

Tutorial T3, EUROGRAPHICS
Saarbrücken, May 8, 2023



Learning with Music Signals: Technology Meets Education

FMP Notebooks

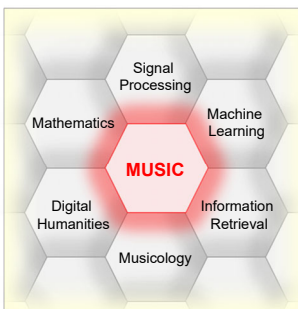
Meinard Müller

International Audio Laboratories Erlangen
meinard.mueller@audiolabs-erlangen.de

Music Processing



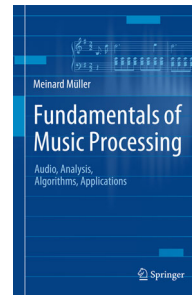
Music Processing: A Multifaceted Research Area



Music ...

- important part of our lives ...
- ... Spotify, Pandora, iTunes, ...
- interdisciplinary research
- intuitive entry point to education

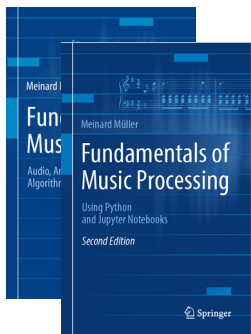
Fundamentals of Music Processing (FMP)



Meinard Müller
Fundamentals of Music Processing
Audio, Analysis, Algorithms, Applications
Springer, 2015

Accompanying website:
www.music-processing.de

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2nd edition
Meinard Müller
Fundamentals of Music Processing
Using Python and Jupyter Notebooks
Springer, 2021

Fundamentals of Music Processing (FMP)

Chapter	Music Processing Scenario
1	Music Representations
2	Fourier Analysis of Signals
3	Music Synchronization
4	Music Structure Analysis
5	Chord Recognition
6	Tempo and Beat Tracking
7	Content-Based Audio Retrieval
8	Musically Informed Audio Decomposition

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FMP Notebooks: Education & Research

FMP Notebooks
Python Notebooks for Fundamentals of Music Processing

The FMP notebooks offer a collection of educational material closely following the textbook [Fundamentals of Music Processing \(FMP\)](#). This is the starting website, which is opened when calling <https://www.audiolabs-erlangen.de/FMP>. Besides giving an [overview](#), this website provides information on the license, the main contributors, and some links.

<https://www.audiolabs-erlangen.de/FMP>

FMP Notebooks: Education & Research

- ... provide educational material for teaching and learning fundamentals of music processing.
- ... combine textbook-like explanations, technical concepts, mathematical details, Python code examples, illustrations, and sound examples.
- ... bridge the gap between theory and practice being based on interactive Jupyter notebook framework.
- ... are freely accessible under a Creative Commons license.

<https://www.audiolabs-erlangen.de/FMP>

FMP Notebooks

Part	Title	Notions, Techniques & Algorithms	HTML	IPYNB
B	Basics	Basic information on Python, Jupyter notebooks, Anaconda package management system, Python environments, visualizations, and other topics	html	ipynb
0	Overview	Overview of the notebooks (https://www.audiolabs-erlangen.de/FMP)	html	ipynb
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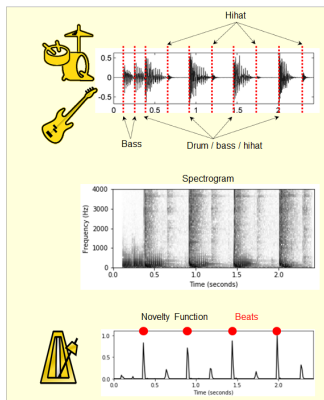
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Part 6: Tempo and Beat Tracking

Part 6: Tempo and Beat Tracking

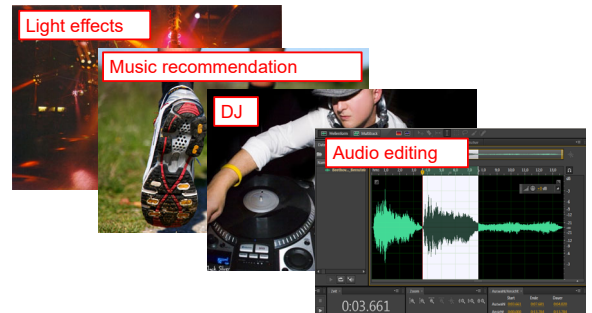


- When listening to a piece of music, we as humans are often able to tap along with the musical beat
- Automated beat tracking: Simulate this cognitive process by a computer



