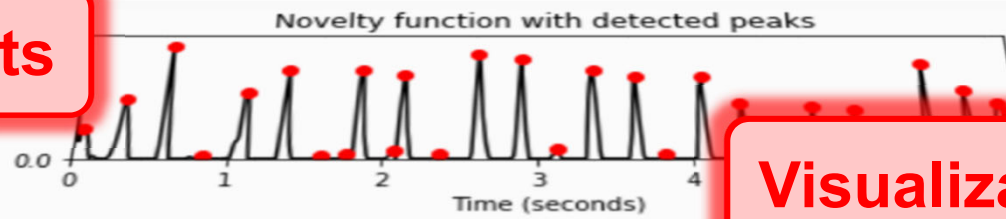
# Tempo and Beat Tracking



# Tempo and Beat Tracking

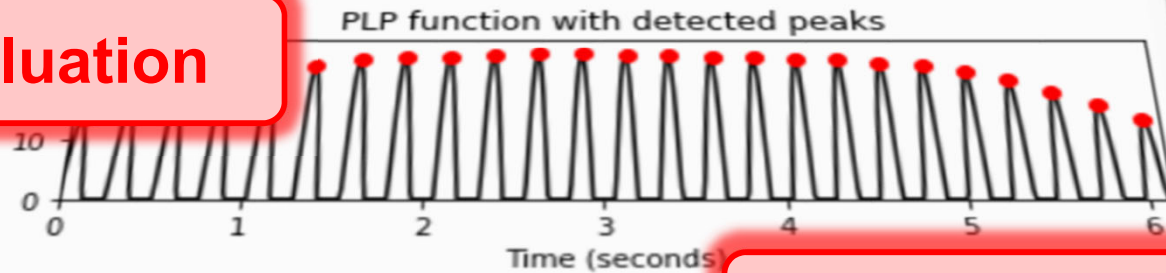


Results



Visualization

Evaluation



Sonification

# FMP Notebooks

Teaching

Understanding

Programming

Baselines

Research

Multimedia

# References

- Meinard Müller: Fundamentals of Music Processing – Using Python and Jupyter Notebooks. 2nd Edition, Springer, 2021.  
<https://www.springer.com/gp/book/9783030698072>
- Meinard Müller and Frank Zalkow: libfmp: A Python Package for Fundamentals of Music Processing. Journal of Open Source Software (JOSS), 6(63): 1–5, 2021.  
<https://joss.theoj.org/papers/10.21105/joss.03326>
- Meinard Müller: An Educational Guide Through the FMP Notebooks for Teaching and Learning Fundamentals of Music Processing. Signals, 2(2): 245–285, 2021.  
<https://www.mdpi.com/2624-6120/2/2/18>
- Meinard Müller and Frank Zalkow: FMP Notebooks: Educational Material for Teaching and Learning Fundamentals of Music Processing. Proc. International Society for Music Information Retrieval Conference (ISMIR): 573–580, 2019.  
<https://zenodo.org/record/3527872#.YOhEQOgzaUk>
- Meinard Müller, Brian McFee, and Katherine Kinnaird: Interactive Learning of Signal Processing Through Music: Making Fourier Analysis Concrete for Students. IEEE Signal Processing Magazine, 38(3): 73–84, 2021.  
<https://ieeexplore.ieee.org/document/9418542>

# Resources (Group Meinard Müller)

- FMP Notebooks:

<https://www.audiolabs-erlangen.de/FMP>

- libfmp:

<https://github.com/meinardmueller/libfmp>

- synctoolbox:

<https://github.com/meinardmueller/synctoolbox>

- libtsm:

<https://github.com/meinardmueller/libtsm>

- Preparation Course Python (PCP) Notebooks:

<https://www.audiolabs-erlangen.de/resources/MIR/PCP/PCP.html>

<https://github.com/meinardmueller/PCP>

# Resources

- librosa:  
<https://librosa.org/>
- madmom:  
<https://github.com/CPJKU/madmom>
- Essentia Python tutorial:  
[https://essentia.upf.edu/essentia\\_python\\_tutorial.html](https://essentia.upf.edu/essentia_python_tutorial.html)
- mirdata:  
<https://github.com/mir-dataset-loaders/mirdata>
- open-unmix:  
<https://github.com/sigsep/open-unmix-pytorch>
- Open Source Tools & Data for Music Source Separation:  
<https://source-separation.github.io/tutorial/landing.html>