Tempo and Beat Tracking

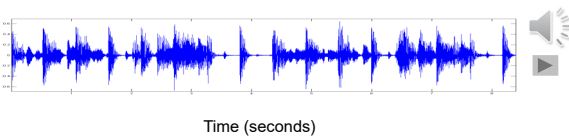
Basic task: "Tapping the foot when listening to music"



Tempo and Beat Tracking

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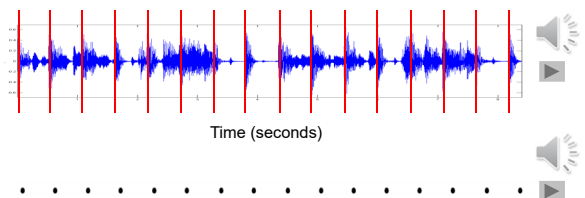
Example: Queen – Another One Bites The Dust



Tempo and Beat Tracking

Basic task: "Tapping the foot when listening to music"

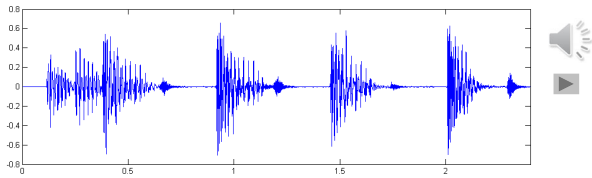
Example: Queen – Another One Bites The Dust



Tempo and Beat Tracking

Tasks

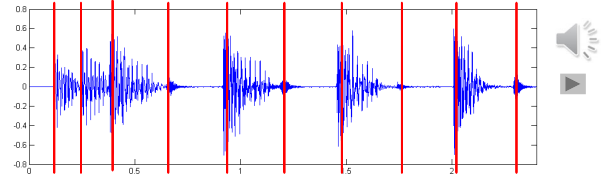
- Onset detection
- Beat tracking
- Tempo estimation



Tempo and Beat Tracking

Tasks

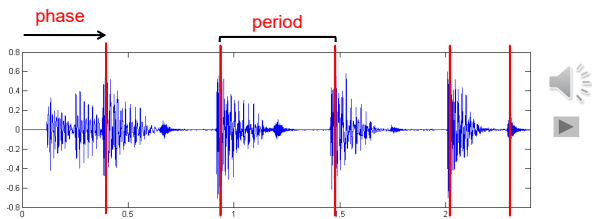
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Tempo and Beat Tracking

Tasks

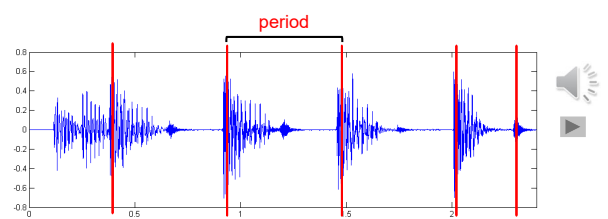
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Tempo and Beat Tracking

Tasks

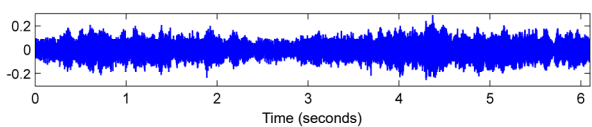
- Onset detection
 - Beat tracking
 - Tempo estimation
- Tempo := 60 / period
Beats per minute (BPM)



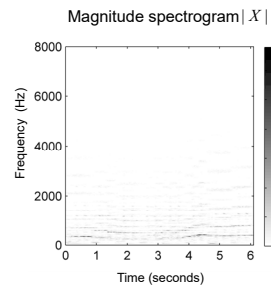
Onset Detection (Spectral Flux)



Audio recording



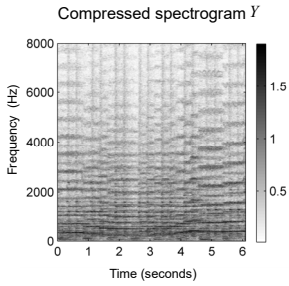
Onset Detection (Spectral Flux)



Steps:

- Spectrogram

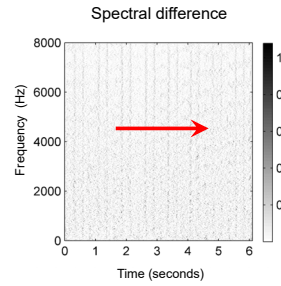
Onset Detection (Spectral Flux)



Steps:

1. Spectrogram
2. Logarithmic compression

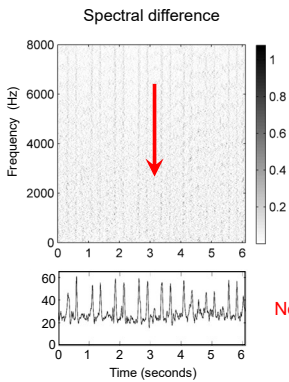
Onset Detection (Spectral Flux)



Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification

Onset Detection (Spectral Flux)

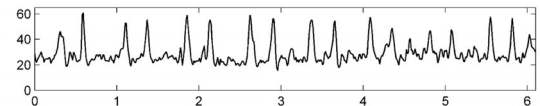


Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation

Onset Detection (Spectral Flux)

Novelty function



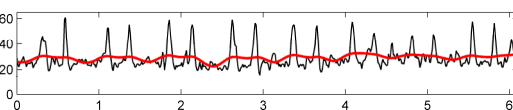
Onset Detection (Spectral Flux)

Steps:

1. Spectrogram
2. Logarithmic compression
3. Differentiation & half wave rectification
4. Accumulation
5. Normalization

Novelty function

Substraction of local average

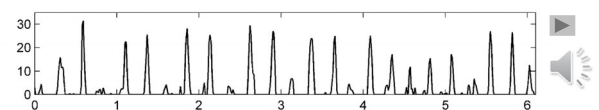


Onset Detection (Spectral Flux)

Steps:

1. Spectrogram
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4. Accumulation
5. Normalization

Normalized novelty function



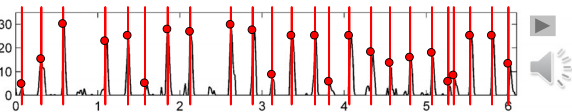
Onset Detection (Spectral Flux)

Steps:

1. Spectrogram
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5. Normalization

Normalized novelty function

Peak positions indicate beat candidates



Onset Detection (Spectral Flux)

Deep Learning

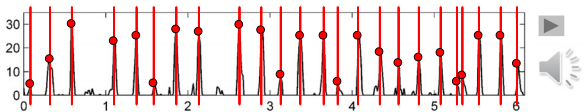
1. Input representation
2. Sigmoid activation
3. Convolution & rectified linear unit (ReLU)
4. Pooling
5. Convolution & ReLU

Steps:

1. Spectrogram
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3. Differentiation & half wave rectification
4. Accumulation
5. Normalization

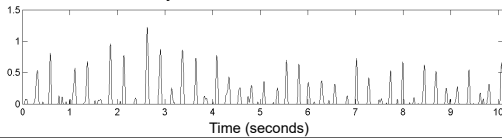
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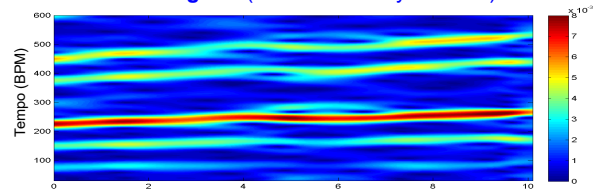
Local Pulse and Tempo Tracking

Normalized novelty function

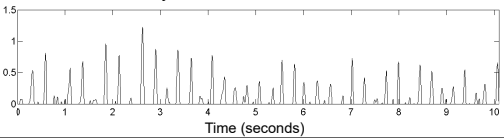


Local Pulse and Tempo Tracking

Fourier temogram (STFT of novelty function)

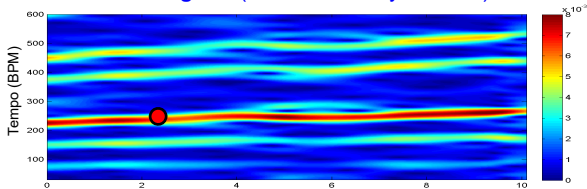


Normalized novelty function

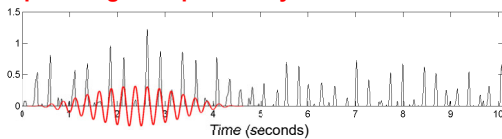


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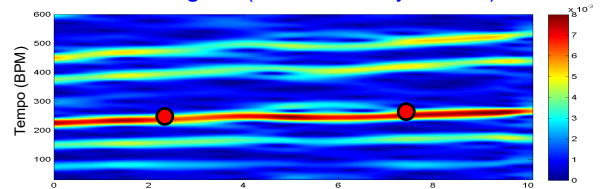


Optimizing local periodicity kernel

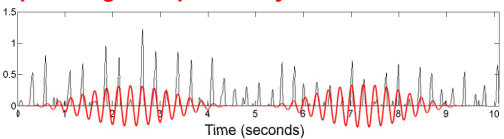


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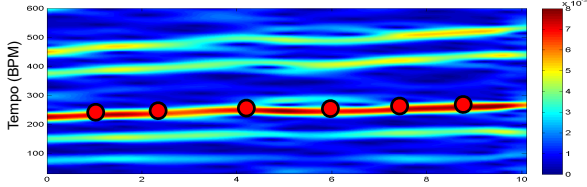


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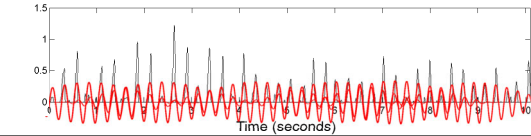


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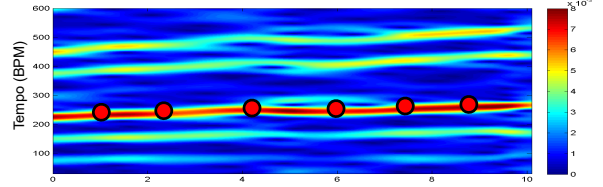


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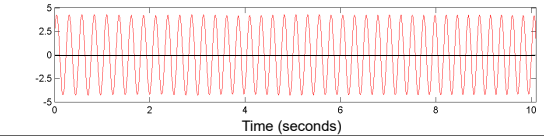


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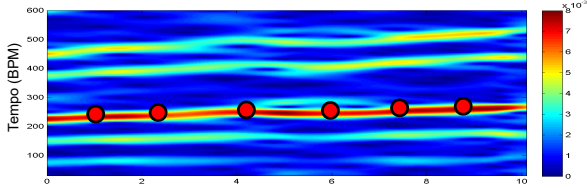


Accumulation of kernels

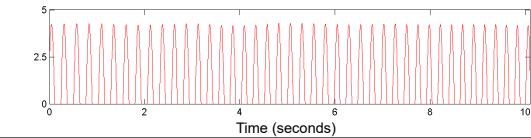


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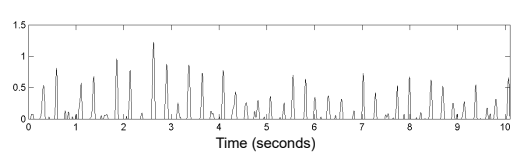


Halfwave rectification

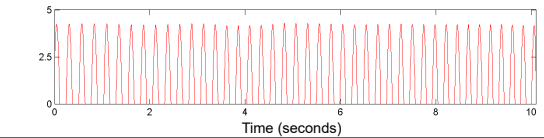


Local Pulse and Tempo Tracking

Novelty Curve



Predominant Local Pulse (PLP)



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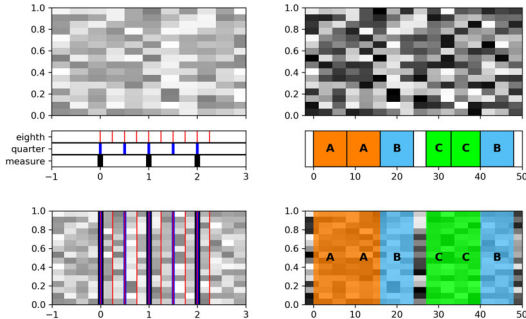
Part B: Basics

Topic	Description
Get Started	Explanation on how to install and use the FMP notebooks
Installation	Installation of Python using Conda
Jupyter Notebook	Usage of Jupyter notebook framework
Python Basics	Introduction of data types, control structures, and functions
Python Style Guide	Recommendations for programming style
Multimedia	Integration of multimedia objects into notebooks
Python Visualization	Generation of figures and images
Python Audio	Reading and writing audio files
Numba	Acceleration of Python functions via JIT compilation
Annotation Visualization	Visualization of annotations (single value, segments)
Sonification	Sonification methods (onsets, F0 trajectories, pitch, chroma)
libfmp	Library of FMP-specific Python functions
MIR Resources	Links to resources that are useful for MIR

Part B: Basics

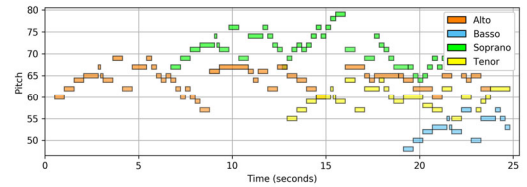
Annotation Visualization

Examples for visualizing annotations of time positions and segments.



Part 1: Music Representations

Symbolic Format: CSV

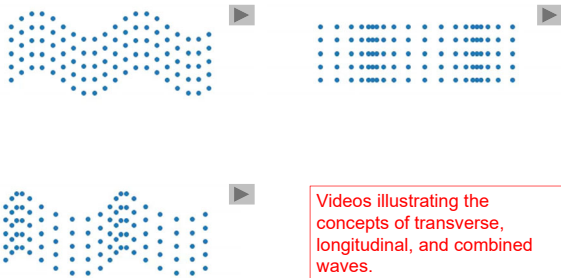


Visualization of a piano-roll representation (Fugue BWV 846 by Bach).



Part 1: Music Representations

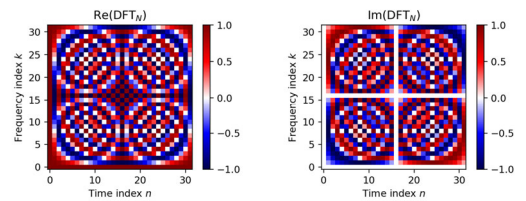
Waves and Waveforms



Videos illustrating the concepts of transverse, longitudinal, and combined waves.

Part 2: Fourier Analysis of Signals

Discrete Fourier Transform (DFT)

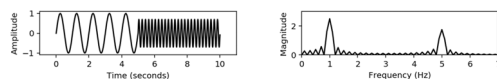


The matrix DFT_N and a visualization of its real and imaginary parts for the case $N = 32$

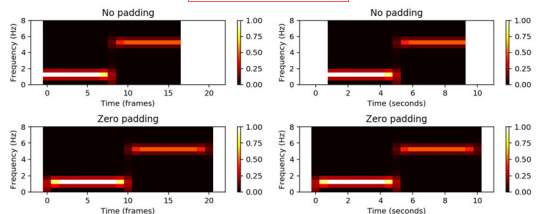
Part 2: Fourier Analysis of Signals

STFT: Padding

Time-domain signal and magnitude Fourier transform.



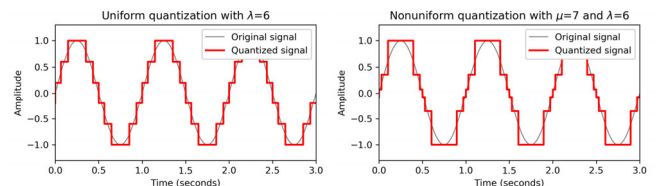
Magnitude STFT.



Part 2: Fourier Analysis of Signals

Digital Signals: Quantization

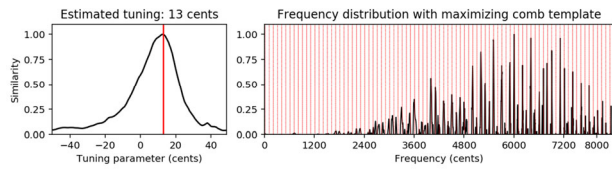
Uniform and nonuniform quantization (based on μ -law encoding) using $\lambda = 6$ quantization levels.



Part 3: Music Synchronization

Transposition and Tuning

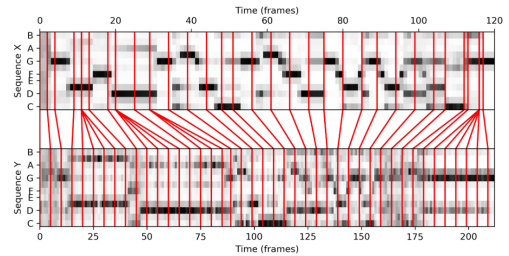
Tuning procedure using a comb-filter approach.



Part 3: Music Synchronization

Music Synchronization

Music synchronization result obtained for two input chromagrams (obtained from two recordings of the beginning of Beethoven's Fifth Symphony).

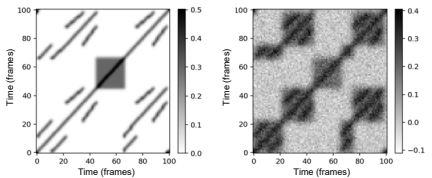
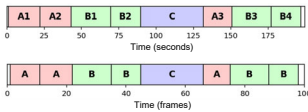


Part 4: Music Structure Analysis

SSM: Synthetic Generation

Structure annotation and different synthetically generated SSMs.

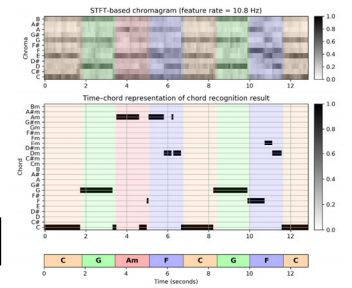
	start	end	label
0	0.00	1.01	
1	1.01	22.11	A1
2	22.11	43.06	A2
3	43.06	69.42	B1
4	69.42	89.57	B2
5	89.57	131.64	C
6	131.64	150.84	A3
7	150.84	176.96	B3
8	176.96	196.90	B4
9	196.90	199.64	



Part 5: Chord Recognition

Template-Based Chord Recognition

Chord recognition task illustrated by the first measures of the Beatles song "Let It Be."

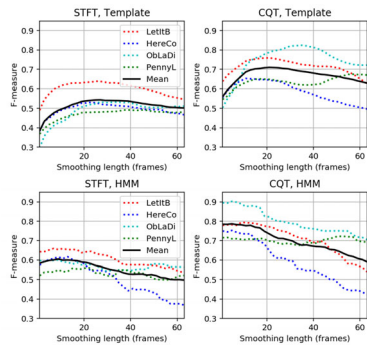


Part 5: Chord Recognition

Experiments: Beatles Collection

Prefiltering experiments for a template-based and an HMM-based chord recognizer applied to two different input chroma representations (STFT, CQT).

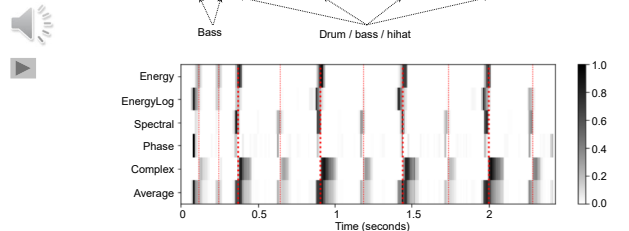
The evaluation is performed on the basis of four Beatles songs (LetItB, HereCo, ObLaDi, PennyL).



Part 6: Tempo and Beat Tracking

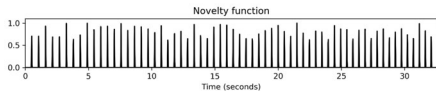
Novelty: Comparison of Approaches

Comparison of novelty detectors using a matrix-based visualization.

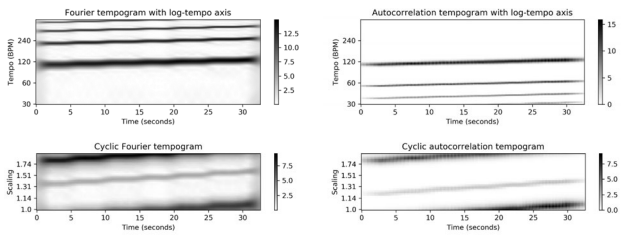


Part 6: Tempo and Beat Tracking

Cyclic Tempogram



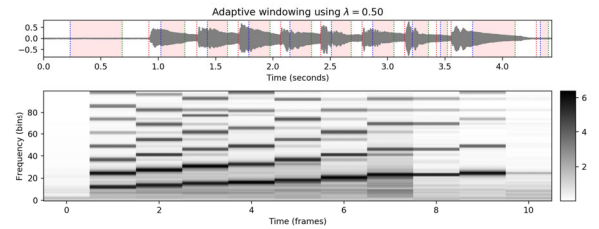
Different tempogram representations of a click track with increasing tempo.



Part 6: Tempo and Beat Tracking

Adaptive Windowing

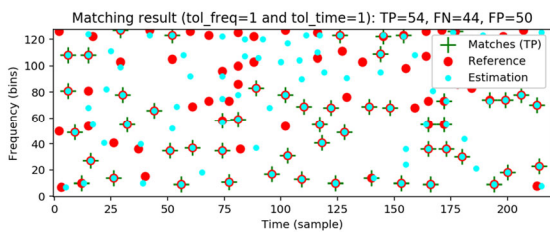
Example of adaptive windowing using a parameter λ to control the neighborhood's relative size to be excluded.



Part 7: Content-Based Audio Retrieval

Audio Identification

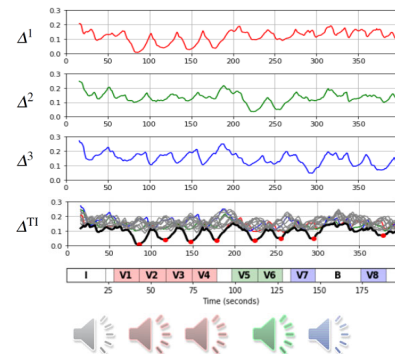
Evaluation measures that indicate the agreement between two constellation maps computed for an original version (Reference) and a noisy version (Estimation).



Part 7: Content-Based Audio Retrieval

Audio Matching

Transposition-invariant matching function illustrated by Zager and Evans' song "In the Year 2525."



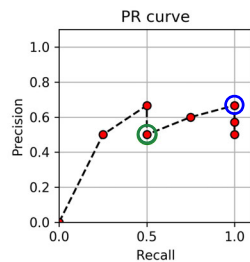
Part 7: Content-Based Audio Retrieval

Evaluation Measures

Various evaluation metrics applied to a toy example.

Rank	ID	Score	χ_0	P(r)	R(r)	F(r)
1	6	3.70	False	0.00	0.00	0.00
2	3	3.60	True	0.50	0.25	0.33
3	4	3.50	True	0.67	0.50	0.57
4	5	3.20	False	0.50	0.50	0.50
5	8	3.10	True	0.60	0.75	0.67
6	2	2.60	True	0.67	1.00	0.80
7	7	1.50	False	0.57	1.00	0.73
8	1	0.70	False	0.50	1.00	0.67

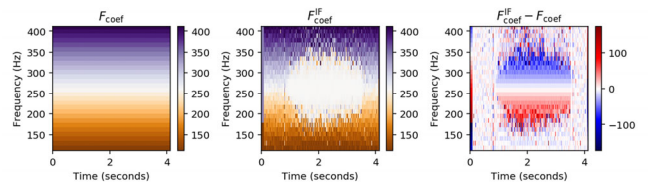
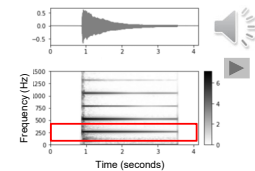
Break-even point = 0.50
F_max = 0.80
Average precision = 0.60833



Part 8: Audio Decomposition

Instantaneous Frequency Estimation

Interpretation of time-frequency bins of an STFT using (frame-dependent) instantaneous frequency values.



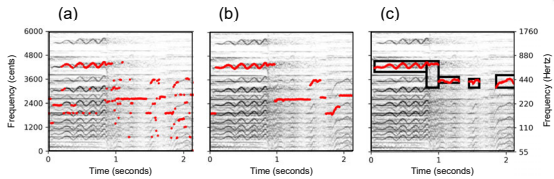
Part 8: Audio Decomposition

Fundamental Frequency Tracking

- Saliency representation with trajectories computed by
- (a) a frame-wise approach,
 - (b) an approach using continuity constraints, and
 - (c) a score-informed approach.



Figure 8.10a from [Müller, FMP, Springer 2015]

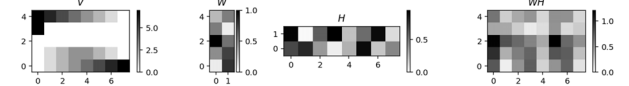


Part 8: Audio Decomposition

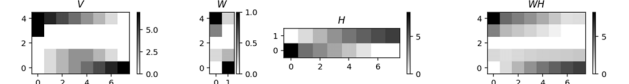
Nonnegative Matrix Factorization (NMF)

NMF procedure applied to a toy example.

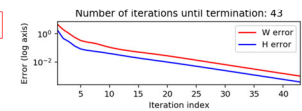
Matrix V and randomly initialized matrices W and H .



Matrix V and matrices W and H after training.



Error terms over iteration.



<https://www.audiolabs-erlangen.de/FMP>

<https://www.audiolabs-erlangen.de/FMP>

Basics + 8 Chapters

<https://www.audiolabs-erlangen.de/FMP>

Basics + 8 Chapters

Tempo and Beat Tracking

Tempo and Beat Tracking

Tempo and Beat Tracking



Definition

We assume that we are given a discrete-time novelty function $\Delta: \mathbb{Z} \rightarrow \mathbb{R}$ to indicate note onset candidates. The idea of Fourier analysis is to detect local periodicity in the novelty curve by comparing it with windowed sinusoids. A high correlation of Δ with a windowed sinusoid indicates a periodicity of the novelty curve (given a suitable phase). This correlation (along with the phase) can be computed via the short-time Fourier transform. To this end, we fix a window function $w: \mathbb{Z} \rightarrow \mathbb{R}$ of length centered at $n = 0$ (e.g., a sampled Hann window). Then, for a frequency parameter $\omega \in \mathbb{R}_{>0}$ and time parameter $m \in \mathbb{Z}$, the complex Fourier coefficient is defined by

$$F(n, \omega) := \sum_{m \in \mathbb{Z}} \Delta(m) w(m - n) \exp(-2\pi i \omega m).$$

Tempo and Beat Tracking



Definition

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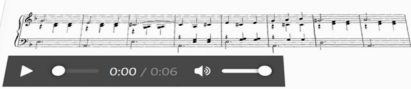
Explanations
Mathematics
Theory

Tempo and Beat Tracking



Example: Shostakovich

In the following example, we consider an excerpt of a recording of Dmitri Shostakovich's Suite for Variety Orchestra No. 1. The score version of the excerpt.



We start with a spectral-based novelty function resampled to F_n^{Δ} . Furthermore, we use a window size corresponding to 5 seconds (1

Annotations
Music Examples
Audio
Links

Tempo and Beat Tracking



```
In [2]: def compute_sinusoid_optimal(c, tempo, n, Fs, N)
        """Compute windowed sinusoid with optimal phase
        Notebook: C6/C6S2_TempogramFourier.ipynb

        Args:
        c: Coefficient of tempogram (c=X(k,n))
        tempo: Tempo parameter corresponding to
        _coef_BPM[k].
        n: Frame parameter of c
        Fs: Sampling rate
        N: Window length
        H: Hop size
```

```
In [2]: def compute_sinusoid_optimal(c, tempo, n, Fs, N)
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        H: Hop size
```

Python Code
Algorithms
Functions

Tempo and Beat Tracking



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Tempo and Beat Tracking

6

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6

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FMP Notebooks

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References

- Meinard Müller: Fundamentals of Music Processing – Using Python and Jupyter Notebooks. 2nd Edition, Springer, 2021. <https://www.springer.com/gp/book/9783030698072>
- Meinard Müller and Frank Zalkow: libfmp: A Python Package for Fundamentals of Music Processing. Journal of Open Source Software (JOSS), 6(63): 1–5, 2021. <https://joss.theoj.org/papers/10.21105/joss.03326>
- Meinard Müller: An Educational Guide Through the FMP Notebooks for Teaching and Learning Fundamentals of Music Processing. Signals, 2(2): 245–285, 2021. <https://www.mdpi.com/2624-6120/2/2/18>
- Meinard Müller and Frank Zalkow: FMP Notebooks: Educational Material for Teaching and Learning Fundamentals of Music Processing. Proc. International Society for Music Information Retrieval Conference (ISMIR): 573–580, 2019. https://zenodo.org/record/3527872#_YQHhEQqzaUk
- Meinard Müller, Brian McFee, and Katherine Kinnaird: Interactive Learning of Signal Processing Through Music: Making Fourier Analysis Concrete for Students. IEEE Signal Processing Magazine, 38(3): 73–84, 2021. <https://ieeexplore.ieee.org/document/9418542>

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Resources (Group Meinard Müller)

- FMP Notebooks: <https://www.audiolabs-erlangen.de/FMP>
- libfmp: <https://github.com/meinardmueller/libfmp>
- synctoolbox: <https://github.com/meinardmueller/synctoolbox>
- libtsm: <https://github.com/meinardmueller/libtsm>
- Preparation Course Python (PCP) Notebooks: <https://www.audiolabs-erlangen.de/resources/MIR/PCP/PCP.html>
<https://github.com/meinardmueller/PCP>

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Resources

- librosa: <https://librosa.org/>
- madmom: <https://github.com/CPJKU/madmom>
- Essentia Python tutorial: https://essentia.upf.edu/essentia_python_tutorial.html
- mirdata: <https://github.com/mir-dataset-loaders/mirdata>
- open-unmix: <https://github.com/sigsep/open-unmix-pytorch>
- Open Source Tools & Data for Music Source Separation: <https://source-separation.github.io/tutorial/landing.html>

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